TRANSVERSE SHUNT IMPEDANCE IN THE SINGLE MODE CAVITY

DESY-TEMF Collaboration Meeting 12/2024

Leon Kronshorst, Wolfgang F. O. Müller

Institute for Accelerator Science and Electromagnetic Fields

2024-12-03

TABLE OF CONTENT

MOTIVATION

Section 1

MOTIVATION

UPGRADE TO PETRA IV

Active planning process of the upgrade

PETRA III ⟶ PETRA IV

Goal - 4th generation light source:

- Low emittance
- High beam current
- Long beam lifetime
- Stable particle acceleration and storage

Challenges:

- Toucheck effect
- Intrabeam scattering

Solution - Bunch lengthening:

- Active 3rd harmonic cavity

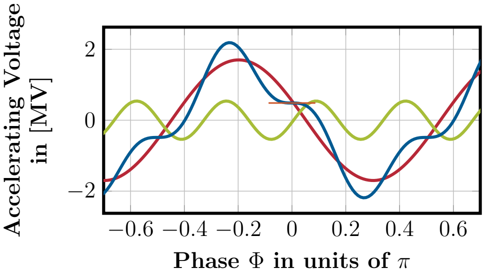
MOTIVATION

ACTIVE 3rd HARMONIC CAVITY

$$V(t) = V_1 \cos(\omega_{RF}\, t + \Phi_1) + V_2 \cos(3\,\omega_{RF}\, t + \Phi_2)$$

Requirements of the 3rd harmonic cavity

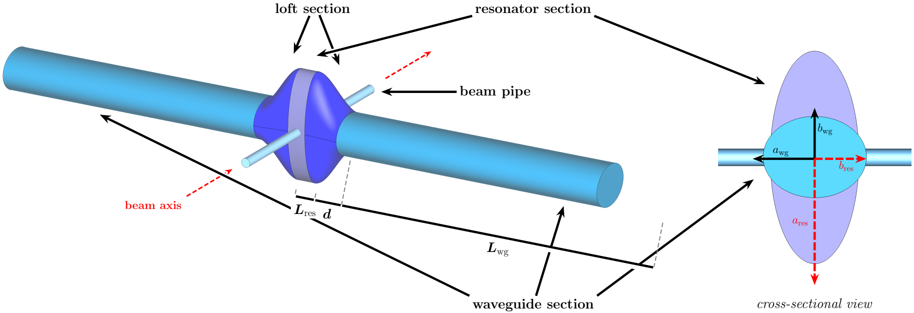
- No phase dependency of the voltage
- Inexpensive and simple manufacturing
- Mitigation of higher order modes (HOM)



THE SINGLE MODE CAVITY

Section 2

THE SINGLE MODE CAVITY



[1]

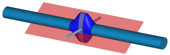
- Resonator Section: resonant frequency, $f_{res} = f_1$
  - Desired accelerating mode resonates around the beam axis
- Waveguide Section: Connected to damper to attenuate HOMs
  - Cutoff frequency between resonant mode and next higher, $f_1 \ll f_c \lesssim f_2$

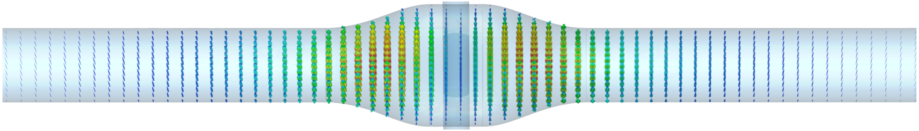
[1] Kronshorst et al.: *Design of a single mode 3rd harmonic cavity for PETRA IV*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-TUPG52

THE SINGLE MODE CAVITY

UNDESIRED HIGHER ORDER MODE

$\mathbf{qTE_{112,even}}$
$f_{13} = 2.2499\,\mathrm{GHz}$
$Q_0 = 259206.4$





- Not all HOM couple to the waveguide section
- These modes have to be studied
  - Either their influence is negligible
  - Or their occurrence has to be suppressed

How to gauge the different transverse modes?

$\Rightarrow$ **Through the kick factor $k_\perp$ and shunt impedance $R_{S,n,\perp}$**

CALCULATION METHODS FOR THE TSI

Section 3

CALCULATION METHODS FOR THE TSI

AND WHAT IT IS

- 3 different approaches to obtain the transverse shunt impedance

frequency domain impedance solver	time domain wakefield solver	frequency domain eigenmode solver

- It gauges the interaction of the particle beam and the cavity wall in transverse direction
- Relation to the kick factor in accordance with [2, 3]
  - $k_\perp = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega\, \underline{\vec{Z}}_\perp(\omega)\, h(\omega,\sigma) = \sum_n k_{n,\perp}$
    - $\underline{\vec{Z}}_\perp(\omega) = \sum_n \frac{\omega_{r,n}}{\omega} \frac{\vec{R}_{\mathrm{S,n},\perp}}{1 + jQ\left(\frac{\omega}{\omega_{r,n}} - \frac{\omega_{r,n}}{\omega}\right)}$

$$\Rightarrow k_{n,\perp} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\omega_{r,n}}{\omega} \frac{\vec{R}_{\mathrm{S,n},\perp}}{1 + jQ\left(\frac{\omega}{\omega_{r,n}} - \frac{\omega_{r,n}}{\omega}\right)} h(\omega,\sigma)$$

[2] Mosnier: *Analyse de la stabilite de faisceau dans un accelerateur lineaire...*, Nucl. Instruments and Methods in Ph. Research, 1987

[3] Zotter, Kheifets: *Impedances and wakes in high-energy particle accelerators*, 2000, World Scientific

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - k_0^2 \underline{\vec{E}} = -jk_0 Z_0 \underline{\vec{J}}(\vec{r}_1^{\perp}, \omega)$$

$$\underline{Z}_{||}(\omega, \vec{r}_2^{\perp}) = -\frac{1}{q_1\, q_2} \int_0^l \mathrm{d}z \underline{\vec{E}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, \omega) \cdot \underline{\vec{J}}_s^*(\vec{r}_2^{\perp}) \qquad [5]$$

Panofsky-Wenzel theorem

$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\perp}) = \frac{c}{\omega_{r,n}} \nabla_{\perp} \underline{Z}_{||}(\omega_{r,n}, \vec{r}_2^{\perp})$$

[5] Quetscher, Gjonaj: *Impedance computation for large accelerator strucures using a domain decomposition method*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-THPC62

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - k_0^2 \underline{\vec{E}} = -j k_0 Z_0 \underline{\vec{J}}(\vec{r}_1^{\,\perp}, \omega)$$

$$\underline{Z}_{\parallel}(\omega, \vec{r}_2^{\,\perp}) = -\frac{1}{q_1\, q_2} \int_0^l \mathrm{d}z\, \underline{\vec{E}}(\vec{r}_1^{\,\perp}, \vec{r}_2^{\,\perp}, z, \omega) \cdot \underline{\vec{J}}_s^{*}(\vec{r}_2^{\,\perp}) \qquad [5]$$

not implemented in CST

Panofsky-Wenzel theorem

$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\,\perp}) = \frac{c}{\omega_{r,n}} \nabla_{\perp} \underline{Z}_{\parallel}(\omega_{r,n}, \vec{r}_2^{\,\perp})$$

[5] Quetscher, Gjonaj: *Impedance computation for large accelerator strucures using a domain decomposition method*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-THPC62

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

to solve

$$\frac{\partial}{\partial t}\vec{B} = -\nabla \times \vec{E}$$
$$\frac{\partial}{\partial t}\vec{E} = \frac{1}{c^2}\nabla \times \vec{B} - \frac{1}{\varepsilon_0}\vec{J}(\vec{r}_1^{\perp}, t)$$

$$\vec{w}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, s) = \frac{1}{q}\int_{-\infty}^{\infty} \mathrm{d}z \left[\vec{E}\left(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, t\right) + c\vec{e}_z \times \vec{B}\left(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, t\right)\right] \quad [6]$$

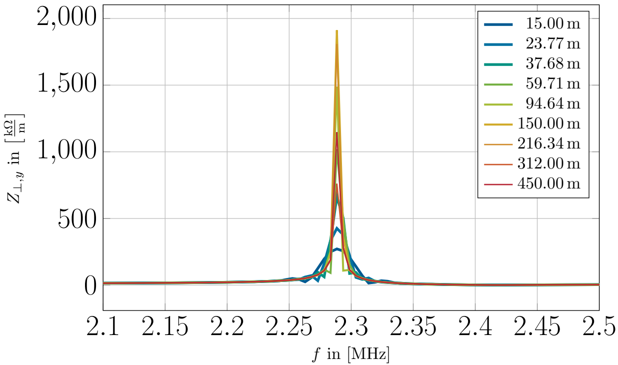
Fourier transform

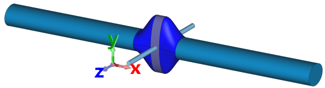
$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\perp}) = \mathcal{F}\{\vec{w}_{\perp}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, s)\}$$

[6] Weiland, Wanzenberg: *Wake fields and impedances*, Frontiers of Particle Beams 1992, 10.1007/3-540-55250-2_26

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver | **time domain wakefield solver** | frequency domain eigenmode solver





CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - \underline{\vec{k}}^2 \underline{\vec{E}} = 0$$

$$\underline{V}_{\mathrm{acc},\parallel} = \int_0^l \mathrm{d}z \underline{\vec{E}}_n \, e^{-j\,\omega_{r,n}\frac{z}{c}}$$

$$R_{\mathrm{S,n},\parallel} = \frac{|\underline{V}_{\mathrm{acc}\,\parallel}|^2}{2P_d}$$

Panofsky-Wenzel

$$\underline{E}_z \approx \frac{\partial}{\partial r}\underline{E}_z\, r$$

$$\vec{R}_{\mathrm{S,n},\perp} = \frac{|\nabla_\perp \underline{V}_{\mathrm{acc},\parallel}|^2}{2\,k\,P_d} \qquad [2]$$

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - \underline{\vec{k}}^2 \underline{\vec{E}} = 0$$

$$\underline{V}_{\mathrm{acc},\|} = \int_0^l \mathrm{d}z \underline{\vec{E}}_n \, e^{-j\,\omega_{r,n}\frac{z}{c}}$$

$$R_{\mathrm{S,n},\|} = \frac{|\underline{V}_{\mathrm{acc}\,\|}|^2}{2P_d}$$

Panofsky-Wenzel

$$\underline{E}_z \approx \frac{\partial}{\partial r} \underline{E}_z \, r$$

$$\vec{R}_{\mathrm{S,n},\perp} = \frac{|\nabla_\perp \underline{V}_{\mathrm{acc},\|}|^2}{2\,k\,P_d} \qquad [2]$$

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

frequency domain eigenmode solver

$$\nabla \times \nabla \times \underline{\vec{E}} - \vec{k}^2 \underline{\vec{E}} = 0$$

$$\underline{V}_{\mathrm{acc},\parallel} = \int_0^l \mathrm{d}z\, \underline{\vec{E}}_n\, e^{-j\,\omega_{r,n} \frac{z}{c}}$$

$$\vec{R}_{\mathrm{S,n},\perp} = \frac{|\nabla_\perp \underline{V}_{\mathrm{acc},\parallel}|^2}{2\,k\,P_d} \qquad [2]$$

beam excitation: $\underline{\vec{J}}(\vec{r}_\perp, z, \omega_{r,n}) = I_0 \delta(\vec{r}_\perp) e^{j k_n z} \vec{e}_z$

Poynting's theorem: $P_n = \int \mathrm{d}V\, \underline{\vec{J}} \cdot \hat{\underline{\vec{E}}}$

steady state $\Rightarrow P_n = \hat{P}_{d,n} = |a_n|^2\, P_{d,n}$

$$a_n = \frac{I_0}{P_{d,n}} \underline{V}^*_{\mathrm{acc}\parallel} \qquad [7]$$

$$\underline{V}_{\mathrm{acc}\perp} = \int_0^l \mathrm{d}z e^{j k_n z} \left[\underline{\vec{E}} + c\vec{e}_z \times \underline{\vec{B}}\right]_\perp$$

$$\vec{R}_{\mathrm{S,n},\perp} = |\underline{\vec{Z}}_\perp(\omega_{r,n})| = \frac{|\hat{\underline{V}}_{\mathrm{acc},\perp}|}{I_0} = \frac{1}{\vec{r}_\perp} \frac{|\underline{V}^*_{\mathrm{acc}\parallel}|\,|\underline{V}_{\mathrm{acc}\perp}|}{2\,P_{d,n}}$$

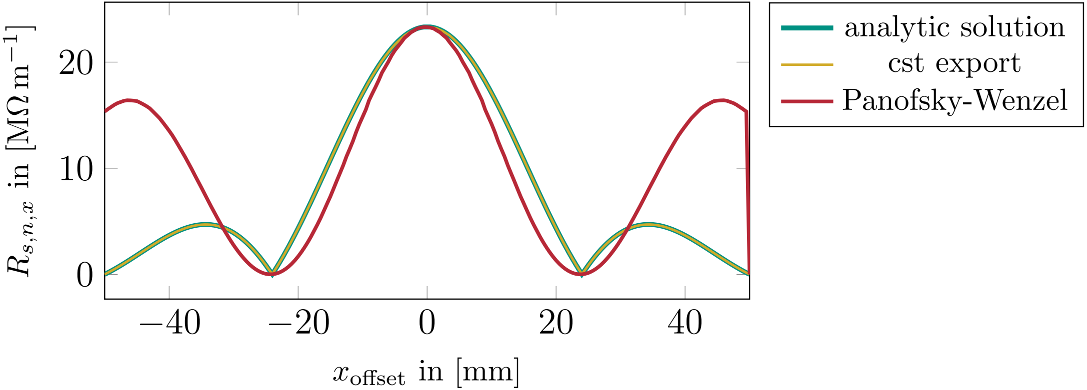
[7] Quetscher, Gjonaj: *unpublished*

EVALUATION OF THE EM ANSATZ

Section 4

EVALUATION OF THE EM ANSATZ

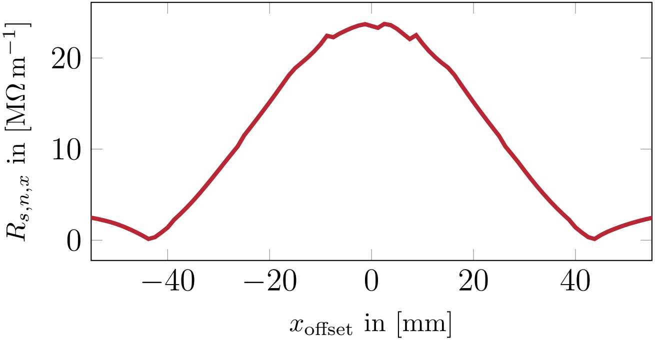
FOR THE TM_{110}-MODE



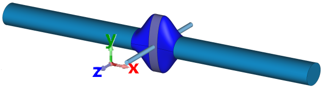
APPLICATION TO THE SINGLE MODE CAVITY

Section 5

APPLICATION TO THE SINGLE MODE CAVITY



- $\mathrm{qTE}_{112,\mathrm{even}}$-Mode
- $f_{13} = 2.2499\,\mathrm{GHz}$



INTEGRATION INTO THE SIMULATION WORKFLOW

Section 6

THE SIMULATION WORKFLOW

THE CONTEMPLATED WORKFLOW

Repeat the following steps until $k_\parallel, k_\perp$ is below the threshold

Step 1: Run wakefield simulation

$\tilde{Z}_\parallel, \tilde{Z}_{\perp,x}, \tilde{Z}_{\perp,y}$

Step 2: Identify frequencies of peaks in coupling impedances

$\tilde{f}_n$

Step 3: Run eigenmode simulation at frequencies of peaks

$\underline{\vec{E}}_n, \underline{\vec{H}}_n$

Step 4: Calculate the modal shunt impedances using the eigenmode ansatz

$R_{s,n,\parallel}, R_{s,n,\perp,x}, R_{s,n,\perp,y}$

Step 5: Calculate the kick and the loss factor from all shunt impedances

THE SIMULATION WORKFLOW

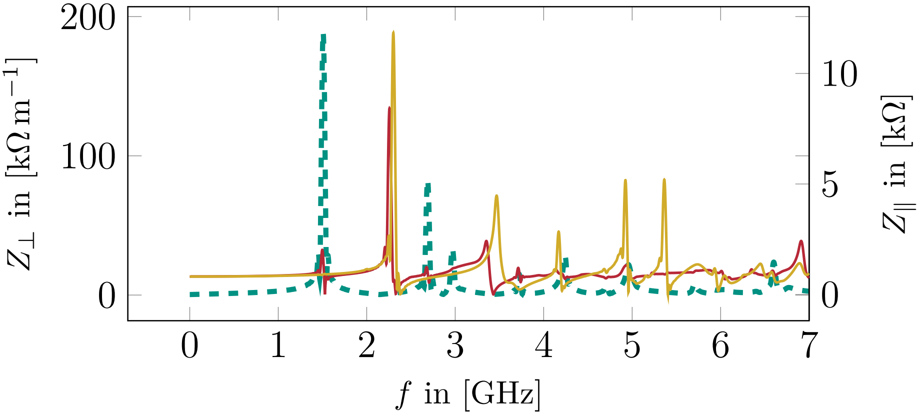
THE REASONING AND ITS ARISING PROBLEMS

- Reasoning
  - Mode frequencies are changed by geometry changes
    - → The wakefield simulation can deliver the frequency of problematic modes
  - Value of coupling impedance is wrong for resonant structures
    - → Eigenmodesimulations were developed to calculate such structures

- Problems
  1. The wakefield simulation seems to find none relevant modes
  2. Some modes seem to be missed even if excited of axis
  3. Long simulation times since cutoff frequency of beam pipe is so high

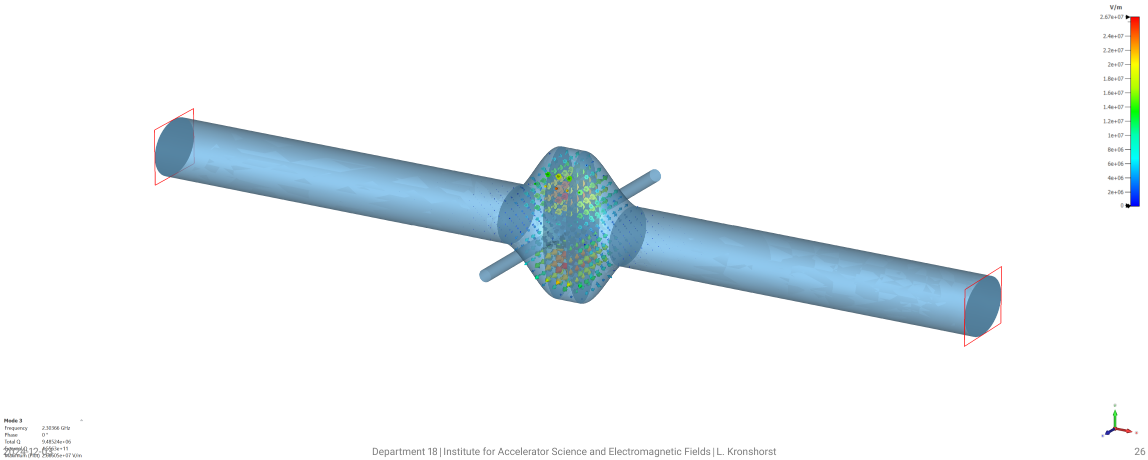
THE SIMULATION WORKFLOW

TESTING WORKFLOW



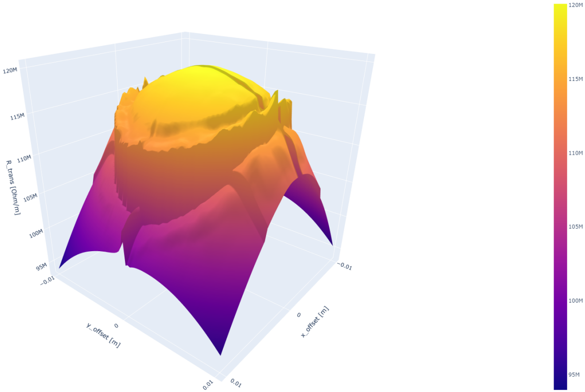
THE SIMULATION WORKFLOW

FOUND PEAK $f = 2.3038$ GHz



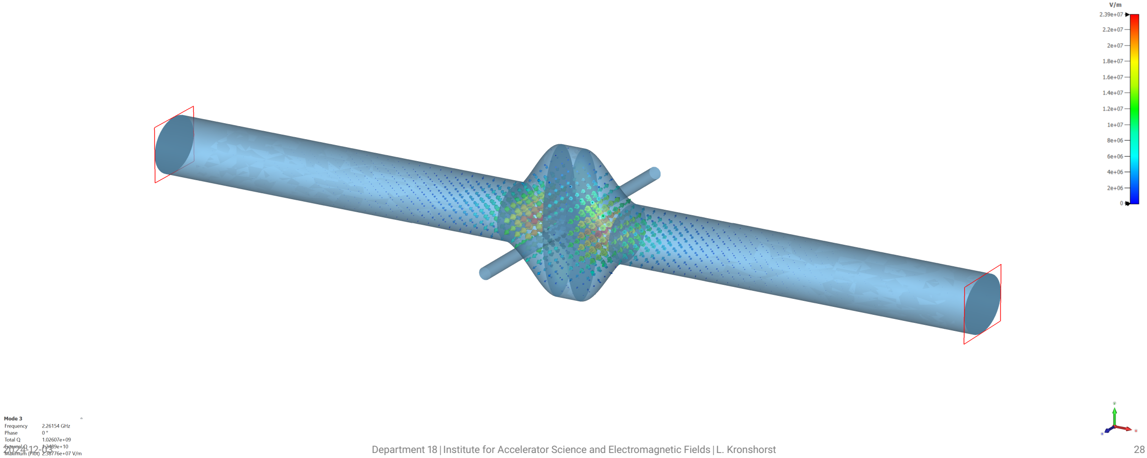
THE SIMULATION WORKFLOW

FOUND PEAK $f = 2.3038\,\mathrm{GHz}$



THE SIMULATION WORKFLOW

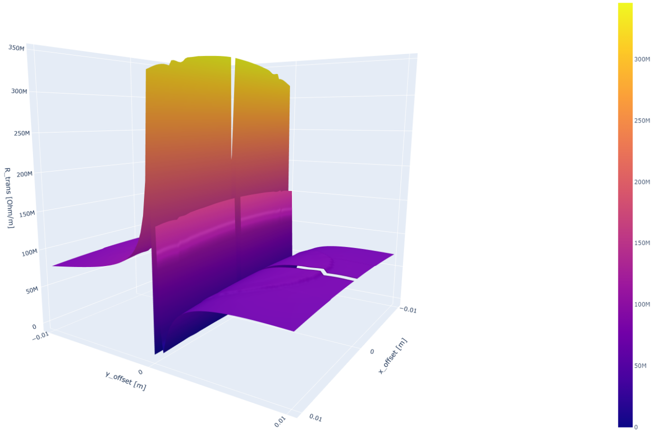
FOUND PEAK $f = 2.2616$ GHz



THE SIMULATION WORKFLOW

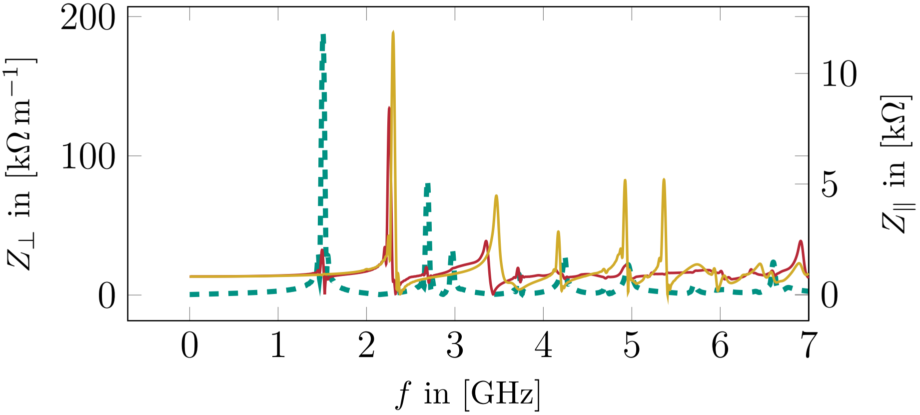
FOUND PEAK $f = 2.2616$ GHz

- The non-linearity at $x = 0$ could mean a focussing transverse influence as mentioned in [3] for elliptic cavities with round beampipes.



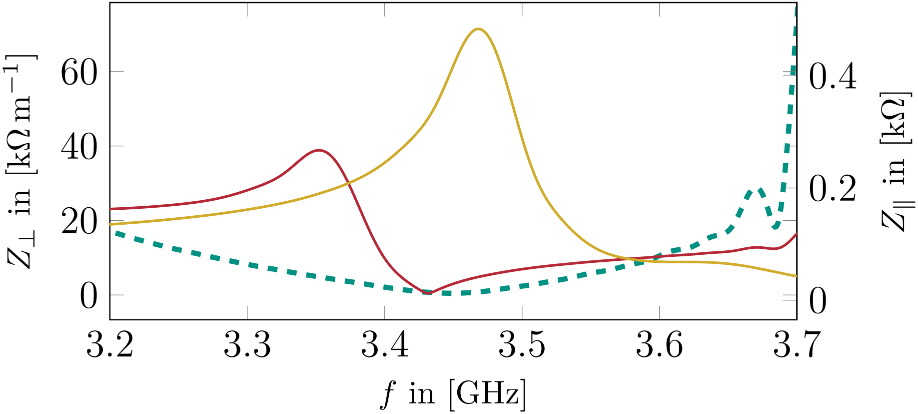
THE SIMULATION WORKFLOW

PROBLEM 1.: NONRELEVANT FREQUENCIES



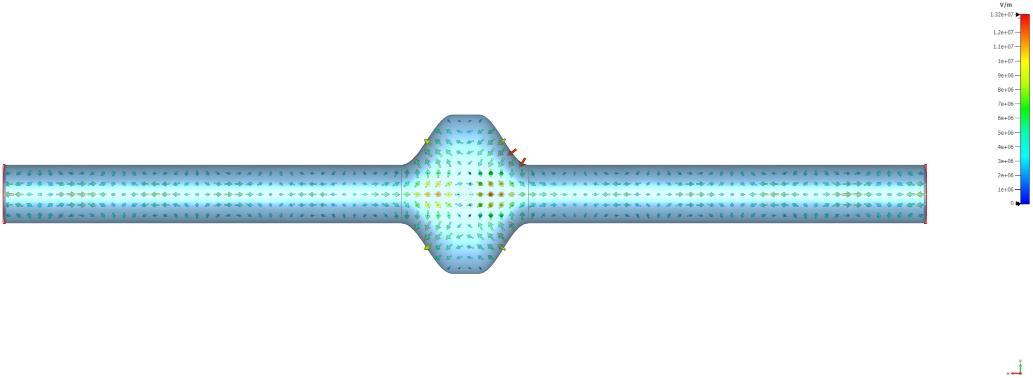
THE SIMULATION WORKFLOW

PROBLEM 1.: NONRELEVANT FREQUENCIES



THE SIMULATION WORKFLOW

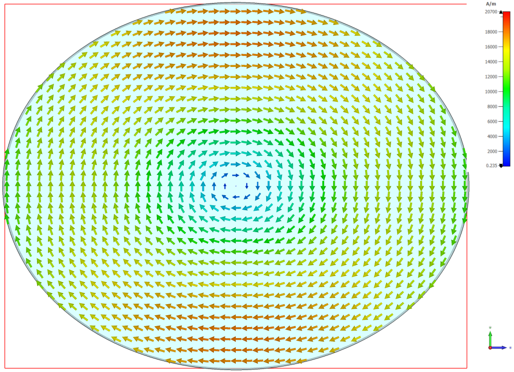
PROBLEM 1.: NONRELEVANT FREQUENCIES



THE SIMULATION WORKFLOW

PROBLEM 1.: NONRELEVANT FREQUENCIES

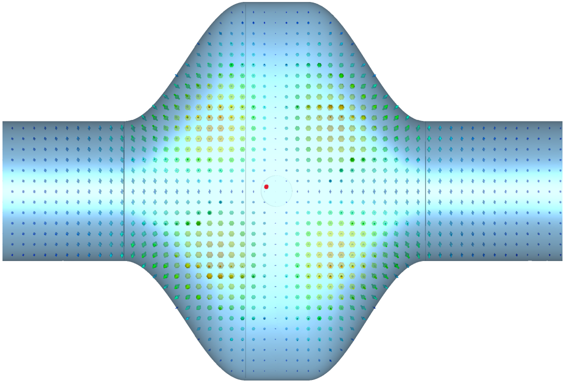
- The mode seems to resonate in cavity section, although it should propagate and leave the cavity out of the port
- The mode in the waveguide section is clearly a $\mathbf{TM}_{01,odd}$



THE SIMULATION WORKFLOW

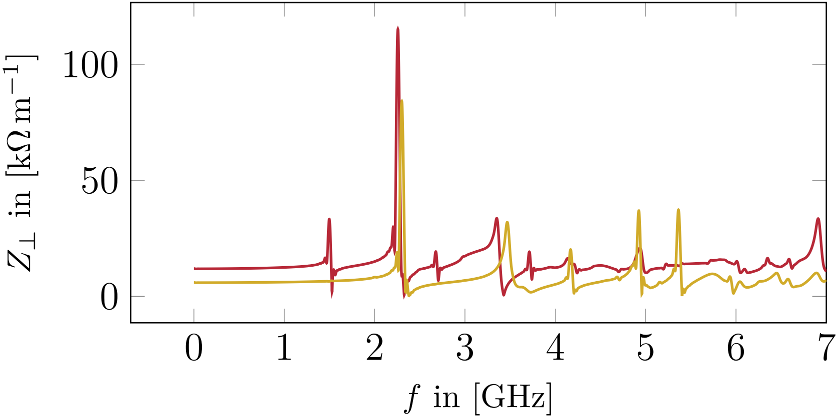
PROBLEM 2.: NOT FOUND FREQUENCIES

- At 3.2231 GHz a mode exsist which is not excited on axis
- Next slide a wakefield simulation with exciting current at $x = 4\,\text{mm}$ and $y = 2\,\text{mm}$



THE SIMULATION WORKFLOW

PROBLEM 2.: NOT FOUND FREQUENCIES



CONCLUSION/OUTLOOK

Section 7

CONCLUSION/OUTLOOK

- Conclusion
  - The eigenmode ansatz to calculate the shunt impedances is implemented.
  - Simulation and post-processing is mostly automated and can be quickly adjusted.
- Outlook
  - Investigation of shunt impedance results
    - Special attention on possible focussing of transverse modes
  - Find a robust and fast way to determine problematic mode
  - **Use improved workflow to gauge HOM of cavity and further optimize it**