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SURFACE IMPEDANCE OF MULTILAYERED SUPERCONDUCTING CAVITIES

DESY-TEMF July 2025

2025-07-03

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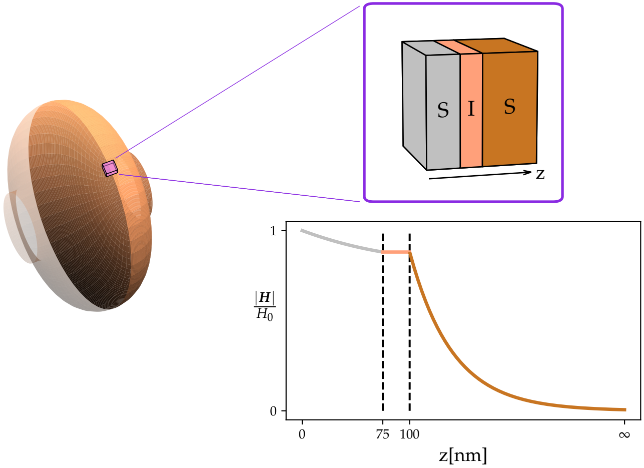
MOTIVATION

Section 1



Why ?*

1. Niobium is expensive
2. Increase in energy efficiency
3. Higher accelerating fields

How ?

1. Replace bulk Nb with Cu, Al, etc. substrates
2. Better thermal conducting substrate
3. Shield with higher H_c material

* Valente-Feliciano, A.-M. *Superconductor Science and Technology* (2016)

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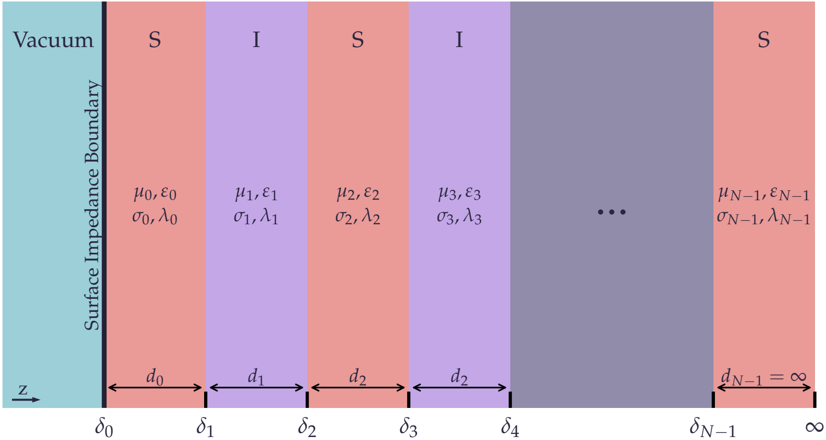
REDUCTION TO SIBC

Section 2

ANALYTICAL SOLUTION



Solve Maxwell's equations in every layer $k \in \{0, \ldots, N-1\}$:

$$E_x^{(k)} = \begin{pmatrix} C_{2k} \\ C_{2k+1} \end{pmatrix} \cdot \begin{pmatrix} e^{j\alpha_k z} \\ e^{-j\alpha_k z} \end{pmatrix} := \boldsymbol{C}_k \cdot \boldsymbol{u}_k$$

$$H_y^{(k)} = -\frac{\alpha_k}{\omega\mu_k} \begin{pmatrix} C_{2k} \\ C_{2k+1} \end{pmatrix} \cdot \begin{pmatrix} e^{j\alpha_k z} \\ -e^{-j\alpha_k z} \end{pmatrix} := -\frac{\alpha_k}{\omega\mu_k} \boldsymbol{C}_k \cdot \boldsymbol{v}_k$$

where $\alpha \in \mathbb{C}$ depends on material type:

$$\text{Insulator} : \alpha^2 = \mu\varepsilon\omega^2$$

$$\text{Normal Conductor} : \alpha^2 = \mu\varepsilon\omega^2 - j\mu\sigma\omega$$

$$\text{Superconductor (two-fluid)} : \alpha^2 = \mu\varepsilon\omega^2 - j\mu\sigma\omega - \frac{1}{\lambda^2}$$

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Surface impedance boundary condition

$$\boldsymbol{n} \times \boldsymbol{E} = Z(\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{H}))$$

yields surface impedance

$$Z(\omega) = \frac{E_x^{(0)}(\delta_0)}{H_y^{(0)}(\delta_0)} = -\frac{\omega\mu_0}{\alpha_0} \frac{\boldsymbol{C}_0 \cdot \boldsymbol{u}(\delta_0)}{\boldsymbol{C}_0 \cdot \boldsymbol{v}(\delta_0)}$$

$\Rightarrow$ We need the coefficient vector $\boldsymbol{C}_0$

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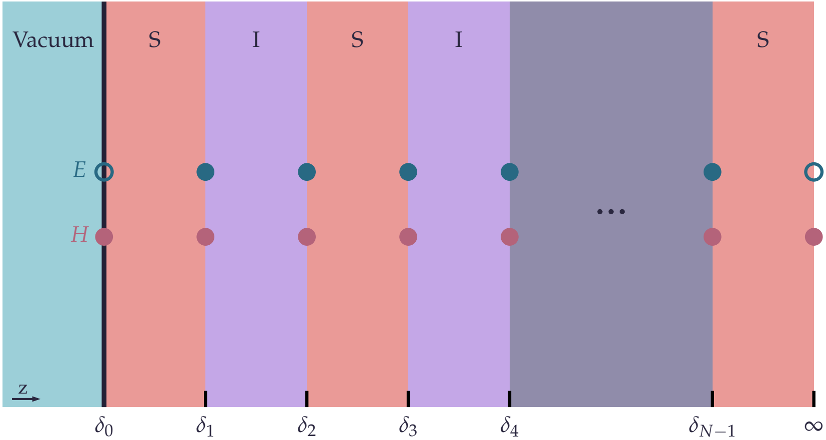

$2N$ Unknows = 1 vacuum constraint + $2(N-1)$ interface constraints + 1 infinity constraint

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Mathematically the constraints are:

- Vacuum constraint:

$$H_y^{(0)}(\delta_0) = H_0$$

- Interface constraints:

$$E_x^{(k)}(\delta_{k+1}) = E_x^{(k+1)}(\delta_{k+1})$$
$$H_y^{(k)}(\delta_{k+1}) = H_y^{(k+1)}(\delta_{k+1})$$

- Infinity constraint:

$$\lim_{z \to \infty} H_y^{(N-1)}(z) = 0$$

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Implied relations between coefficients:

- Vacuum constraint:

$$\boldsymbol{C}_0 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -\frac{\omega\mu_0}{\alpha_0} H_0$$

- Interface constraints:

$$\boldsymbol{C}_k \cdot \boldsymbol{u}_k(\delta_{k+1}) = \boldsymbol{C}_{k+1} \cdot \boldsymbol{u}_{k+1}(\delta_{k+1})$$
$$\boldsymbol{C}_k \cdot \boldsymbol{v}_k(\delta_{k+1}) = \gamma_k \boldsymbol{C}_{k+1} \cdot \boldsymbol{v}_{k+1}(\delta_{k+1}), \quad \gamma_k = \frac{\mu_k}{\mu_{k+1}} \frac{\alpha_{k+1}}{\alpha_k}$$

- Infinity constraint:

$$\boldsymbol{C}_{N-1} = C \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Recursion relations via linear combination of interface constraints:

$$\boldsymbol{C}_k \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2} e^{-j\alpha_k \delta_{k+1}} \boldsymbol{C}_{k+1} \cdot \left(\Gamma_k \boldsymbol{u}_{k+1}(\delta_{k+1}) \right), \qquad \Gamma_k = \mathrm{diag}(1+\gamma_k, 1-\gamma_k)$$

$$\boldsymbol{C}_k \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2} e^{j\alpha_k \delta_{k+1}} \boldsymbol{C}_{k+1} \cdot \left(\Gamma_k^\tau \boldsymbol{u}_{k+1}(\delta_{k+1}) \right), \qquad \Gamma_k^\tau = \mathrm{diag}(1-\gamma_k, 1+\gamma_k)$$

yields characteristic matrix equation for each layer:

$$\boldsymbol{C}_k = \tau_{k,k+1} \boldsymbol{C}_{k+1}, \quad \tau_{k,k+1} = \frac{1}{2} \begin{pmatrix} e^{-j\alpha_k \delta_{k+1}} \left[\Gamma_k \boldsymbol{u}_{k+1}(\delta_{k+1}) \right]^T \\ e^{j\alpha_k \delta_{k+1}} \left[\Gamma_k^\tau \boldsymbol{u}_{k+1}(\delta_{k+1}) \right]^T \end{pmatrix}$$

and consequently a single transfer matrix:

$$\boldsymbol{C}_0 = T \boldsymbol{C}_{N-1} := \left[\prod_{k=0}^{N-2} \tau_{k,k+1} \right] \boldsymbol{C}_{N-1}$$

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Now notice:

$$\boldsymbol{C}_0 = T\boldsymbol{C}_{N-1} = CT\begin{pmatrix}1\\0\end{pmatrix} = C\begin{pmatrix}T_{00}\\T_{01}\end{pmatrix}$$

So we find the surface impedance:

$$Z(\omega) = -\frac{\omega\mu_0}{\alpha_0}\frac{T_{00}+T_{01}}{T_{00}-T_{01}}$$

- $Z(\omega)$ is determined by transfer matrix, which we can compute
- $Z(\omega)$ is a non-linear function of ω
- **Difficulty**: Eigenvalueproblem is complex and non-linear

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SELF-CONSISTENCY

- ✓ Surface impedance uses all material parameters from all layers
- ✓ Reduces to bulk superconductor formula when $N = 1$
- ✓ Reduces to bulk formula when all layers are identical

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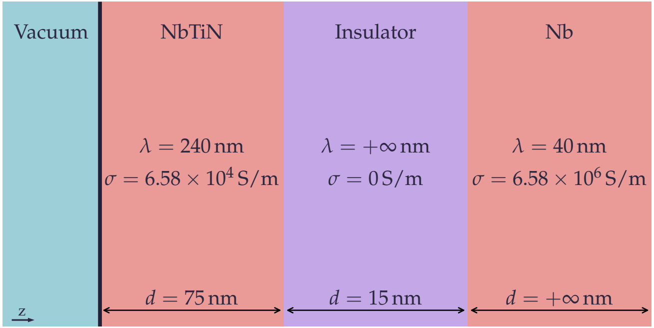
EXAMPLE

Bulk Nb with generic insulator layer and NbTiN sputtering at $f = 1.3\,\text{GHz}^*$



$$Z(\omega) = 21.462 \times 10^{-9}\,\Omega + 1.225 \times 10^{-3} j\,\Omega$$

* Keckert S., *PhD Dissertation, Universität Siegen* (2020)

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CURRENT PROGRESS & FUTURE WORK

Section 3

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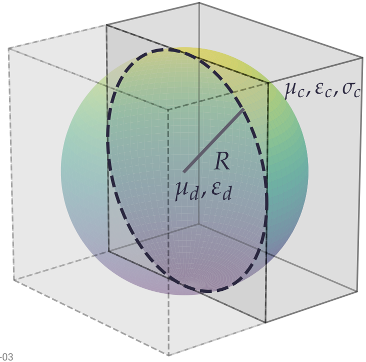
LOSSY SPHERICAL RESONATOR*



Transcendental equation for TM_{mnp} modes

$$\eta_d \left[\frac{j_{n-1}(\beta_d R)}{j_n(\beta_d R)} - \frac{n}{\beta_d R} \right] - \eta_c \left[\frac{h^{(2)}_{n-1}(\beta_c R)}{h^{(2)}_n(\beta_c R)} - \frac{n}{\beta_c R} \right] = 0$$

where

$$\eta_d = \sqrt{\frac{\mu_d}{\varepsilon_d}}, \beta_d = \omega\sqrt{\mu_d \varepsilon_d},$$

$$\eta_c = \sqrt{\frac{\mu_c}{\varepsilon_c}}, \beta_c = \omega\sqrt{\mu_c \varepsilon_c},$$

* Papantonis S. *et al.*, *Progress in Electromagnetics Research* (2015)

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SEARCH FOR SOFTWARE

Software	Pros	Cons
CST	▪ Reliable ▪ Easy to use	▪ Limited access to base structures ▪ Convergence difficulties when $Q \geq 10^{11}$
GetDP	▪ Open source ▪ More access than CST	▪ SLEPc is difficult to tune ▪ **.pro* files become difficult for complicated problems
FEniCSx	▪ Open source ▪ More access than GetDP	▪ SLEPc is difficult to tune ▪ Steep learning curve
CEM-3D	▪ Developed for RF cavities ▪ Custom eigensolver	▪ Proprietary ▪ Requires bug fix

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FUTURE WORK

Study global surface impedance from multilayer computation

- Decide on which software to use
- Check that the result is stable across multiple iterations
- Check that it reduces to bulk result in appropriate limits

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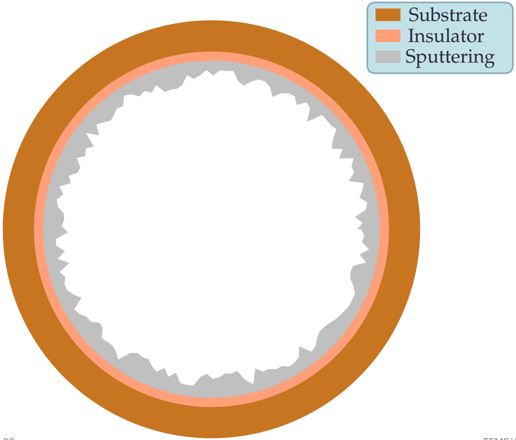
Study effects of sputtering distributions



- Thickness of sputtered layer becomes stochastic
- Model sputtering with Karhunen-Loève expansion
- $Z(\omega) \rightarrow Z(\omega, \boldsymbol{r})$
- Software must provide fine control
- Eigenvalues must have sufficient precision

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CONCLUSION

Section 4

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CONCLUSION

- Multilayered cavities can be modelled via SIBC
- Resulting eigenvalue problem is complex and non-linear
- Software must allow sufficient control to model sputtering
- Eigensolver must have sufficient precision to see sputtering effects