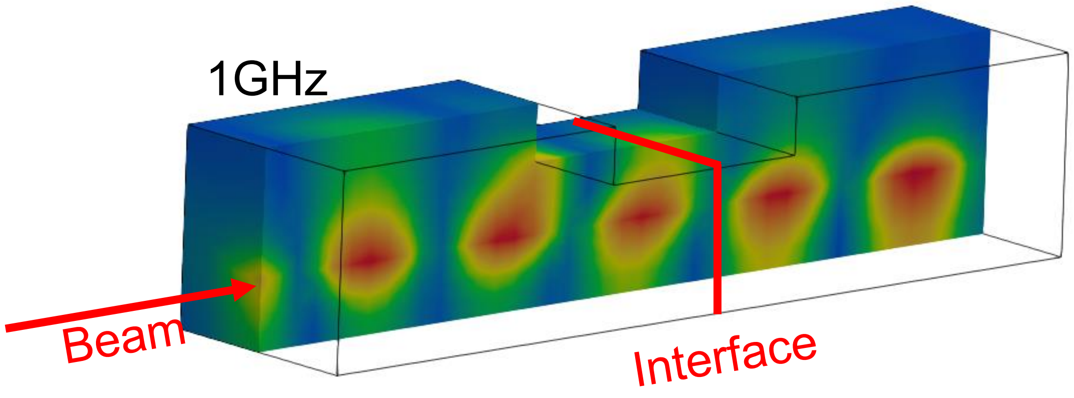
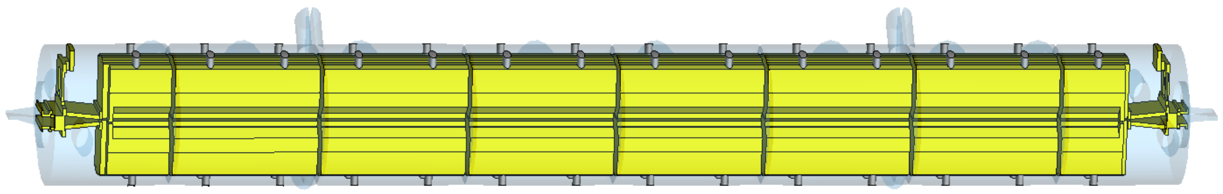
TWO-LEVEL DDM FOR FAST COMPUTATION OF CAVITY IMPEDANCES

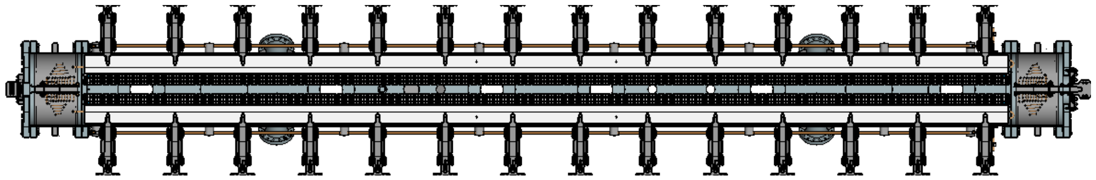
CONTENT

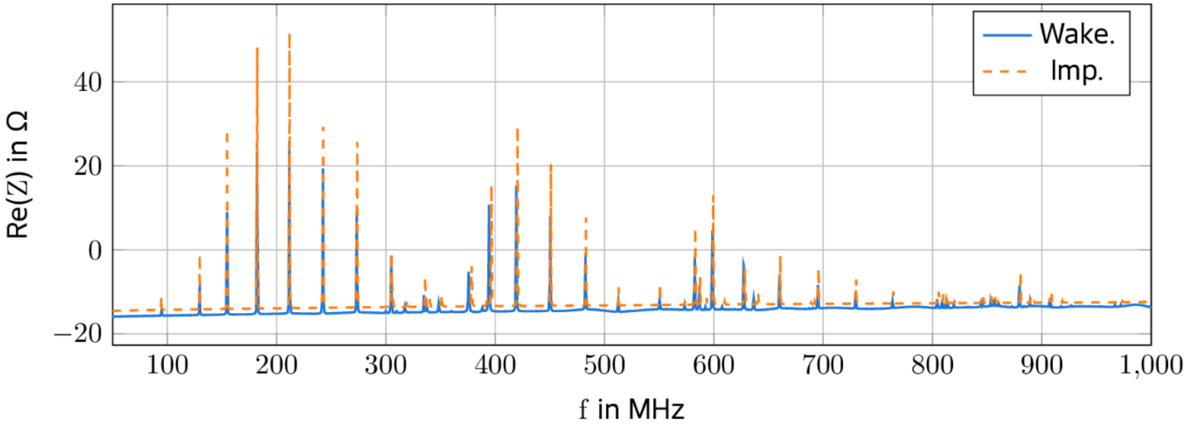
- Introduction
- Two-level Schwarz DDM
- Application
  - Waveguide transition
  - In-vacuum undulator
- Multi-grid approach





INTRODUCTION





- 1 day with FEM (CST eigenmode Solver/FELIS), 500 GB RAM
- More than 1 week with the CST wakefield solver

- HPC resources available (Lichtenberg Cluster)
- Parallelization strategy Schwarz domain decomposition
  - Problematic overhead due to large number of iterations, especially
    - In resonant problems
    - For large numbers of subdomains
- Solution: extension by a two-level approach

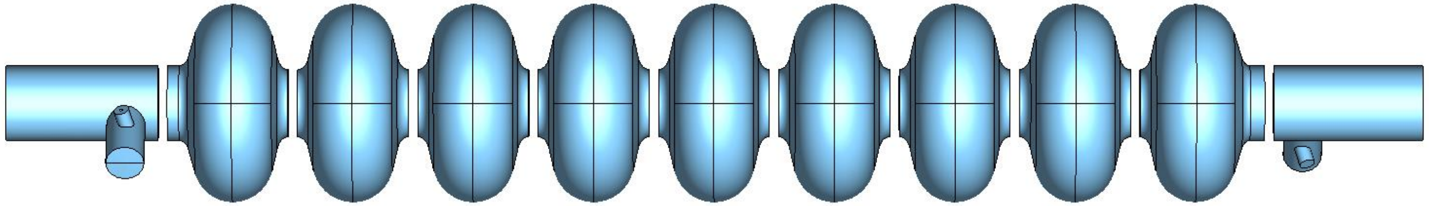
DOMAIN DECOMPOSITION
PROBLEM STATEMENT

- Weak formulation per subdomain: find $E \in H(\mathrm{curl})$ such that $\forall v \in H(\mathrm{curl})$:

$$\int_{\Omega_i} \mu^{-1} \nabla \times E \cdot \nabla \times v \,\mathrm{d}V - \omega^2 \int_{\Omega_i} \varepsilon\, E \cdot v \,\mathrm{d}V =$$

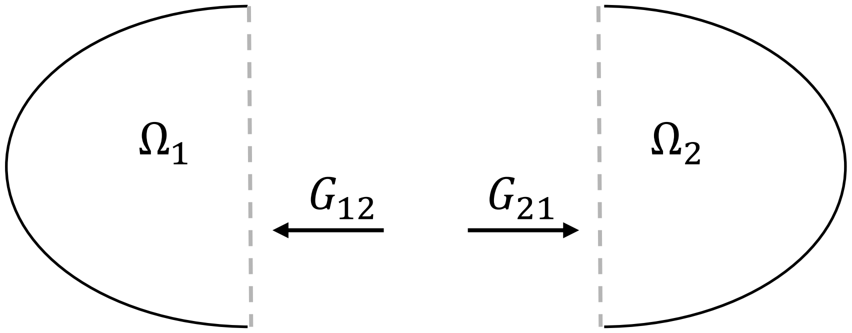
$$\underbrace{-j\omega \int_{\Omega_\mathrm{i}} J \cdot v \,\mathrm{d}V}_{\text{Current excitation}} - \underbrace{\int_{\Sigma_\mathrm{SI}} \mu^{-1}\gamma^\mathrm{t}(\nabla \times E) \cdot v \,\mathrm{d}S}_{\text{Resistive wall}} - \underbrace{\int_{\Sigma_\mathrm{WG}} \mu^{-1}\gamma^\mathrm{t}(\nabla \times E) \cdot v \,\mathrm{d}S}_{\text{Waveguides}} - \underbrace{\int_{\Sigma_\mathrm{DDM}} \mu^{-1}\gamma^\mathrm{t}(\nabla \times E) \cdot v \,\mathrm{d}S}_{\text{DDM interface}}$$

- Resistive wall BC: $\mu^{-1}\gamma^\mathrm{t}(\nabla \times E) = \mathrm{j}\omega Y(\omega)\gamma^\mathrm{T}(E)$
- Waveguide BC: $\mu^{-1}\gamma^\mathrm{t}(\nabla \times E) = \mu^{-1}\gamma^\mathrm{t}(\nabla \times E^\mathrm{Inc}) - j\omega \sum_{m=1}^{\infty} Y_m^\mathrm{TX}(\omega) a_m (E - E^\mathrm{Inc})\, e_{m,\perp}^\mathrm{TX})$



DOMAIN DECOMPOSITION
DOMAIN COUPLING

- Domain coupling: transmission condition from domain 2 to domain 1
  - $j\mathbf{k}\gamma_1^{\mathrm{T}}(E_1) + \gamma_1^{\mathrm{t}}(\nabla \times E_1) = j\mathbf{k}\gamma_2^{\mathrm{T}}(E_2) - \gamma_1^{\mathrm{t}}(\nabla \times E_2) \coloneqq G_{12}$
- Boundary operators
  - OTC0: $\mathbf{k}\gamma^{\mathrm{T}}(E) \coloneqq k_0\gamma^{\mathrm{T}}(E)$
  - OTC0c: $\mathbf{k}\gamma^{\mathrm{T}}(E) \coloneqq \frac{1}{p}(k_0^2\mathbf{I} - \nabla_\perp \times \nabla_\perp \times)\gamma^{\mathrm{T}}(E)$
  - MTC: $\mathbf{k}\gamma^{\mathrm{T}}(E) \coloneqq \sum_{m=1}^{\infty} k_m^{\mathrm{TX}}(\omega)a_m^{\mathrm{TX}}(E)e_{\perp,m}^{\mathrm{TX}}$



DOMAIN DECOMPOSITION
CONDENSED MATRIX FORMULATION

- $a_1(E_1, v) = \int_{\Omega_1} \nabla \times E_1 \cdot \nabla \times v \, \mathrm{d}V - k^2 \int_{\Omega_1} E_1 \cdot v \, \mathrm{d}V - \int_{\Sigma_{\mathrm{DDM}}} j\mathbf{k}\gamma_1^{\mathrm{T}}(E_1) \cdot v \, \mathrm{d}S - \mathrm{BCs}$
  - $\rightarrow A_1$
- Reformulated coupling condition
  - $b_1(E_2, v) = \int_{\Sigma} j2\mathbf{k}\gamma_2^{\mathrm{T}}(E_2) \cdot v \mathrm{d}V$
    - $\rightarrow B_1$
- Yields DD preconditioned matrix form (for two domain decomposition)
  - $\begin{bmatrix} I & I - B_1 A_1^{-1} \\ I - B_2 A_2^{-1} & I \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} B_1 A_1^{-1} l_{J,1} \\ B_2 A_2^{-1} l_{J,2} \end{bmatrix}$
  - $Mg = l_g$
- $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} A_1^{-1}(g_1 + l_{J,1}) \\ A_2^{-1}(g_2 + l_{J,2}) \end{bmatrix}$

DOMAIN DECOMPOSITION
ONE-LEVEL SCHWARZ METHOD

- Fixed point iteration
- Start
  - g_k
- Compute residual
  - $r_k = b - M g_k$
- Update
  - $g_{k+1} = g_k + r_k$
- Problems
  - Number of DDM iterations increases with number of subdomains
  - No use of knowledge obtained in previous solves (at other frequencies)

DOMAIN DECOMPOSITION
TWO-LEVEL SCHWARZ METHOD

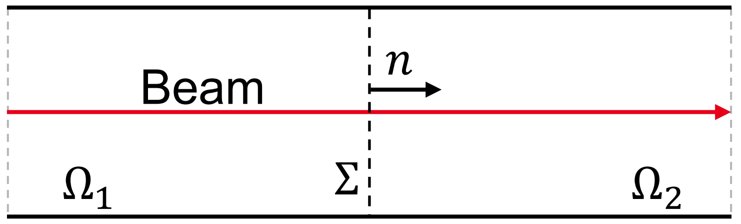
- Idea
  - Use coarse problem to reduce the error cheaply
- Start
  - g_k
- Compute residual
  - $r_k = b - M g_k$
- Correction with coarse model
  - (analytical approximation [Lee, et al.], coarser discretization [Gander, et al.], ROM [Floch, et al.],…)
  - $y_k = C r_k$
- Update
  - $g_{k+1} = g_k + r_k + y_k$

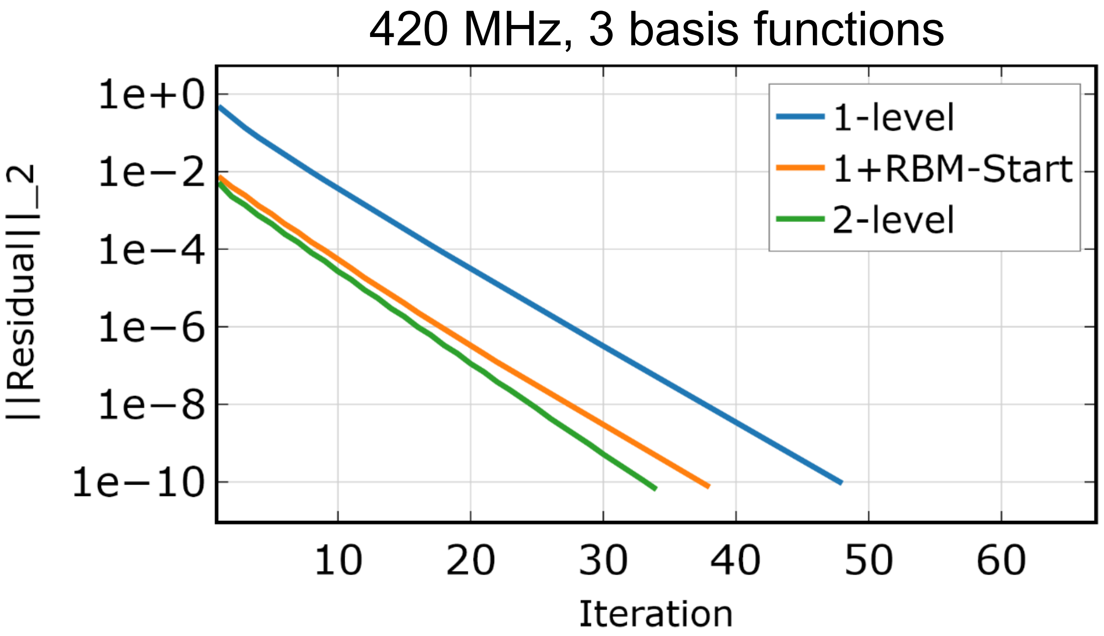
DOMAIN DECOMPOSITION
TWO-LEVEL CONDENSED RBM-DDM

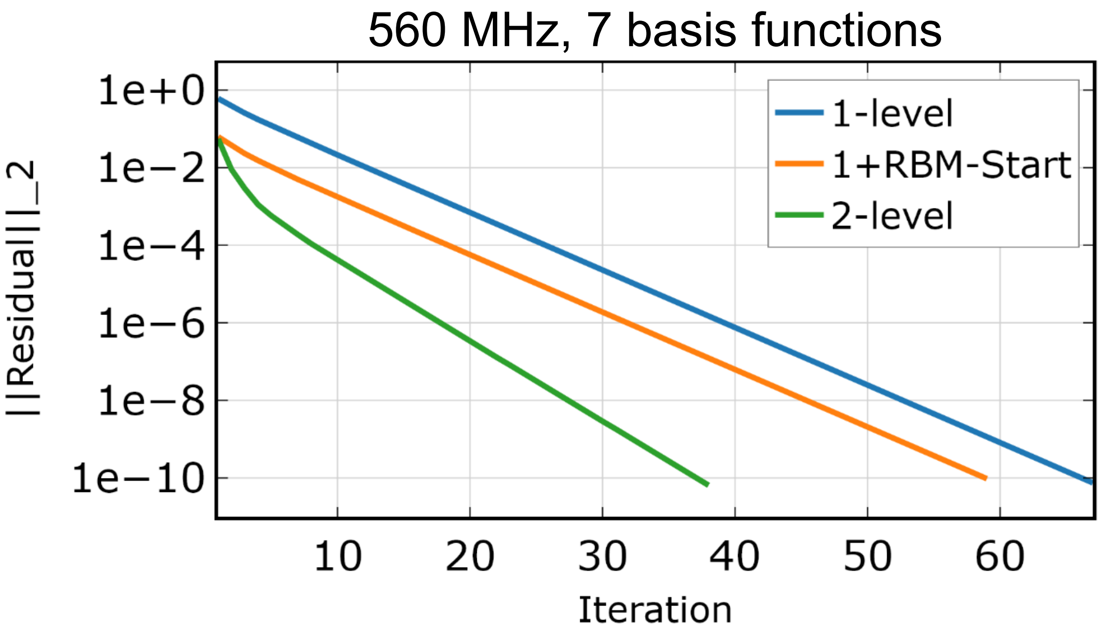
- Start
  - g_k, $G = [g(f_1), g(f_2), \dots]$
- Compute residual
  - $r_k = b - Mg_k$
- Correction with flux-ROM $\widehat{M} = (MG)^H MG$
  - $\Delta g_k = G\widehat{M}^{-1}(MG)^H r_k + (I - MG\widehat{M}^{-1}(MG)^H) r_k$
  - Coarse ROM correction: $G\widehat{M}^{-1}(MG)^H r_k$
  - Project MG component to 0: $(I - MG\widehat{M}^{-1}(MG)^H) r_k$
- Update
  - $g_{k+1} = g_k + \Delta g_k$
- No extra cost besides computing MG

VALIDATION

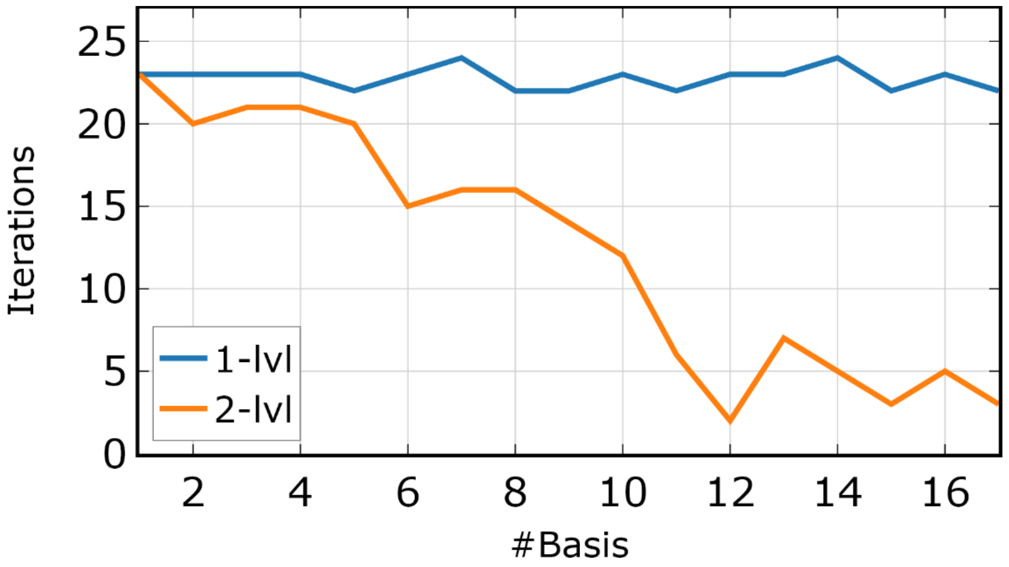
- Homogeneous waveguide with surface impedance
- OTC0c as TC, FPI
- RBM for 400 to 600 MHz

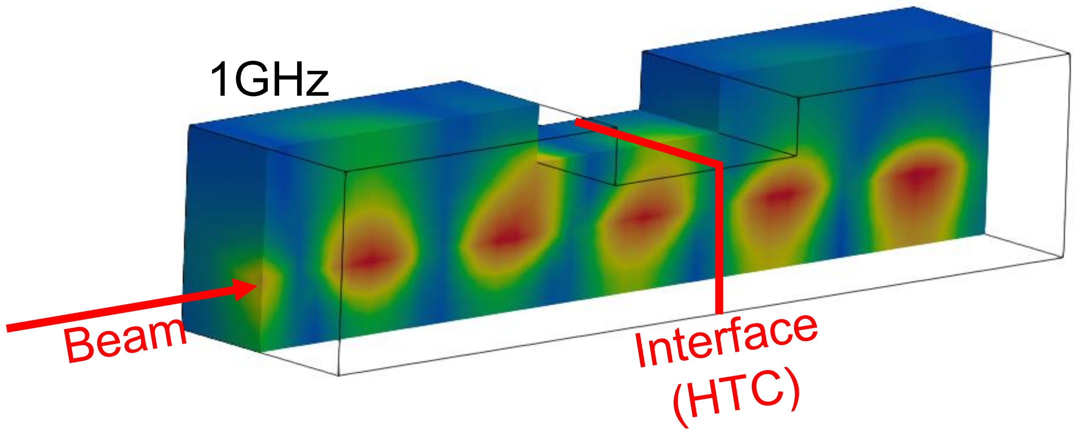


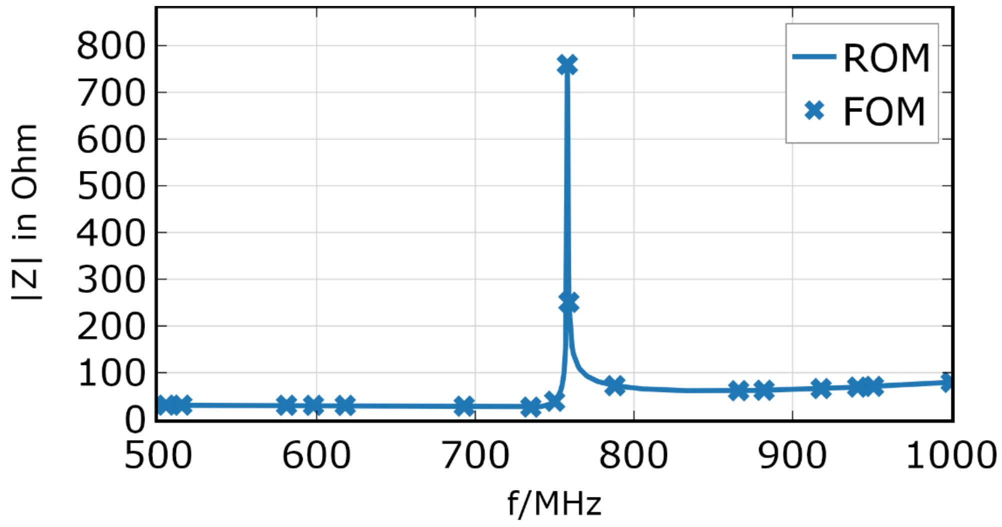




WAVEGUIDE TRANSITION



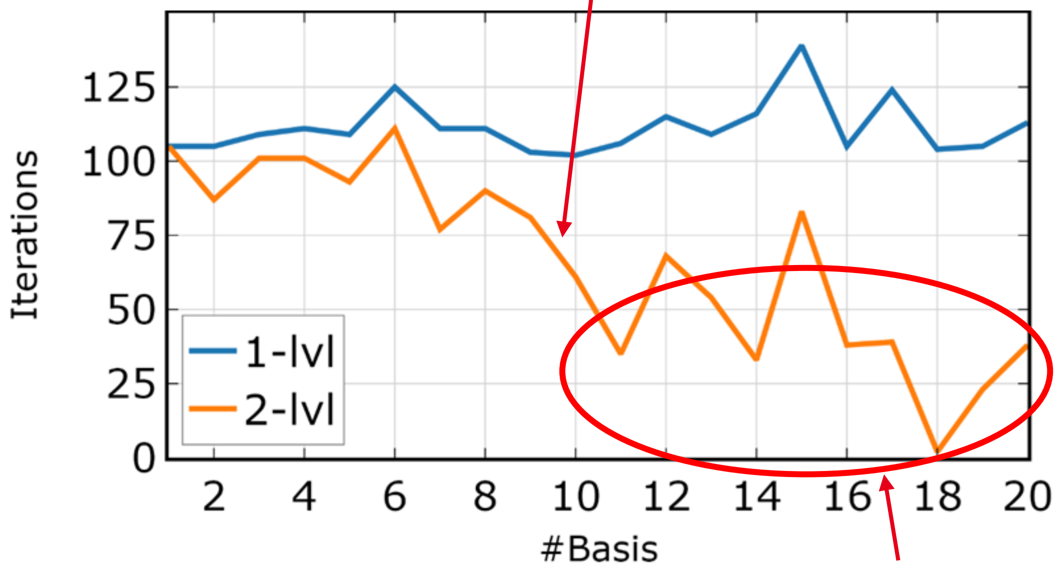




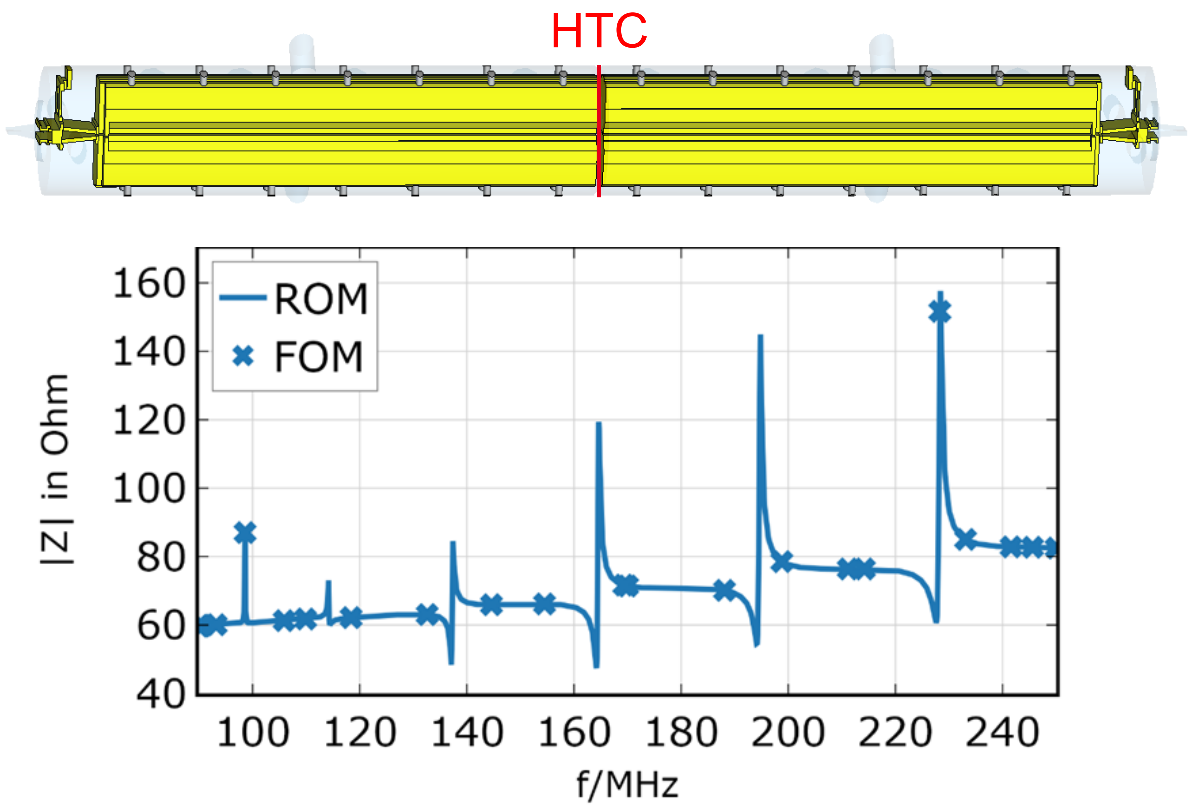
- Number of iterations decrease as ROM converges
- Very few iterations required for last 7 basis functions
- In total 44% less iterations

IN VACUUM UNDULATOR
2-DOMAIN DECOMPOSITION

Improved convergence rate after ≈1 basis function per resonance



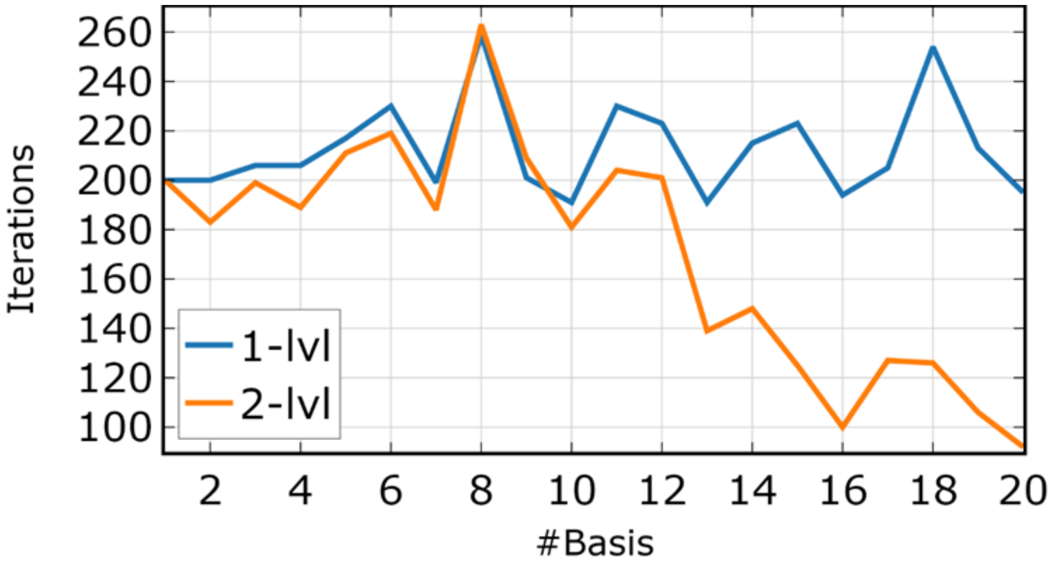
Only few iterations for later basis functions

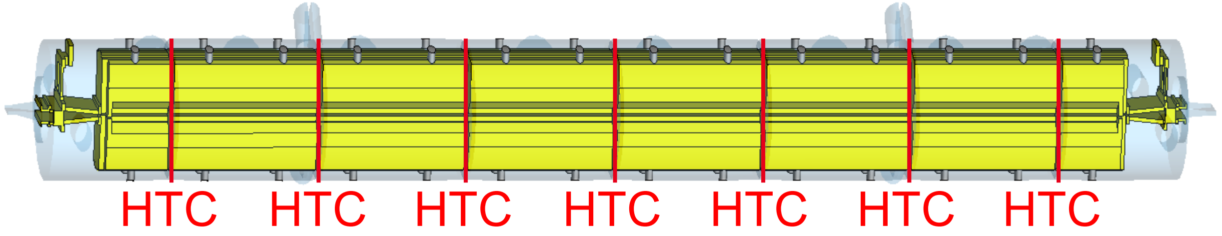


In total 40% less iterations

IN VACUUM UNDULATOR
8-DOMAIN DECOMPOSITION

- Significant improvement in required number of iterations for later basis functions
- In total 20% less iterations
- Improvement decreases for larger numbers of subdomains





→ New formulation of flux-ROM for more domains
  - Bases for each side of each interface
  - Improves overall convergence

→ Reduction of iterations for early basis functions required
  - Multi-grid approach

MULTI GRID

- Idea: Initialize ROM with basis functions from solution with lower polynomial degree

Classical multi-grid mapping

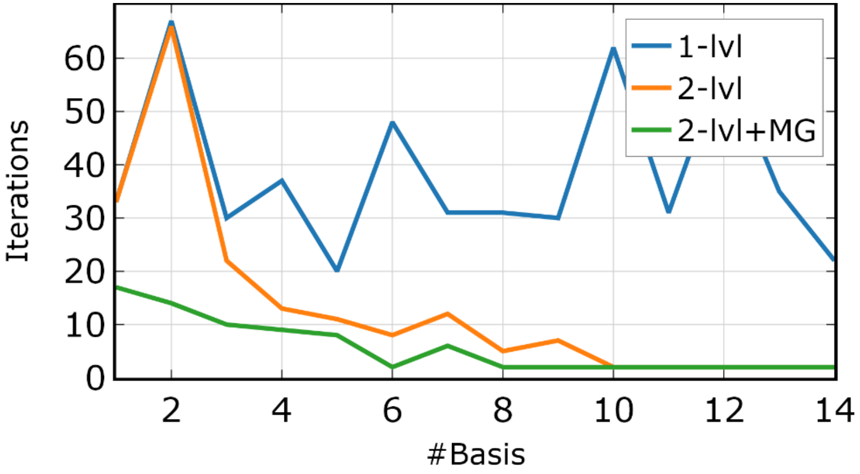
- $C^{p,p+1}: V^p \to V^{p+1}$
- $e^{p+1} = C^{p,p+1} e^p$
- Leads to insufficient accuracy (without smoothing step)
- Observation: S-Parameters converge much faster than FEM coefficients

Mode-MG

- Solve at order p
- Compute modal coefficients for interface fluxes
  - $\alpha_{\mathrm{g},m}^p = e_m^p \cdot g^p$
- Compute interface fluxes for fine problem
  - $g^{p+1} = M^{2\mathrm{D}} \sum_{m=1}^{M} \alpha_{\mathrm{g},m}^p e_m^{p+1}$
- Currently only for non-degenerate modes

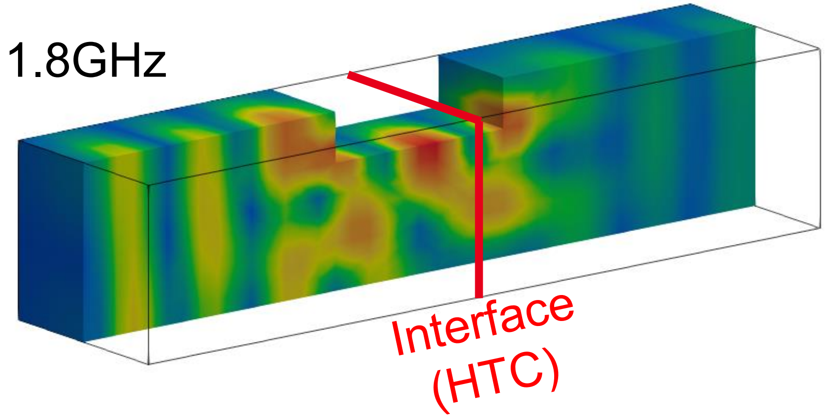
MODE MULTI GRID

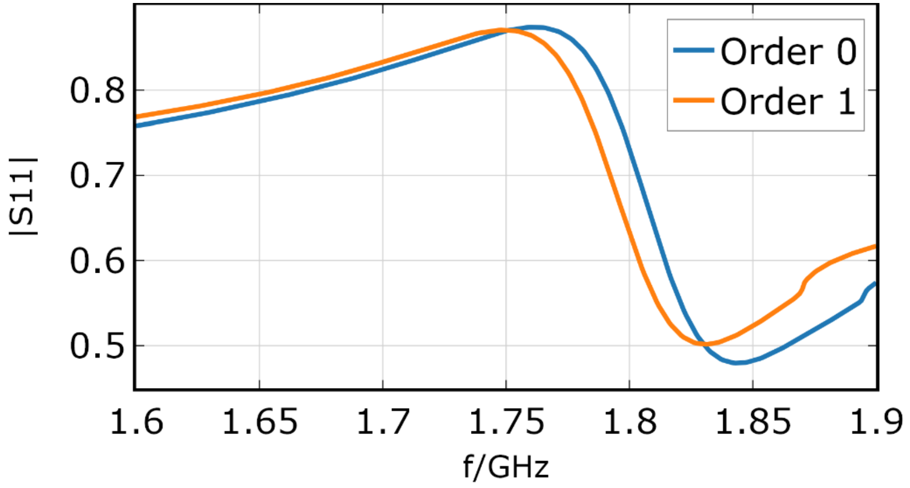
- Excitation: TE1; FPI
- Coarse: 0^{th} order; fine: 1^{st} order



- Reduction of iterations (for fine problem)

	1-lvl	2-lvl
2-lvl	67%	-
2-lvl+MG	87%	62%





SUMMARY

- Progress presented on improving FELIS for large scale impedance computations
- Extended existing one-level Schwarz DDM to two-level implementation
  - ROM as coarse space [Floch 2017]
  - New: ROM for the interface fluxes instead of electric field
    - Small and accurately known computational overhead
    - Expected to have superior convergence properties
  - Significantly improves convergence rates for two-domain decomposition
  - Further improvements required for larger decompositions
  - Still slow convergence for first RBM snapshots
- Multi-grid approach
  - Precompute approximate RBM basis functions with coarser discretization
  - Significantly reduces required iterations in preliminary test