TECHNISCHE UNIVERSITÄT DARMSTADT

COMPARING GEOMETRY VARIANTS OF THE SINGLE MODE CAVITY

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OBJECTIVE

Section 1

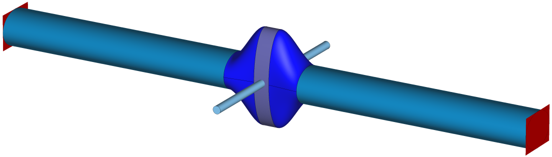
OBJECTIVE

ACTIVE 3^{rd} HARMONIC CAVITY

Requirements of the 3^{rd} harmonic cavity

- No phase dependency of the voltage
- Inexpensive and simple manufacturing
- Mitigation of higher order modes (HOM)

Test different geometry variants of the current cavity



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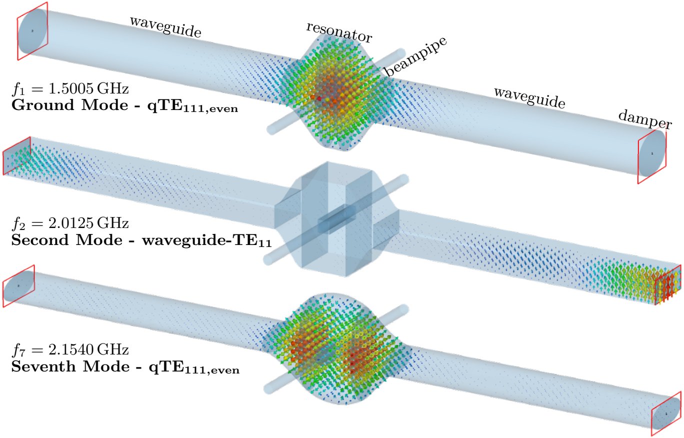
OBJECTIVE

ACTIVE 3^{rd} HARMONIC CAVITY



- elliptical waveguide, elliptical resonator
- rectangular waveguide, rectangular resonator
- elliptical waveguide, ellipsoidal resonator

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COMPARATIVE CHARACTERISTICS

Section 2

COMPARATIVE CHARACTERISTICS

THE EFFICIENCY ε AS A FIGURE OF MERIT

- The efficiency $\varepsilon := \frac{V_\text{acc}}{P_\text{loss}}$ as a figure of merit
  - suggested last year in [1]
- Relating to the energy conversion efficiency η

$$\eta = \frac{P_\text{out}}{P_\text{in}} = \frac{P_\text{out}}{P_\text{out} + P_\text{loss} + P_\text{misc}}$$

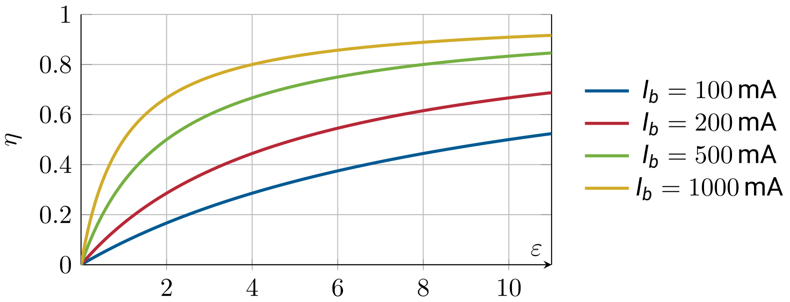
- with the output power $P_\text{out} = I_b V_\text{acc}$

$$\eta \approx \frac{I_b\,\varepsilon}{I_b\,\varepsilon + 1} \qquad \text{and thus} \qquad \partial_\varepsilon \eta = \frac{I_b}{(I_b\,\varepsilon + 1)^2}$$

[1] Kronshorst et al.: *Design of a single mode 3rd harmonic cavity for PETRA IV*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-TUPG52

COMPARATIVE CHARACTERISTICS

THE EFFICIENCY ε AS A FIGURE OF MERIT



- The efficiency ε is a good figure of merit for the cavity
  - η is a strictly monotonic function of ε for any I_b
  - It is independent of the beam current I_b
  - For *epsilon* $\rightarrow \infty$ the energy conversion efficiency η approaches its limit of 1

COMPARATIVE CHARACTERISTICS

THE TRANSVERSE SHUNT IMPEDANCE

- Shunt impedances and in particular the transverse shunt impedance $R_{\mathrm{s,\,n},\perp}$ are important figures of merit
- Calculatiung them can at times be difficult
- proposed altenative to wakefield simulations:

$$R_{\mathrm{s,\,n},\perp} = |\underline{\vec{Z}}_{\perp}(\omega_{r,n})| = \frac{|\hat{\underline{V}}_{\mathrm{acc},\perp}|}{I_0} = \frac{1}{\vec{r}_{\perp}} \frac{|\underline{V}^{*}_{\mathrm{acc},\|}|\,|\underline{V}_{\mathrm{acc},\perp}|}{2\,P_{d,n}} \,[2]$$

⇒ Comparison to transverse impedances from Arsenyev et al. [3]

[2] Kronshorst et al.: *Design of a single mode 3rd harmonic cavity for PETRA IV*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-TUPG52
[3] S. Arsenyev et al.: *Traveling wave method for simulating geometric beam coupling impedance of a beamscreen with pumping holes*, Phys. Rev. Accel. Beams **22**, 051002– Published 28 May, 2019

COMPARATIVE CHARACTERISTICS

THE TRANSVERSE SHUNT IMPEDANCE

- Arsenyev proposed a method to calculate the transverse driving and detuning impedance $Z_\perp^{\text{driv}}$ and $Z_\perp^{\text{det}}$
- Given the longitudinal function of x: $\left(\frac{R}{Q}\right)(x) = f(x)$
- The transverse impedances follow as

$$Z_x^{\text{driv}}(\omega, x_0) = \frac{c\,g(\omega)}{4\omega}\,\frac{f'(x_0)^2}{f(x_0)}$$

$$Z_x^{\text{det}}(\omega, x_0) = \frac{c\,g(\omega)}{4\omega}\left(-\frac{f'(x_0)^2}{f(x_0)} + 2f''(x_0)\right)$$

  - with $g_n(\omega) = \frac{Q_n}{1+iQ_n\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega_r}\right)}$

COMPARATIVE CHARACTERISTICS

DRIVING SHUNT IMPEDANCE

Previous

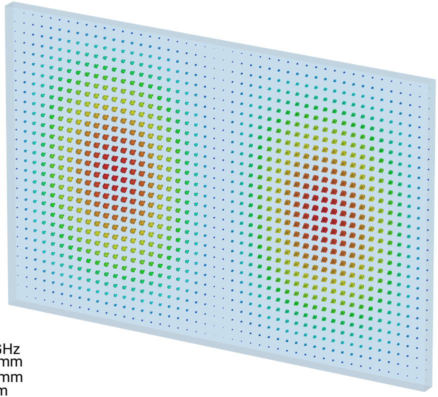
$$|R_{\text{s, n},\perp}(r_\perp)| = \frac{1}{\vec{r}_\perp} \frac{|\underline{V}^*_{\text{acc}\,\|}|\,|\underline{V}_{\text{acc},\perp}|}{2\,P_{d,n}}$$

Arsenyev

$$R^{\text{driv}}_{x,n}(x_0) = \frac{c\,g(\omega_{r,n})\,Q}{4\omega_{r,n}} \frac{f'(x_0)^2}{f(x_0)}$$

New proposition

$$R^{\text{driv}}_{x,n}(x_0) = \frac{k}{2\,P_{d,n}}\,|V_\perp(x_0)|^2$$



TM_{210}
$f = 2.5\,\text{GHz}$
$a = 150\,\text{mm}$
$b = 100\,\text{mm}$
$L = 5\,\text{mm}$

COMPARATIVE CHARACTERISTICS

DRIVING SHUNT IMPEDANCE

Previous

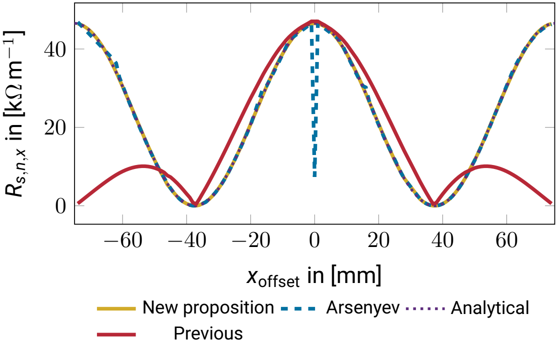
$$|R_{s,n,\perp}(r_\perp)| = \frac{1}{\vec{r}_\perp} \frac{|V^*_{\mathrm{acc}\,\|}|\,|V_{\mathrm{acc},\perp}|}{2\,P_{d,n}}$$

Arsenyev

$$R^{\mathrm{driv}}_{x,n}(x_0) = \frac{c\,g(\omega_{r,n})\,Q_n}{4\omega_{r,n}} \frac{f'(x_0)^2}{f(x_0)}$$

New proposition

$$R^{\mathrm{driv}}_{x,n}(x_0) = \frac{k}{2\,P_{d,n}} |V_x(x_0)|^2$$



COMPARATIVE CHARACTERISTICS

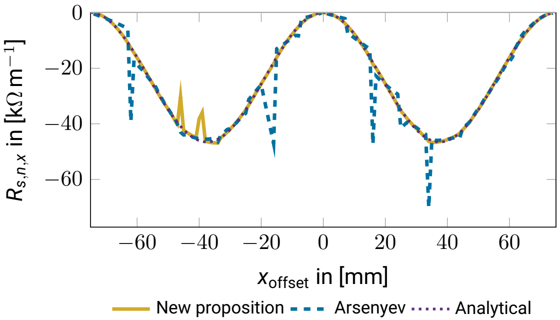
DETUNING SHUNT IMPEDANCE

Arsenyev

$$R^{\mathrm{det}}_{x,n}(x_0) = \frac{c\, g(\omega_{r,n})\, Q_n}{4\omega_{r,n}} \left(-\frac{f'(x_0)^2}{f(x_0)} + 2f''(x_0)\right)$$

New proposition

$$R^{\mathrm{det}}_{x,n}(x_0) = -\frac{1}{2\,P_{d,n}} \left|V_{\parallel}(x_0)\right| \frac{\mathrm{d}}{\mathrm{d}x}\left|V_x(x_0)\right|$$



COMPARISON RESULTS

Section 3

COMPARISON RESULTS

FIGURES OF MERIT

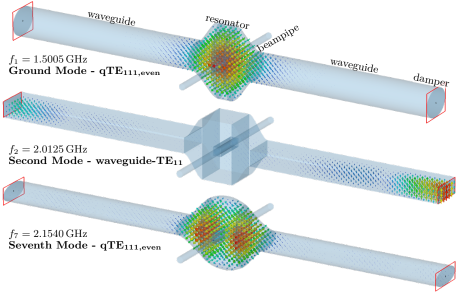
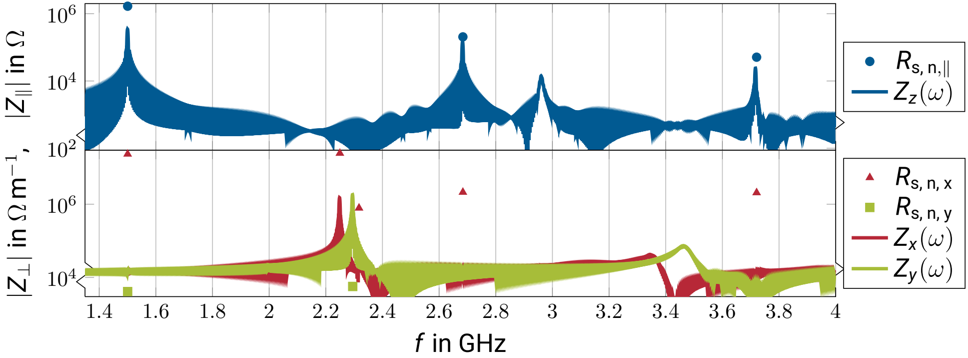


Tabelle: Figures of merit of the single mode cavity variations.

Figure of merit	Unit	Values		
		Elliptical	Rectangular	Ellipsoid
f_1	GHz	1.500 51	1.500 14	1.502 38
Δf	GHz	0.511 97	0.443 18	0.510 17
ε	A^{-1}	3.384	3.261	3.424
Q_0		23 247.3	21 662.1	25 037.2
λ_{TTF}		0.754	0.759	0.694
R_s	MΩ	2.306	4.627	4.391

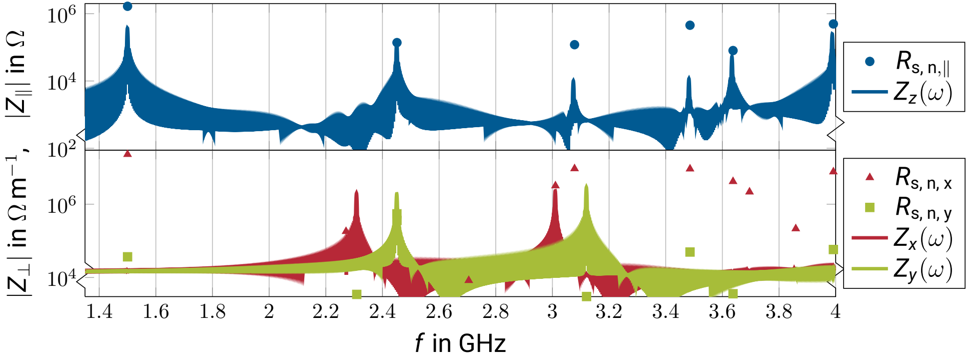
COMPARISON RESULTS

IMPEDANCE OF ELLIPCTICAL VARIANT



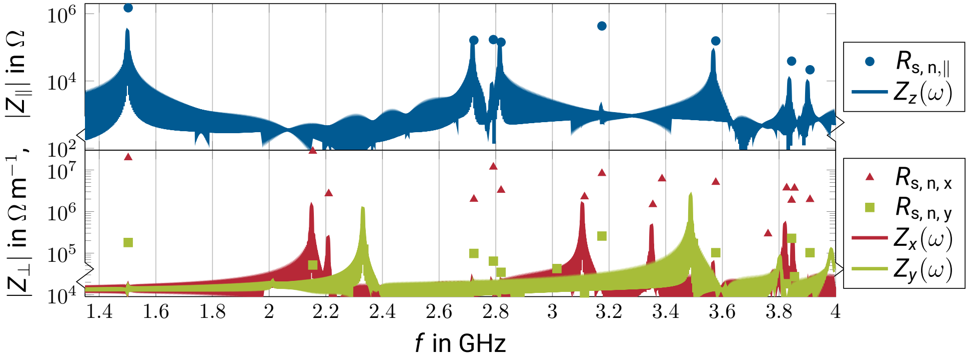
COMPARISON RESULTS

IMPEDANCE OF RECTANGULAR VARIANT



COMPARISON RESULTS

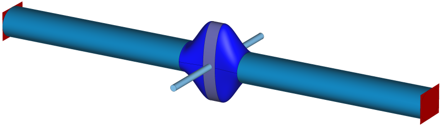
IMPEDANCE OF ELLIPSOID VARIANT



COMPARISON RESULTS

Tabelle: Figures of merit of the single mode cavity variations.

Figure of merit	Values Elliptical	Rectangular	Ellipsoid
ε	3.384	3.261	3.424
Q_0	23 247.3	21 662.1	25 037.2
λ_{TTF}	0.754	0.759	0.694
$n_{\mathrm{HOM},\parallel}$	3	6	8
$n_{\mathrm{HOM},\perp}$	5	9	15



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CONCLUSION & OUTLOOK

Section 4

CONCLUSION & OUTLOOK

- Conclusion
  - The efficiency ε is a good figure of merit for cavities
  - The transverse driving and detuning impedances $Z_{\perp}^{\mathrm{driv}}$ and $Z_{\perp}^{\mathrm{det}}$ can efficiently be calculated from eigenmode results
  - Wakefield simulations are time-consuming if at all possible for resonant structures
  - **The elliptical variant appears to be the best candidate**
- Outlook
  - Addition of a coupler and tuner to the model
  - Increase the beam pipe diameter
  - Find optimal parameter set maximizing the efficiency ε while reducing the number of HOM's