

NUMERICAL QUALITY FACTOR STATISTICS OF COATED SRF CAVITY

DESY-TEMF December 2025

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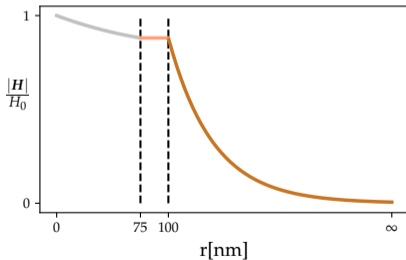
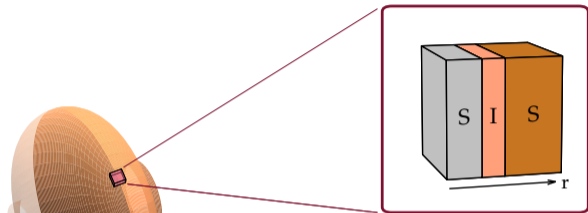
CONTENTS

- 1** Motivation & Outline
- 2** Surface Impedance
- 3** Gaussian Field Coating

- 4** Eigenvalue Problem
- 5** Results
- 6** Conclusion & Outlook

MOTIVATION & OUTLINE

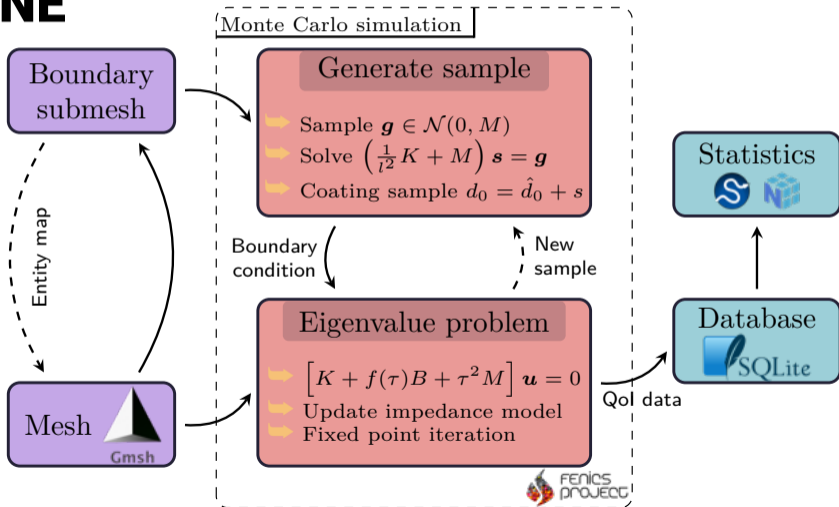
Section 1



Observation: SRF benefits happen in thin layer

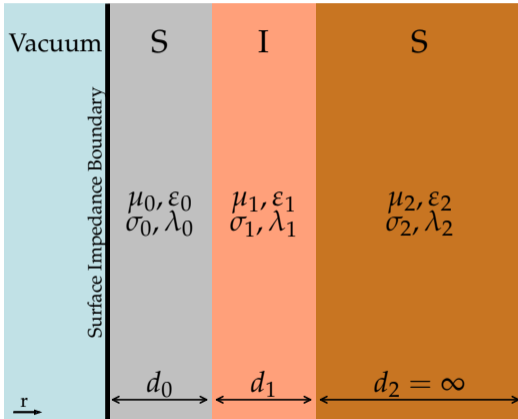
- ➔ Shield substrate from accelerating fields.
- ➔ Substrate with higher H_c
- ➔ Substrate with better thermal properties.

OUTLINE



SURFACE IMPEDANCE

Section 2



Insulator: $\alpha^2 = \mu\epsilon\omega^2$

Normal Conductor: $\alpha^2 = \mu\epsilon\omega^2 - j\mu\sigma\omega$

Superconductor: $\alpha^2 = \mu\epsilon\omega^2 - j\mu\sigma\omega - \frac{1}{\lambda^2}$

Fields in each layer:

$$E_x^{(k)}(r) = C_{2k}e^{j\alpha_k r} + C_{2k+1}e^{-j\alpha_k r}$$

$$H_y^{(k)}(r) = -\frac{\alpha_k}{\mu_k\omega} [C_{2k}e^{j\alpha_k r} - C_{2k+1}e^{-j\alpha_k r}]$$

Leontovich boundary condition:

$$\mathbf{n} \times \mathbf{E} = Z(\omega)(\mathbf{n} \times (\mathbf{n} \times \mathbf{H}))$$

GAUSSIAN FIELD COATING

Section 3

Zero-mean Gaussian random field can be obtained by solving stochastic PDE


Let $s : \mathbb{R}^d \rightarrow \mathbb{R}$ be a Gaussian random field, i.e. $s \sim \text{GP}(0, c_s(\mathbf{x}, \mathbf{y}))$, with Matérn kernel

$$c_s(\mathbf{x}, \mathbf{y}) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)} \left(\frac{\sqrt{2\nu}}{l} \|\mathbf{x} - \mathbf{y}\| \right)^\nu K_\nu \left(\frac{\sqrt{2\nu}}{l} \|\mathbf{x} - \mathbf{y}\| \right)$$

then $s(\mathbf{x})$ is the solution of the stochastic PDE

$$(\kappa^2 - \nabla^2)^\beta s(\mathbf{x}) = \frac{1}{\tau} g(\mathbf{x})$$

where $g(\mathbf{x}) \sim \text{GP}(0, \delta(\mathbf{x} - \mathbf{y}))$ and $\kappa(\nu, l)$, $\beta(\nu, d)$, $\tau(\sigma, \nu, d, l)$.

 Koh K. et al., Stochastic PDE representation of random fields for large-scale Gaussian process regression and statistical finite element analysis, *Comput. Methods Appl. Mech. Engrg.* (2023)

Zero-mean Gaussian random field can be obtained by solving stochastic PDE


For $d = 3$ and $\nu = 1/2$ this simplifies to

$$c_s(\mathbf{x}, \mathbf{y}) = \sigma^2 \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|}{l}\right), \quad \left(\frac{1}{l^2} - \nabla^2\right) s(\mathbf{x}) = \sqrt{\frac{8\pi\sigma^2}{l}} g(\mathbf{x})$$

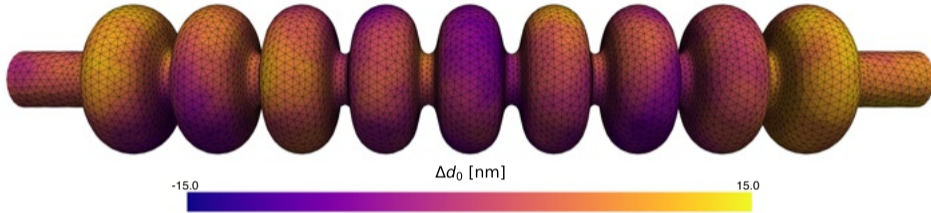
Using a finite element approximation we obtain a discrete linear system

$$s(\mathbf{x}) \approx \sum_{i=1}^N s_i \phi_i(\mathbf{x}), \quad g(\mathbf{x}) \approx \sum_{i=1}^N g_i \phi_i(\mathbf{x}) \quad \Rightarrow \quad \left(\frac{1}{l^2} M + K\right) \mathbf{s} = \mathbf{g}$$

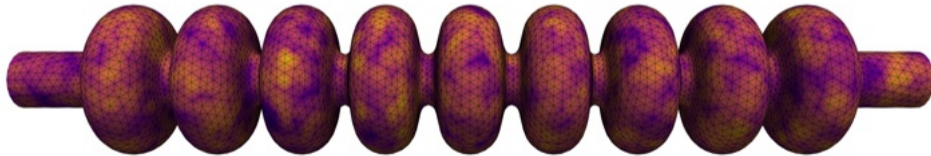
where M is the mass matrix, K the stiffness matrix and $\mathbf{g} \sim \mathcal{N}(\mathbf{0}, M)$

 Koh K. *et al.*, Stochastic PDE representation of random fields for large-scale Gaussian process regression and statistical finite element analysis, *Comput. Methods Appl. Mech. Engrg.* (2023)

Correlation length = 0.1 m



Correlation length = 0.01 m



Note: TESLA geometry from Wanzenberg R.

Wanzenberg, R., Monopole, Dipole and Quadrupole Passbands of the TESLA-cell Cavity, *DESY* (2001)

EIGENVALUE PROBLEM

Section 4

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= \left(\frac{\omega}{c}\right)^2 \mathbf{E} && \text{on } \Omega, \\ \mathbf{n} \times \mathbf{E} &= Z(\omega) [\mathbf{n} \times (\mathbf{n} \times \mathbf{H})] && \text{on } \partial\Omega \end{aligned}$$

yields weak formulation

$$\frac{1}{\kappa^2} \int_{\Omega} dV [(\nabla \times \mathbf{u}) \cdot (\nabla \times \mathbf{v})] + \frac{\tau}{\kappa} \frac{Z_0}{Z(\kappa, \tau)} \int_{\partial\Omega} dS [(\mathbf{n} \times \mathbf{u}) \cdot (\mathbf{n} \times \mathbf{v})] + \tau^2 \int_{\Omega} dV [\mathbf{u} \cdot \mathbf{v}] = 0$$

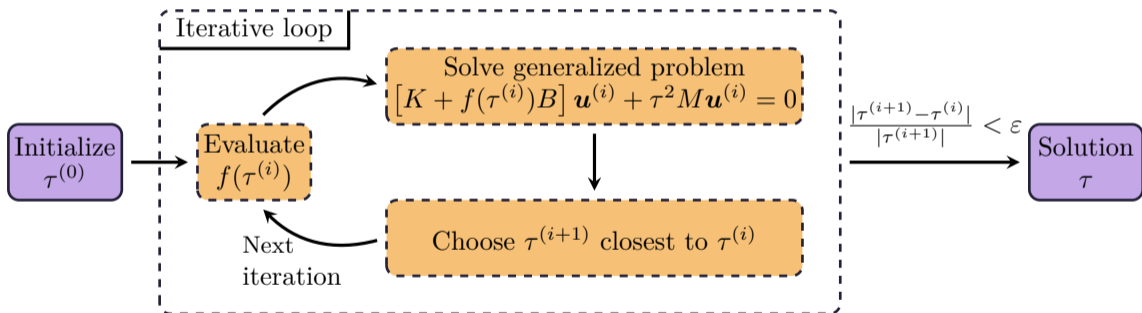
where

$$\kappa = \frac{2\pi f}{c} \text{ and } \tau = -\frac{1}{2Q} + j \Rightarrow \omega = 2\pi f \left(1 + \frac{j}{2Q}\right) = -j\kappa\tau$$


and hence non-linear complex eigenvalue problem of the form

$$[K + f(\tau)B + \tau^2 M] \mathbf{u} = 0$$

Solve non-linear eigenvalue problem with fixed point iteration

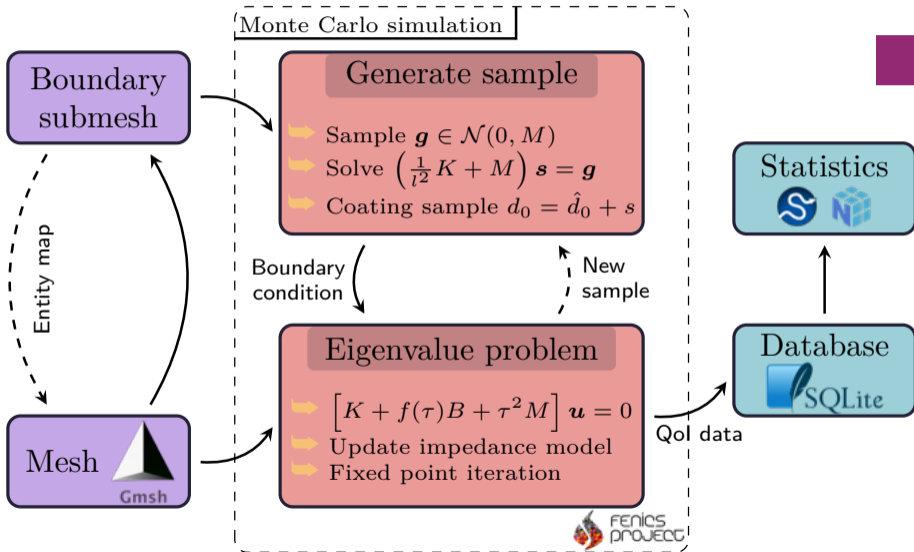


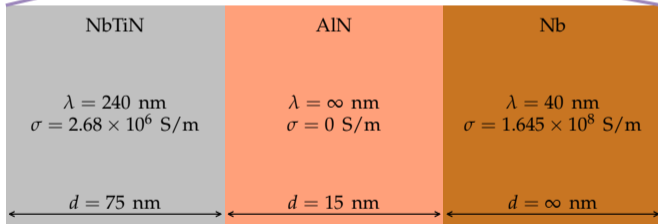
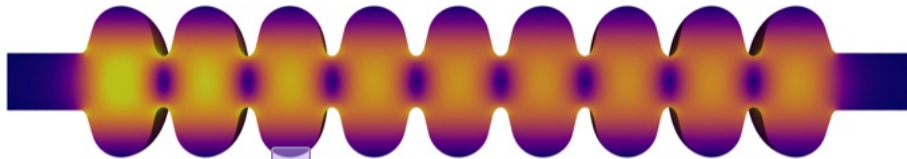
Note: use $\mathbf{u}^{(i)}$ for the initial search space of iteration $i + 1$

 Hernandez V. et al., SLEPc: A scalable and flexible toolkit for the solution of eigenvalue problems, *ACM Trans. Math. Software* (2005)

RESULTS

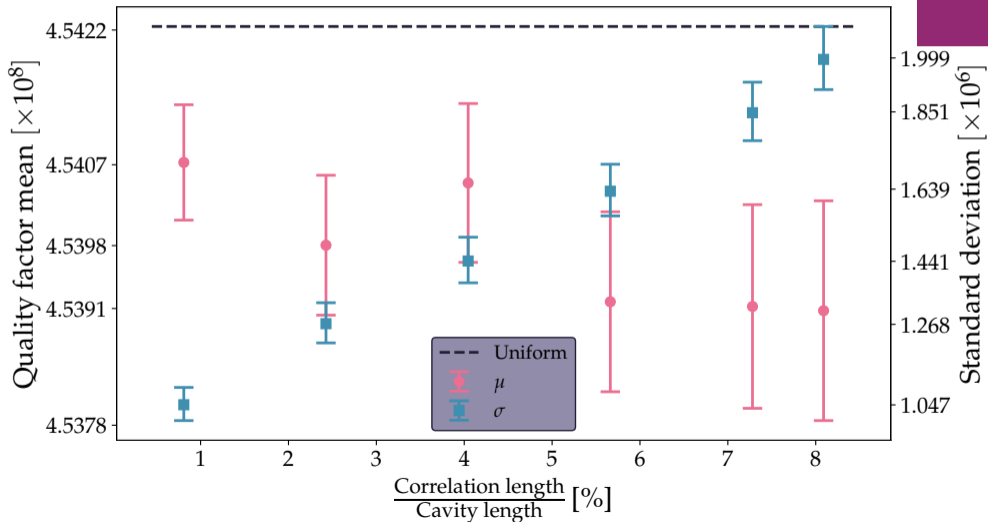
Section 5





$$Z = 596.932 \text{ n}\Omega + 1.225j \text{ m}\Omega$$

- ➔ $f = 1.3 \text{ GHz}$
- ➔ $T = 2 \text{ K}$
- ➔ Operating π -mode
- ➔ 83211 tetrahedra
- ➔ 574740 DOFs
- ➔ 6 correlation lengths l
- ➔ 1024 samples per l



Levene's test

H_0 : The variances of two populations are equal

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	5×10^{-8}				
0.05	2×10^{-20}	8×10^{-5}			
0.07	3×10^{-37}	1×10^{-14}	1×10^{-4}		
0.09	5×10^{-53}	8×10^{-27}	9×10^{-13}	5×10^{-4}	
0.1	2×10^{-62}	4×10^{-35}	2×10^{-19}	5×10^{-8}	0.040

Table: p-values for Levene's test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0 .

Welch's t-test (unequal variance t-test)

H_0 : The means of two populations are equal, the variance may be unequal

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	0.073				
0.05	0.682	0.248			
0.07	0.011	0.330	0.052		
0.09	0.016	0.331	0.061	0.946	
0.1	0.019	0.324	0.065	0.956	0.806

Table: p-values Welch's t-test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0 .

Two-sample Kolmogorov-Smirnov test

H_0 : Two samples are drawn from the same continuous distribution

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	7×10^{-3}				
0.05	1×10^{-3}	4×10^{-2}			
0.07	7×10^{-9}	3×10^{-3}	9×10^{-3}		
0.09	1×10^{-11}	3×10^{-5}	4×10^{-4}	0.588	
0.1	5×10^{-13}	9×10^{-7}	5×10^{-5}	0.116	0.806

Table: p-values for two-sample Kolmogorov-Smirnov test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0 .

CONCLUSION & OUTLOOK

Section 6

Conclusion

- ➔ Impact on Q-factor due to inhomogeneity is minor
- ➔ Differences between different correlation lengths is numerically visible. Sample means are mostly similar, sample variances are statistically different
- ➔ Homogeneous quality factor is higher than all inhomogeneous samples

Outlook

- ➔ Repeat with different materials
- ➔ Repeat with tuned cavity and consider field flatness
- ➔ Assume thicker coating in areas of high induced current to emulate "worst case"
- ➔ Use more realistic coating distribution instead of Gaussian field