

Simulation of CSR Wakefields using a Scattered Field Formulation

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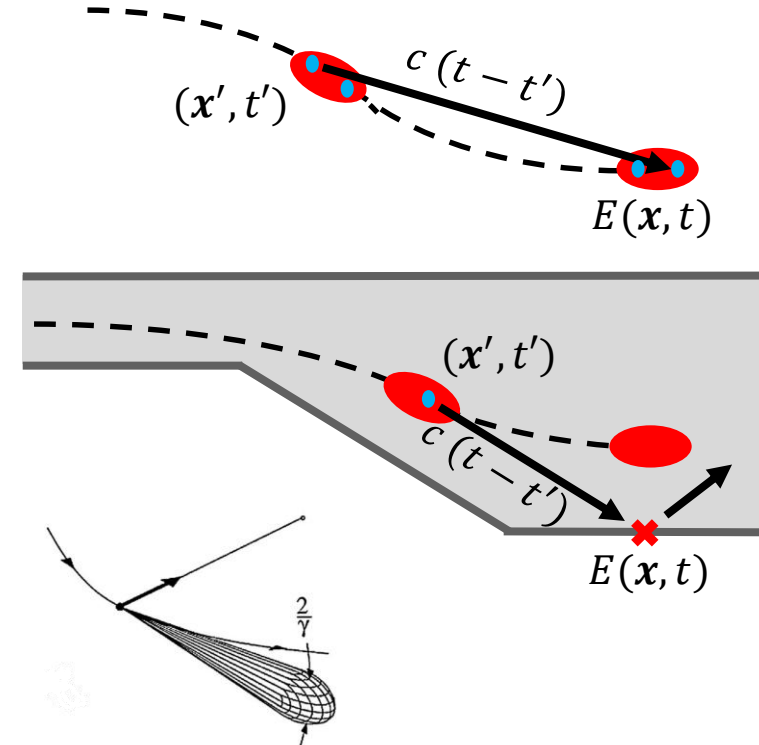
DESY-TEMF Collaboration Meeting
December 11 2025, Hamburg, Germany

Outline

- Motivation and background
- Scattered Field Formulation
 - Wakefield simulations
 - Continuous charge approach
- Application: BC0 @DESY

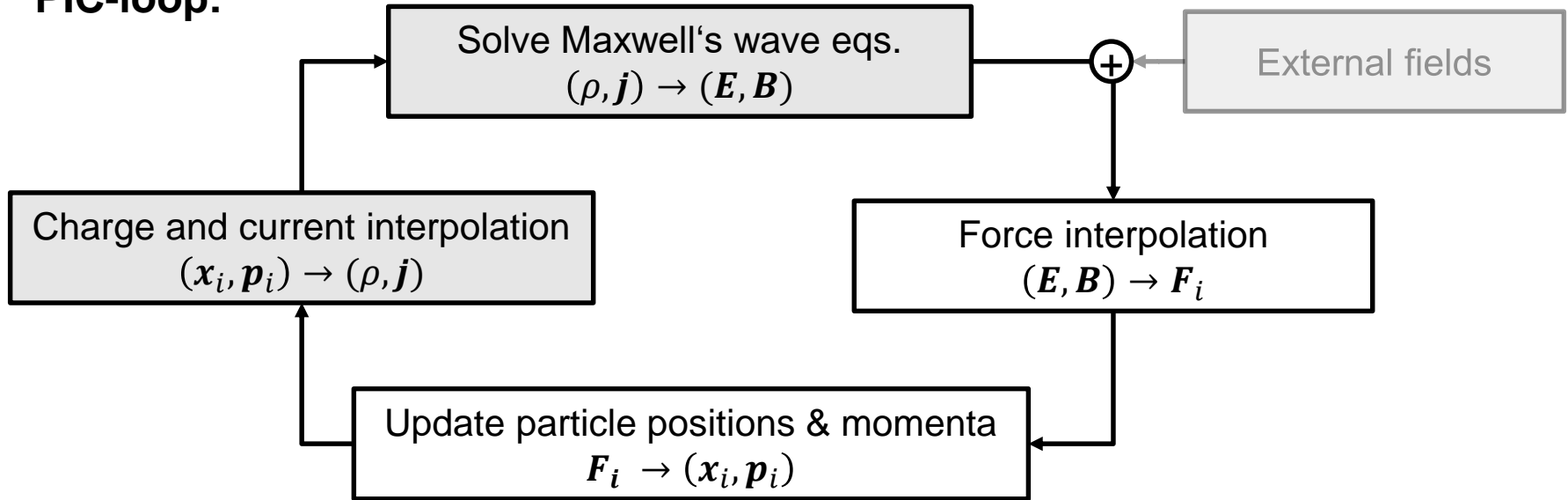
Motivation

- Curved particle trajectories: Coherent Synchrotron Radiation (CSR) effects
- (Semi-) analytical models for simple setups (e.g. free-space, rigid bunch, fixed trajectory)
 - Semianalytical approach with continuous charge (M. Dohlus, 2003)
- Expensive simulations: Radiation cone angle γ^{-1} , frequencies $\sim \gamma^3$
- Our work: CSR wakes from arbitrary chambers



Background: Electromagnetic Particle-in-Cell

- **PIC-loop:**

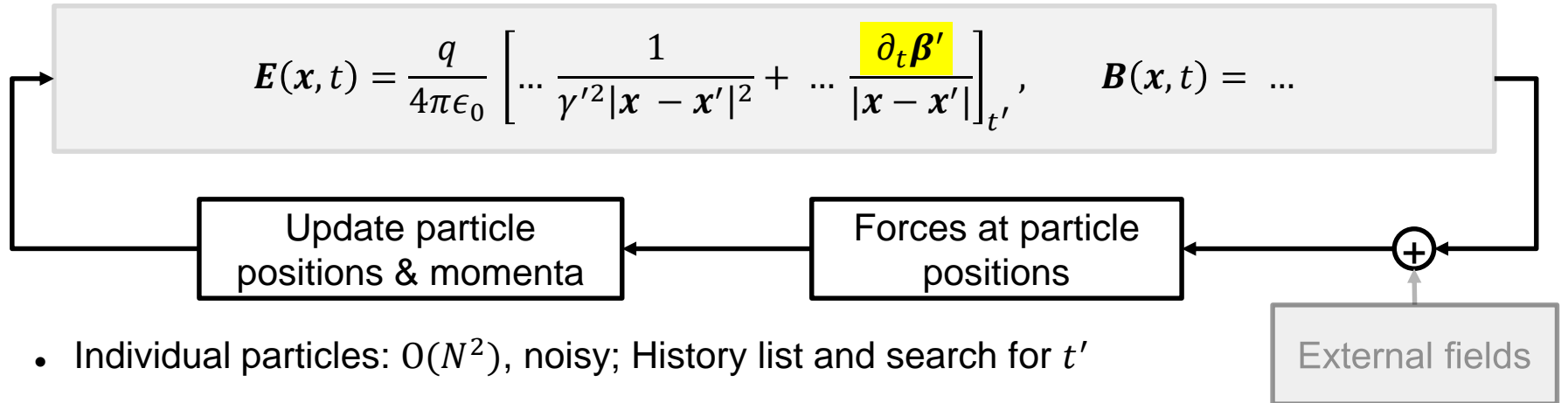
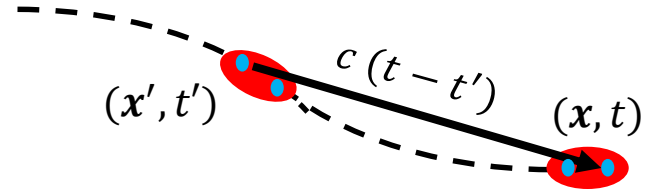


- Fully self-consistent, “brute-force”

Background: Quasistatic Particle Tracking

- **Relativistic Particle Tracker for Injectors and Linacs (REPTIL)**

- Assume point particle in **free space**
- Liénard-Wiechert field equations:



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Scattered Field Formulation

- **Idea: Separate field contributions**

“Incident field”

- Maxwell with beam current

Particle tracking solver REPTIL

$$\frac{d}{dt} E = \epsilon^{-1} \text{curl} H - \epsilon^{-1} J$$
$$\frac{d}{dt} H = -\mu^{-1} \text{curl} E$$

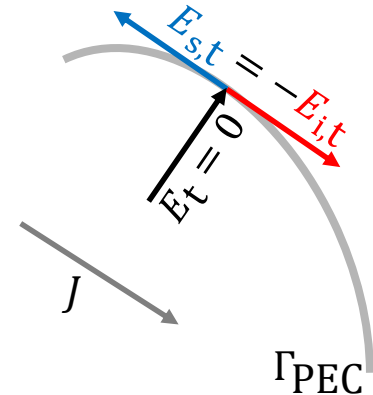
BC: $E_t = 0$ on Γ_{PEC}

“Scattered field”

- homogeneous Maxwell
- modified boundary conditions

Wakefield solver PBCI
(new BC)

$$E = E_i + E_s$$

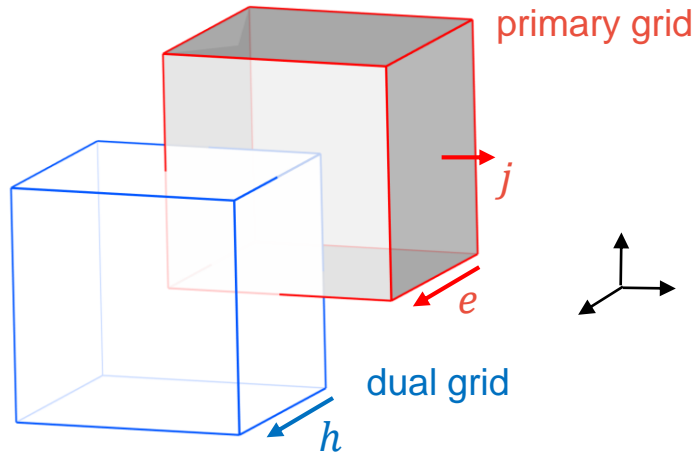


$$E_s(0) = E(0) - E_i(0)$$

Set “incident field” to the free-space field of the particles

Background: Wakefield Simulation

- **Parallel Beam Cavity Interaction (PBCI)**
 - Electromagnetic wave field solver incl. materials
 - Assume beam ultra-relativistic + rigid ($v_z = c$)



$$\begin{aligned} \frac{d}{dt} E &= \varepsilon^{-1} \text{curl} H - \varepsilon^{-1} J \\ \frac{d}{dt} H &= -\mu^{-1} \text{curl} E \end{aligned}$$

Discretization FIT / FDTD

$$\frac{d}{dt} \begin{pmatrix} h \\ e \end{pmatrix} = \begin{pmatrix} 0 & -M_{\mu}^{-1} C \\ M_{\varepsilon}^{-1} C^T & 0 \end{pmatrix} \begin{pmatrix} h \\ e \end{pmatrix} - \begin{pmatrix} 0 \\ M_{\varepsilon}^{-1} j \end{pmatrix}$$

↑ voltages over grid edges
↑
current through grid faces
↑

Scattered Field Formulation in PBCI

- **Modified wakefield solver:**

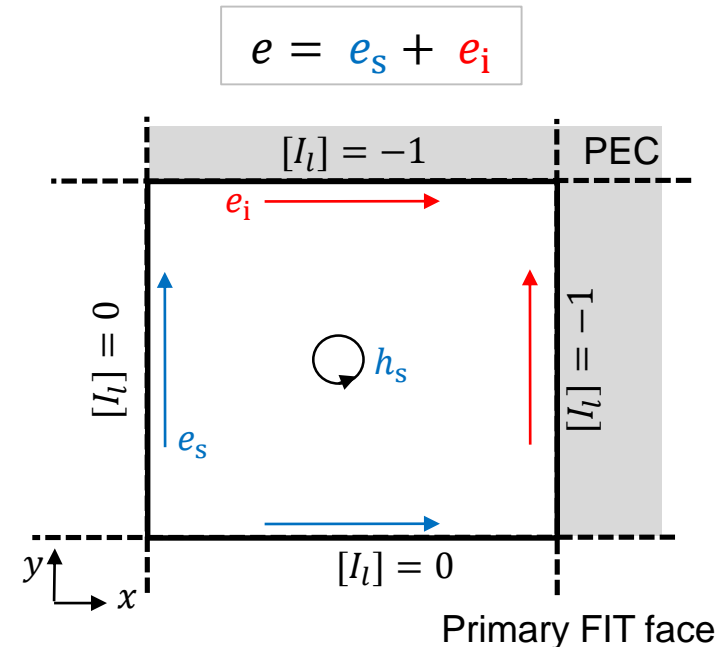
- Discretization of Faraday's eq. at a PEC boundary

$$\frac{d}{dt} \begin{pmatrix} h_s \\ e_s \end{pmatrix} = \begin{pmatrix} 0 & -M_\mu^{-1}C \\ M_\epsilon^{-1}C^T & 0 \end{pmatrix} \begin{pmatrix} h_s \\ e_s \end{pmatrix} - \begin{pmatrix} M_\mu^{-1}j_{\text{mag}} \\ 0 \end{pmatrix}$$

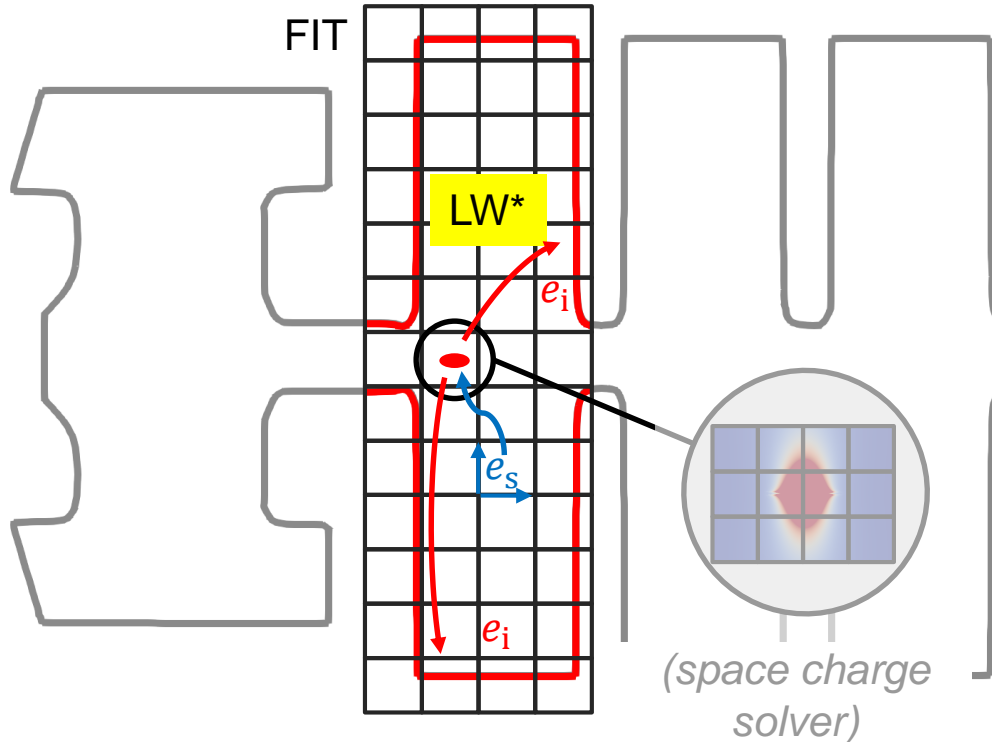
- Equivalent magnetic current at the boundary

$$j_{\text{mag}} = C I_l e_i$$

- With local interpolation matrix $I_l = \begin{cases} -1 & \text{if edge in PEC} \\ 0 & \text{else} \end{cases}$
- Material matrices remain the same as for PEC-boundaries



Coupling: PBCI + REPTIL



Main idea (old):

- couple space charge solver (REPTIL)
- with wake field solver (PBCI)
- via scattered field formulation

New approach:

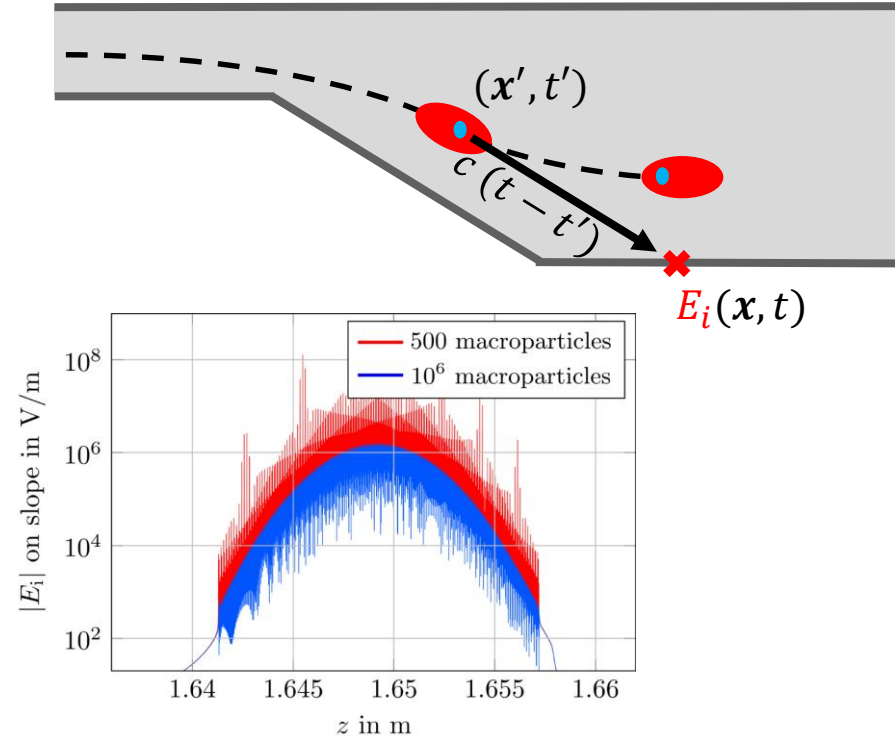
- evaluate space-charge field in free-space at the boundary
- using Liénard-Wiechert* solution
- to include retardation and radiation effects

Smoothing: Continuous Charge Approach

- CSR require LW incident field
 - Resolution of light cone requires unreasonably high number of macroparticles
- Continuous charge approach (M. Dohlus, 2003): 3D Gaussian

$$\rho(\mathbf{x}, t) \sim q \exp\left(-\frac{1}{2} \frac{|\mathbf{x} - \mathbf{x}_t(t)|^2}{\sigma^2}\right)$$

→ semi-analytical fields, 1D integral



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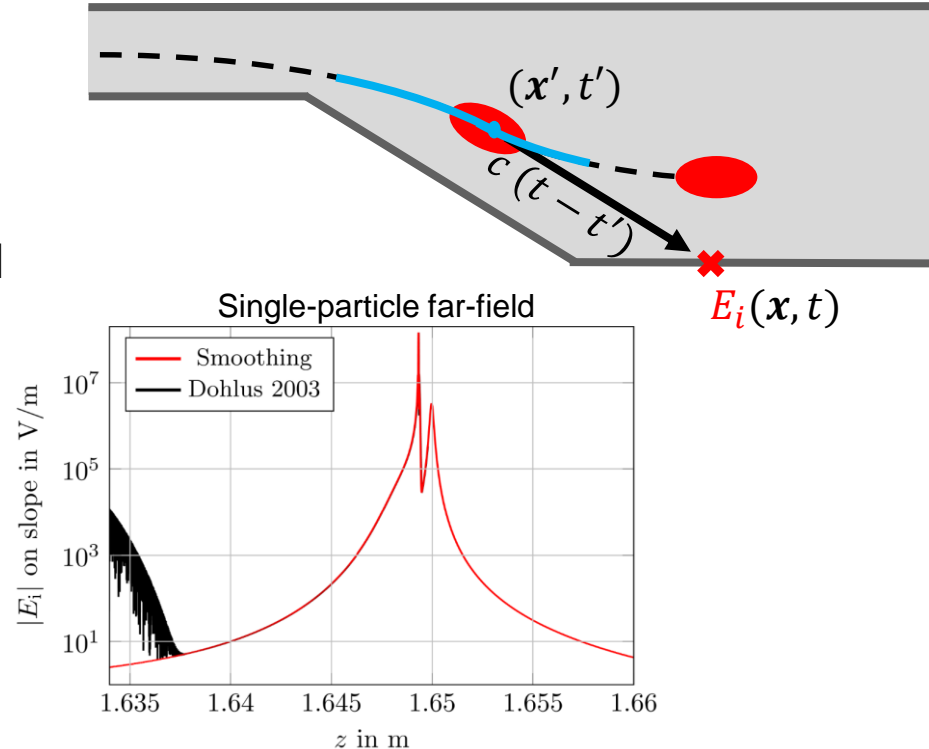
- Our variant:
 - Distribute charge along trajectory \mathbf{x}_t : time shift τ to center
 - Line density $n(\tau) \sim \exp\left(-\frac{1}{2} \frac{\tau^2}{\sigma_\tau^2}\right)$
 - 1D Continuous charge

$$\rho(\mathbf{x}, t) \sim q n(\tau) \delta^3(\mathbf{x} - \mathbf{x}_t(t + \tau))$$

→ weighted sum of LW evaluations

Smoothing: Continuous Charge Approach

- 1D continuous charge:
 - Similar implementation to 3D continuous charge possible: find retarded position, step backward and forward in history list
 - Convergence with larger timestep than 3D



Smoothing: Continuous Charge Approach

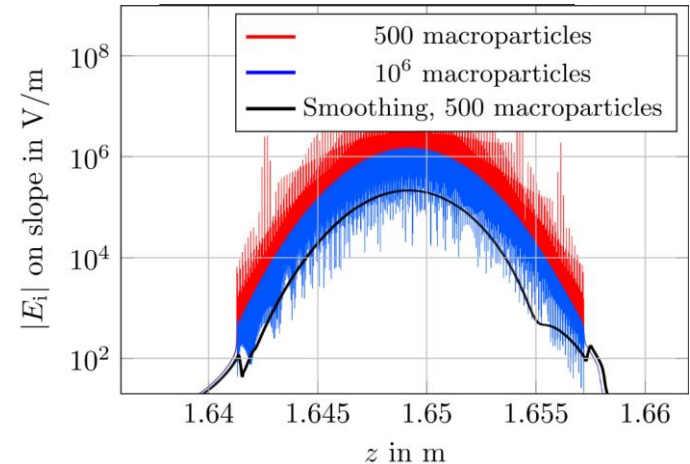
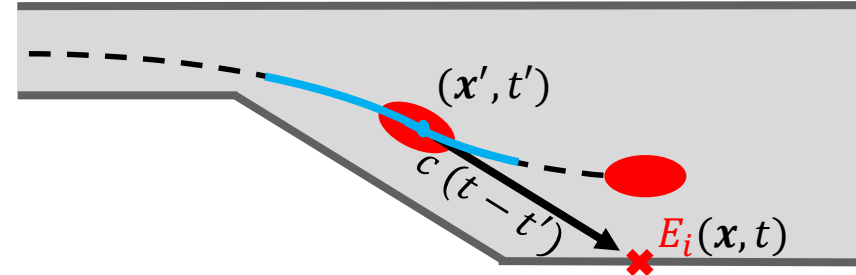
- 1D continuous charge:
 - Combine with averaging over FIT timestep:

$$n(\tau) \sim \exp\left(-\frac{1}{2} \frac{\tau^2}{\sigma_\tau^2}\right)$$



$$\text{FIT: } e^{t_m} \sim \frac{1}{\Delta t_{\text{PBCI}}} \int_{t_m-0.5}^{t_m+0.5} E(t) dt$$

$$N(\tau) \sim \frac{1}{\Delta t_{\text{PBCI}}} \left(\text{erf}_{\sigma_\tau} \left(\tau + \frac{\Delta t_{\text{PBCI}}}{2} \right) - \text{erf}_{\sigma_\tau} \left(\tau - \frac{\Delta t_{\text{PBCI}}}{2} \right) \right)$$

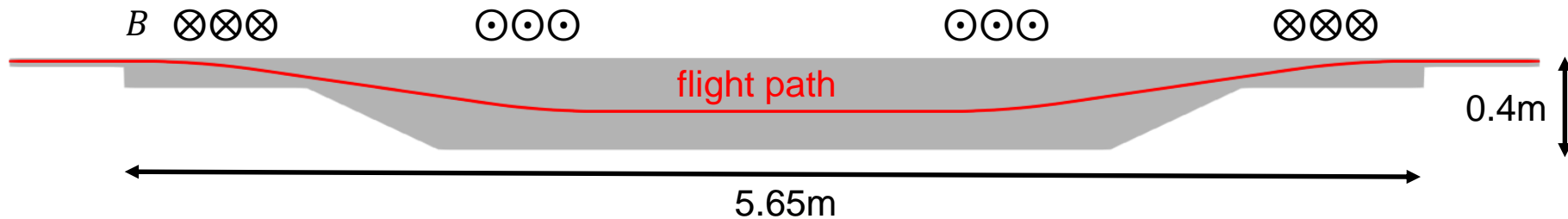


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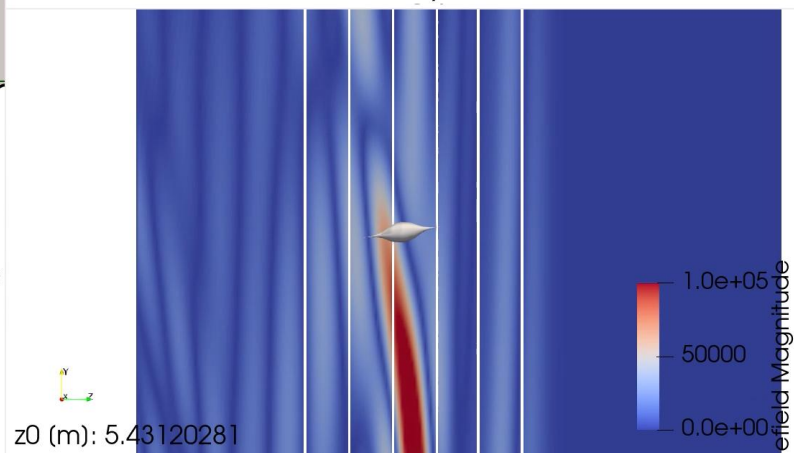
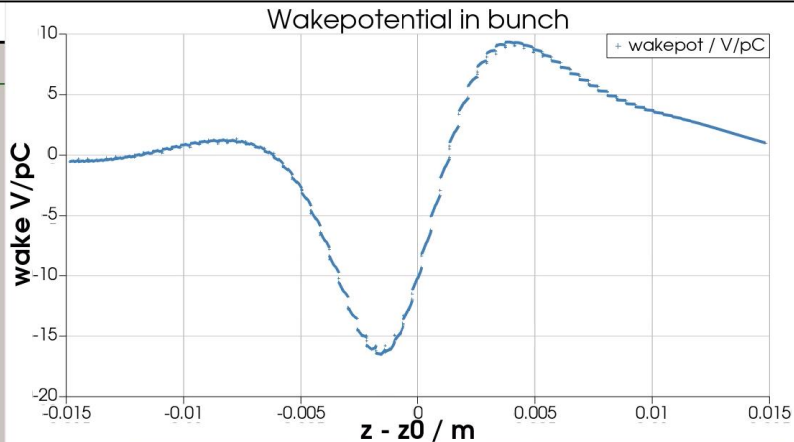
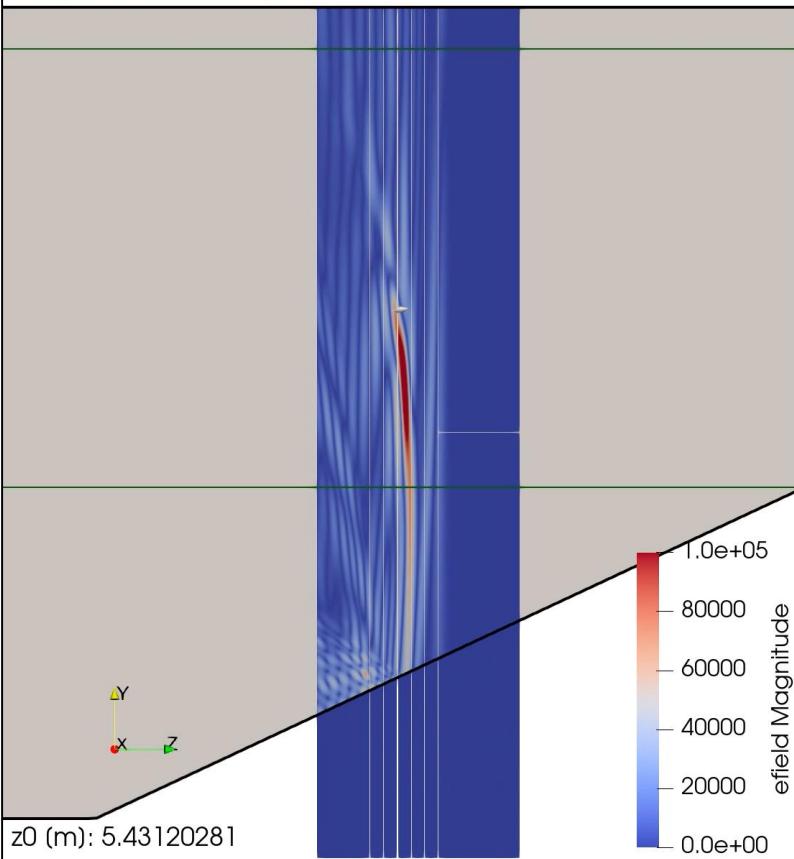
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- BC: imposed energy chirp causes different path lengths
 - BC0: compress bunch length from 3mm to 1mm @130MeV
 - Indirect CSR wakes from chamber



LW_130M\res05-500, full BC0



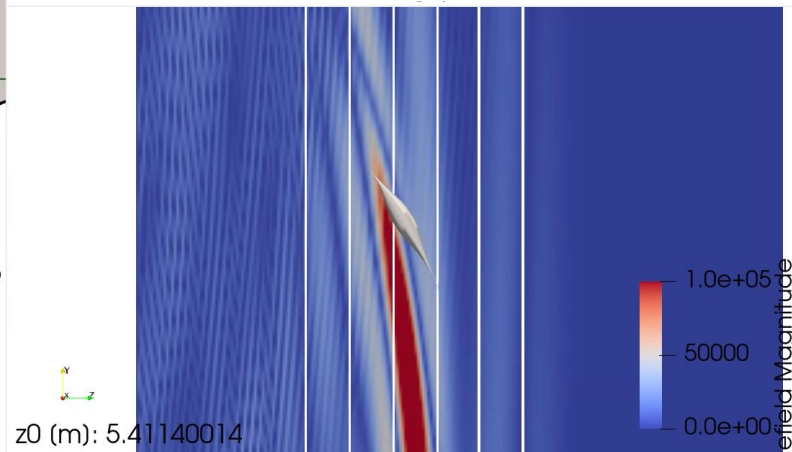
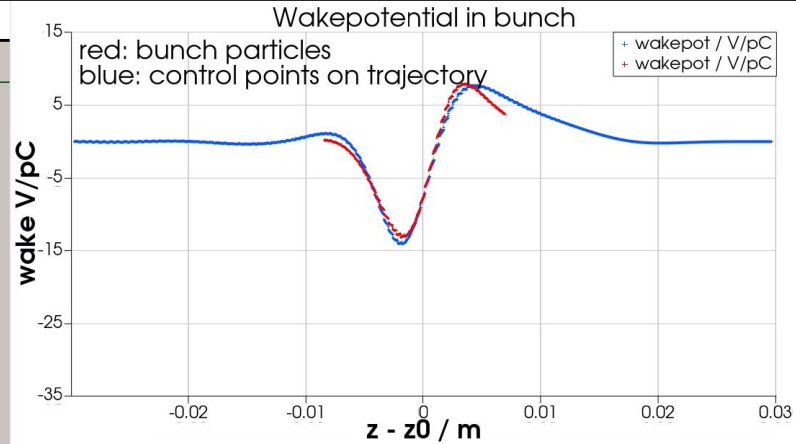
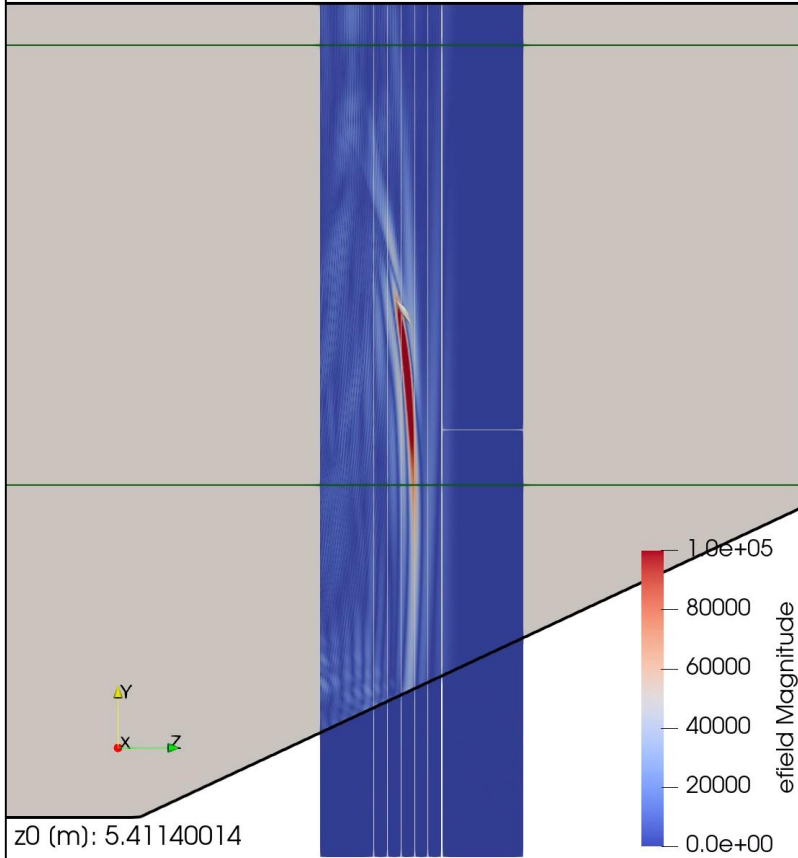
$\sigma_z = 2\text{mm}$,
line charge

0.2mm mesh

500 macro-
particles (+
test points)

Smoothing:
Gaussian

LW_130M\res05-500, full BC0 - compression 3mm - 1.5mm



$\sigma_z = 3\text{mm} / 1.5\text{mm}$, line charge

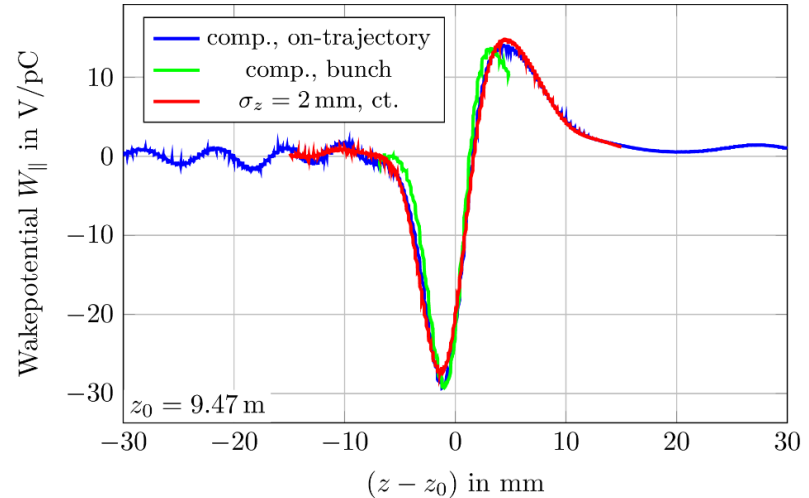
0.2mm mesh

500 macro-particles (+ test points)

Smoothing: Gaussian

Preliminary Results: BC0 @DESY

- BC: imposed energy chirp causes different path lengths
 - BC0: compress bunch length from 3mm to 1mm @130MeV
 - Indirect CSR wakes from chamber
 - Line charge, w/wo compression



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flight path

0.4m

5.65m

Summary

- Simulation of indirect CSR wakes
 - Coupled Scattered Field Formulation
 - Smooth radiation far-field from 1D continuous charge approach
- Ongoing:
 - Performance, mesh refinement
 - Combine with free-space CSR effects

