Updates on the Coupled Simulation of Wakefield and Space Charge Effects

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Task for Beam Dynamics Simulations

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- **Solve Maxwell's eqs. + Eq. of Motion:**
  - # Particles, Geometry, multi-scale → ~~Full EM-Particle in cell~~

space charge / particle tracker

wakefield solver

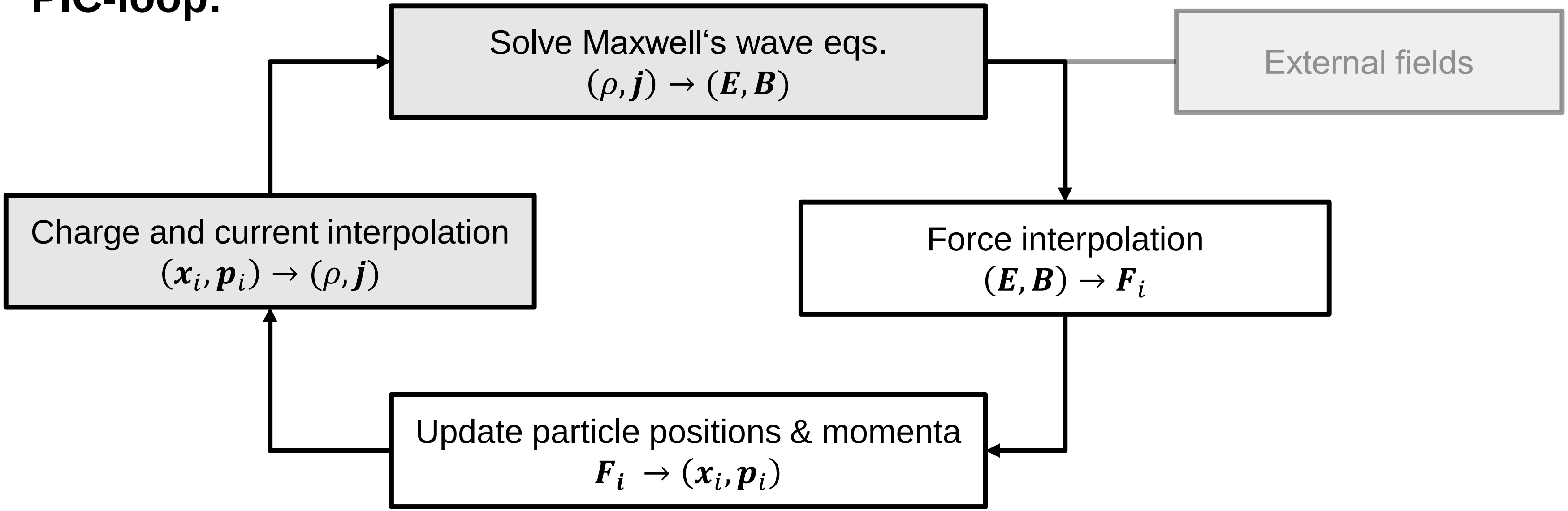
REPTIL

Take the best from both?

PBCI

Background: Electromagnetic Particle-in-Cell

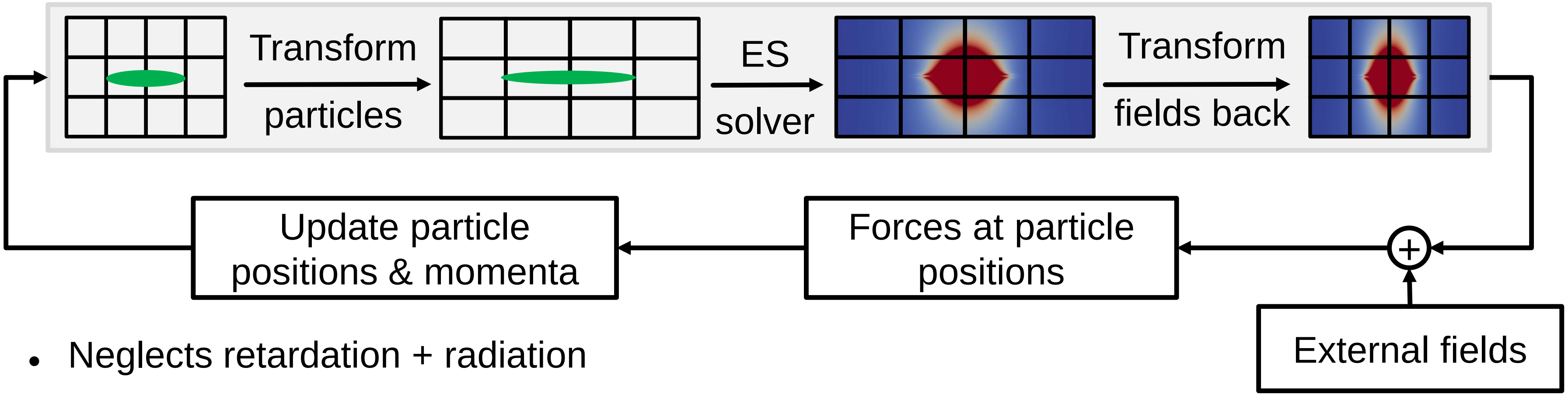
- **PIC-loop:**



- Fully self-consistent, “brute-force”

Background: Quasistatic Particle Tracking

- **Relativistic Particle Tracker for Injectors and Linacs (REPTIL)**
  - Assume particle cloud in free space + nearly-uniform movement
  - Electrostatic field solver (e.g. Green-fct. DFT or FMM) in particle's rest frame



  - Neglects retardation + radiation

Background: ~~Quasistatic~~ Particle Tracking

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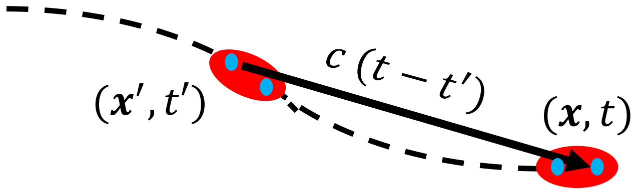
- **Relativistic Particle Tracker for Injectors and Linacs (REPTIL)**
  - Assume particle cloud in free space
  - Liénard-Wiechert field equations:



$$\boldsymbol{E}(\boldsymbol{x}, t) = \frac{q}{4\pi\epsilon_0}\left[\dots \frac{1}{\gamma'^2|\boldsymbol{x} - \boldsymbol{x}'|^2} + \dots \frac{\partial_t \boldsymbol{\beta}'}{|\boldsymbol{x} - \boldsymbol{x}'|}\right], \qquad \boldsymbol{B}(\boldsymbol{x}, t) = \dots$$

Update particle positions & momenta

Forces at particle positions

$\oplus$

External fields

  - Individual particles: $\mathcal{O}(N^2)$, noisy; Search for t'

Task

- **Solve Maxwell's eqs. + Eq. of Motion:**
  - # Particles, Geometry, multi-scale → ~~Full EM-Particle in cell~~

space charge / particle tracker

- Free-space assumption
- Poisson eq. in Lorentz frame
- Green-function 3D DFT
- − No transient fields

wakefield solver

- EM wave eq.
- Particles => current
- FDTD / FIT
- − No intermediate feedback

REPTIL — Take the best from both? — PBCI

Scattered Field Formulation

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- **Idea: Separate field contributions**

"Incident field"
- Mxw. with beam current

$$\frac{d}{dt}E = \varepsilon^{-1}\,\mathrm{curl}\,H - \varepsilon^{-1}J$$
$$\frac{d}{dt}H = -\mu^{-1}\,\mathrm{curl}\,E$$

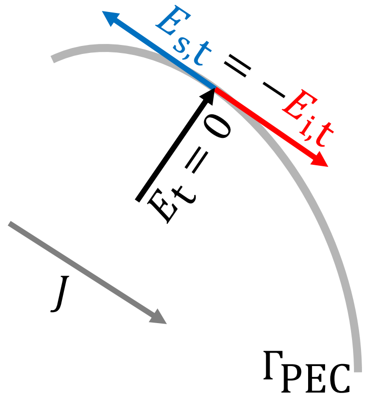
BC: $E_\mathrm{t} = 0$ on Γ_PEC

"Scattered field"
- homogeneous Mxw.
- modified boundary conditions

Particle space charge solver REPTIL

$E = E_\mathrm{i} + E_\mathrm{s}$

Wakefield solver PBCI (new BC)



$$E_\mathrm{s}(0) = E(0) - E_\mathrm{i}(0)$$

Set "incident field" to the space charge field of the particles in free-space

Scattered Field Formulation in PBCI

- **Changes to wakefield solver:**
  - Discretization of Faraday's eq. at a PEC boundary

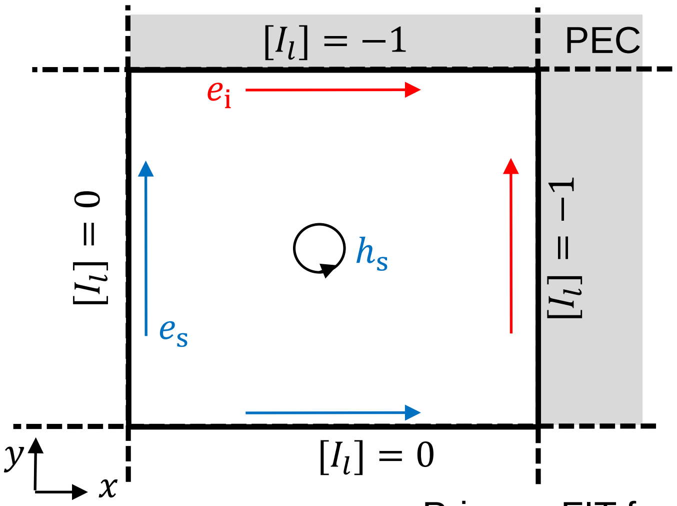
$$\boxed{\frac{d}{dt}\begin{pmatrix} h_\mathrm{s} \\ e_\mathrm{s}\end{pmatrix} = \begin{pmatrix} 0 & -M_\mu^{-1}C \\ M_\varepsilon^{-1}C^T & 0 \end{pmatrix}\begin{pmatrix} h_\mathrm{s} \\ e_\mathrm{s}\end{pmatrix} - \begin{pmatrix} M_\mu^{-1}\, j_\mathrm{mag} \\ 0 \end{pmatrix}}$$

  - Equivalent magnetic current at the boundary

$$\boxed{j_\mathrm{mag} = C\, I_l\, e_\mathrm{i}}$$

  - With local interpolation matrix $I_l = \begin{cases} -1 & \text{if edge in PEC} \\ 0 & \text{else} \end{cases}$
  - Material matrices remain the same as for PEC-boundaries

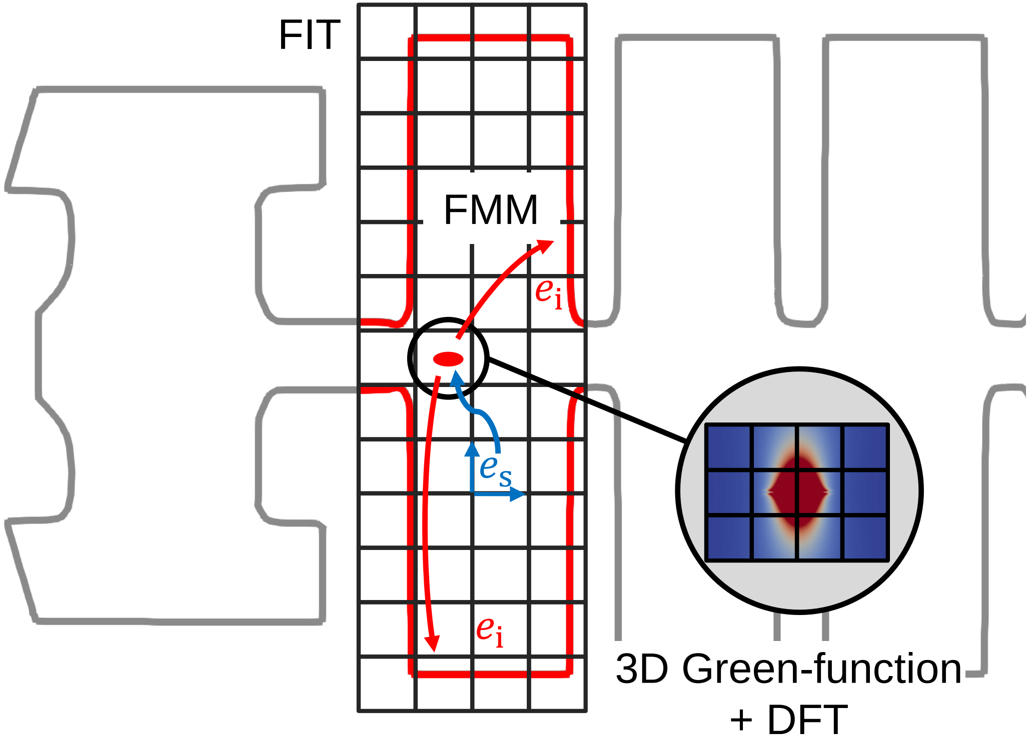
$$\boxed{e = e_\mathrm{s} + e_\mathrm{i}}$$



Primary FIT face

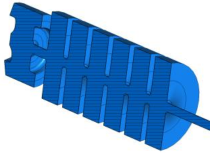
Cmp. previous DESY-TEMF meetings

Coupling: PBCI + REPTIL

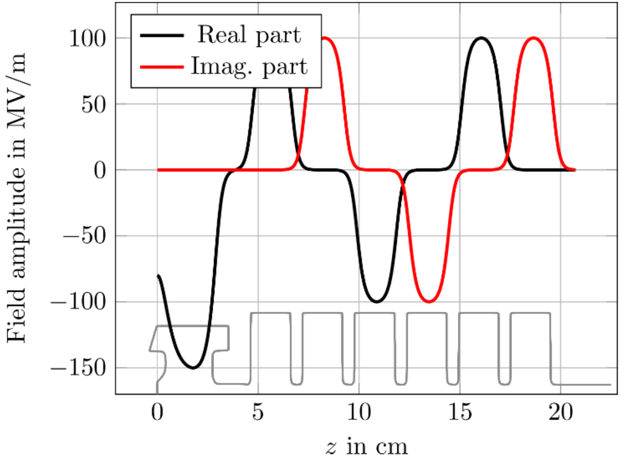


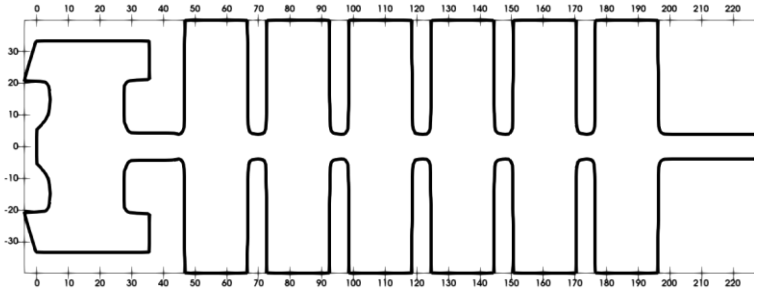
- Idea: couple space charge solver (REPTIL) with wake field solver (PBCI) via scattered field formulation
- Mesh-free, fast evaluation of space-charge field on boundary: Fast Multipole Method (FMM); or Liénard-Wiechert
- Solvers independent (grid, time step, optimization, …)

Quasi-Traveling Wave Gun Model

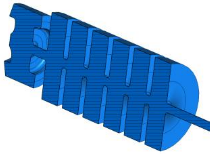


- Motivation: Natsui, Yoshida (IPAC’14)
- 7 Side-Coupled standing wave cavities (alternating field phase)
- Quasi-TW field pattern seen by particles
- Bending of acc. Field → RF-focusing, no solenoid required
- High bunch charge: 5nC
- Narrow, long geometry: 4mm iris radius, ~20cm acceleration path length





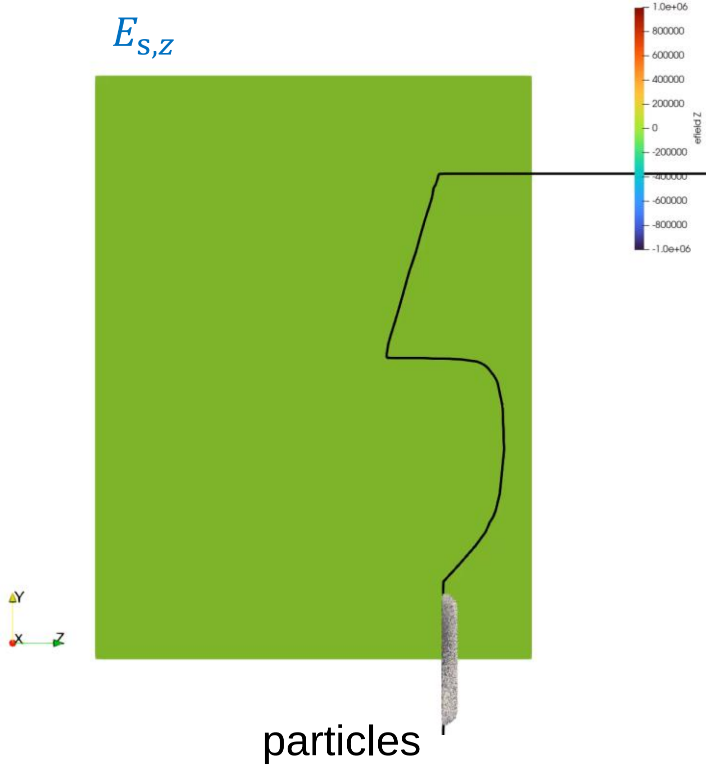
Quasi-Traveling Wave Gun Model



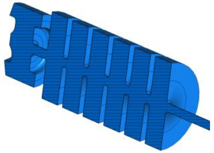
- Narrow, long geometry: 4mm iris radius, ~20cm acceleration path length
- Video: fields build up over time
- Compare EM-PIC (CST) with SFF for incident field provided by FMM or LW

Bunch (simplified model):

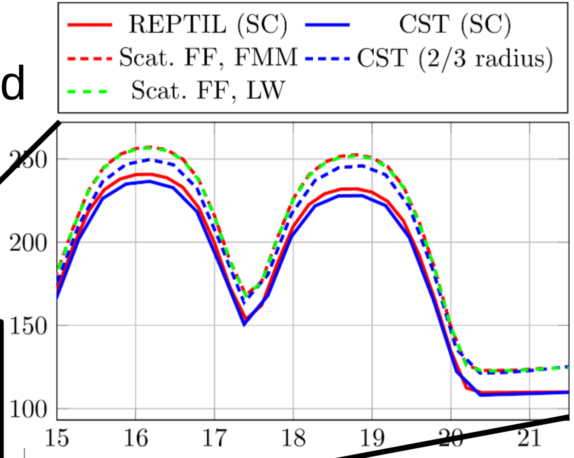
Charge	5nC
Length	~1.5mm
Size	<0.5mm
Energy	14.2MeV at gun exit

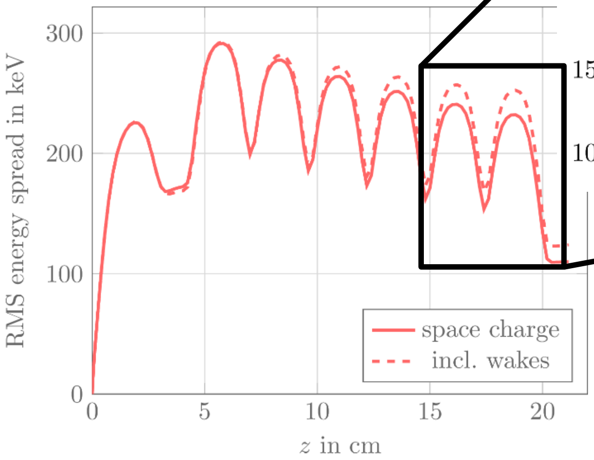


Results for Quasi-TW Gun



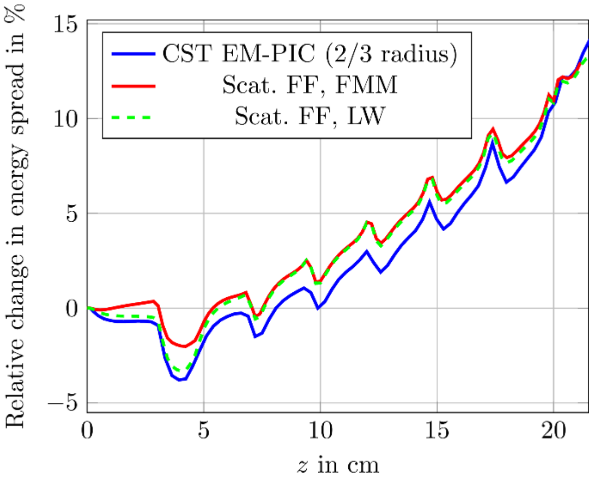
- Longitudinal wakes increase energy spread





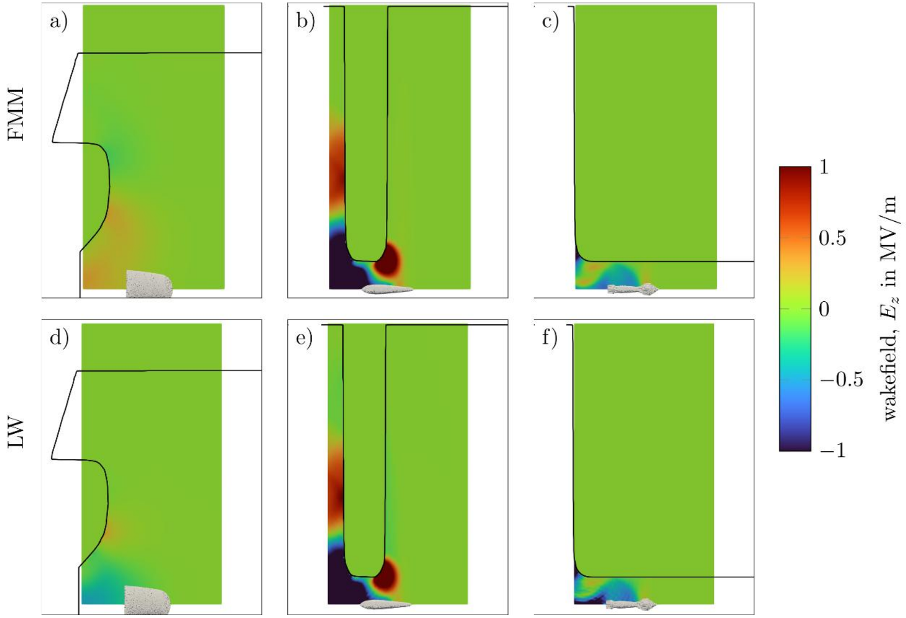
- Good agreement with EM-PIC

- LW + FMM agree after few cm

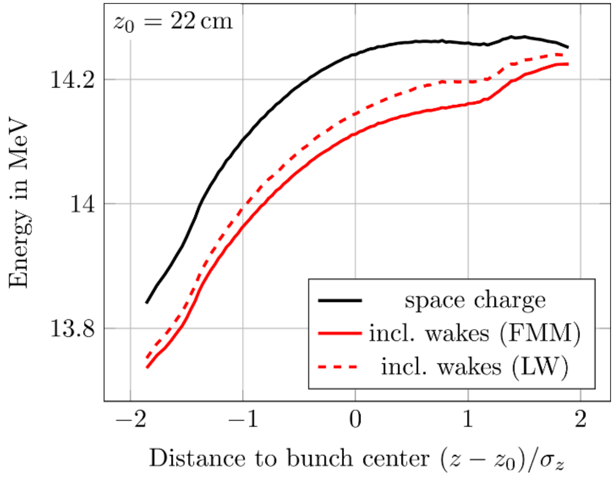


Results for Quasi-TW Gun: Energy Chirp

- Retardation effect in first cell

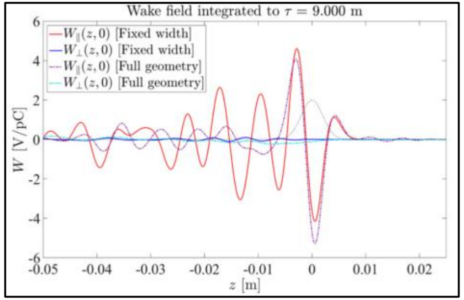


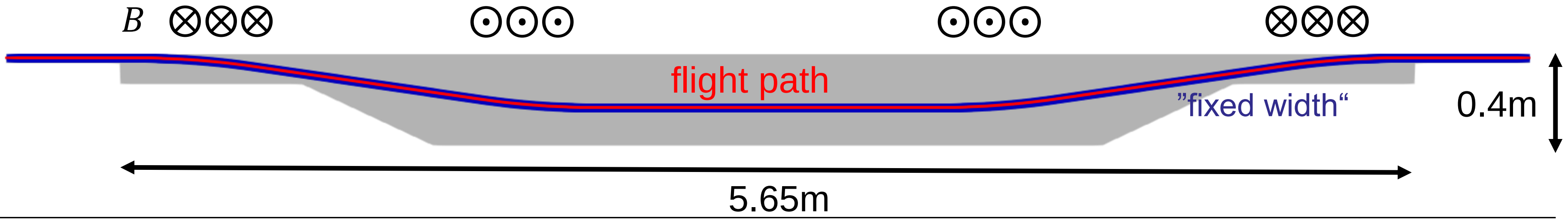
- No retardation: Wakes at bunch head stronger for FMM



Ongoing Work: BC0 @DESY

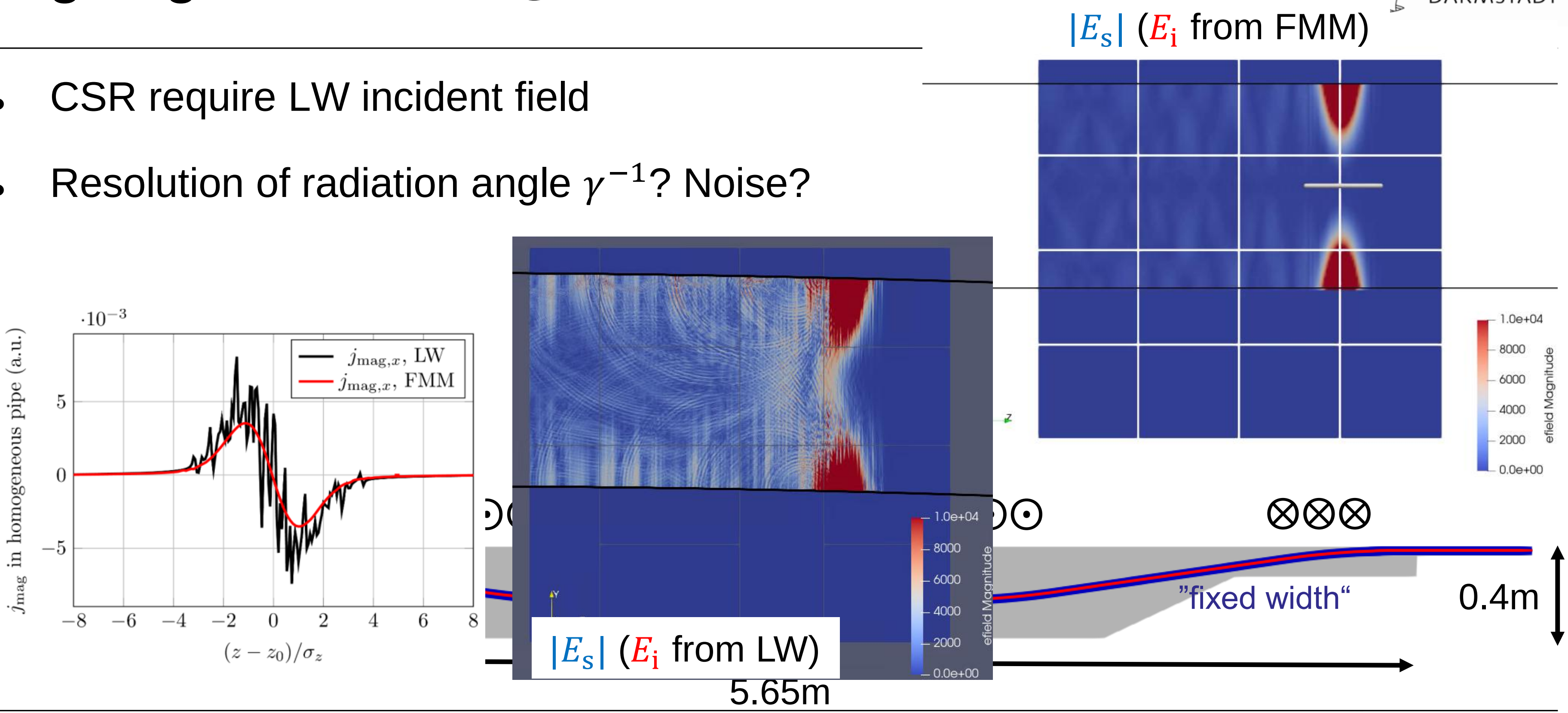
- BC: imposed energy chirp causes different path lengths
  - BC0: compress bunch length from 3mm to 1mm @130MeV
- Bizzozero 2019: CSR wakefield in BC0 with DG-FEM





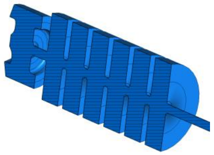
Ongoing Work: BC0 @DESY

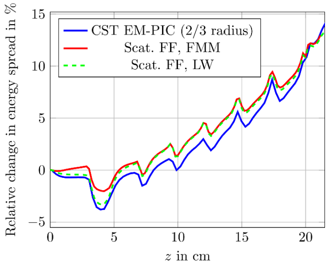
- CSR require LW incident field
- Resolution of radiation angle γ^{-1}? Noise?



Summary

- Validation of coupled space charge and wake field approach for gun simulations
  - Including impact of retardation effects
  - Comparison to EM-PIC (cmp. talk from Dec.)





- Ongoing work:
  - Resolution of CSR wakefields in BC