NONLINEAR SIMULATIONS OF THE FAST CORRECTOR MAGNETS FOR PETRA IV

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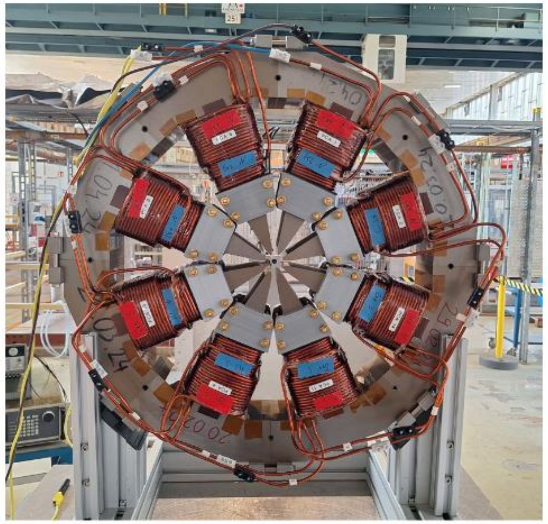
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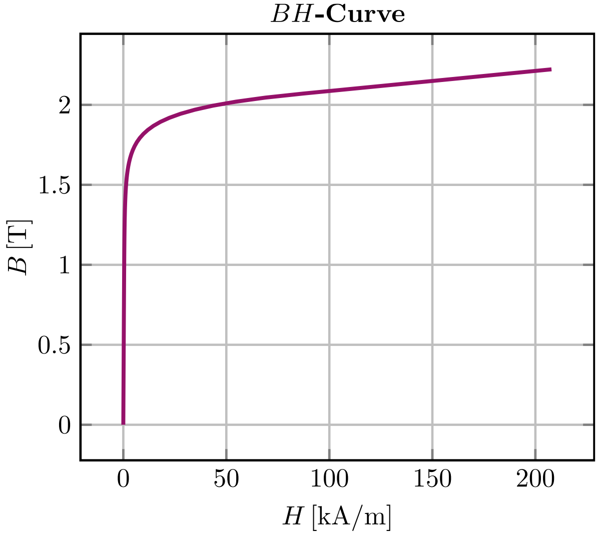

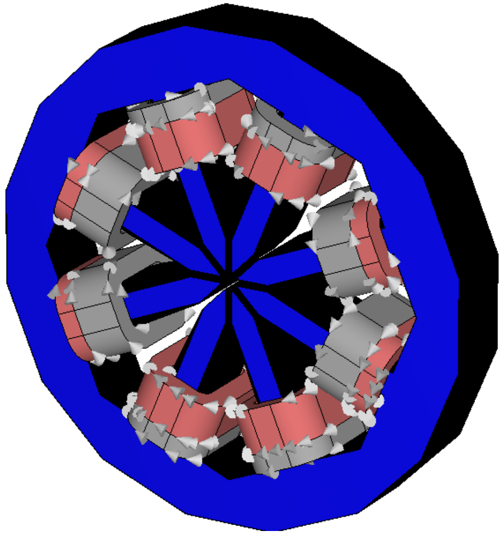

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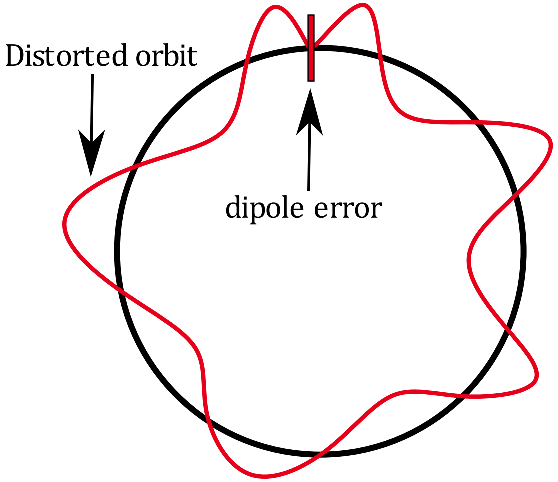
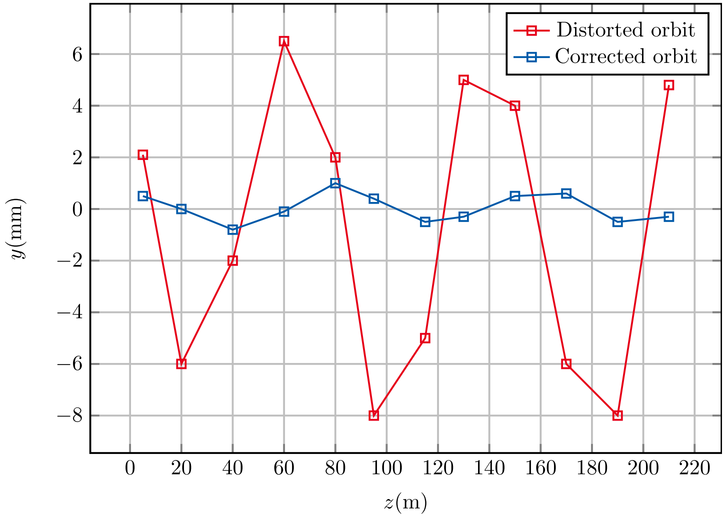
MOTIVATION

- Circular accelerators need dipole magnets to correct orbit distortions
- **PETRA IV**: ultra-low emittance synchrotron radiation source

➔ Fast orbit feedback system, **corrector magnets with frequencies in kHz range**
- **Strong eddy currents ➔** power losses, time delay, and field distortion
- **Simulation challenging** due to small skin depths and laminated yoke

➔ Need for technique to simplify simulations





HOMOGENIZATION

- Magnetoquasistatic PDE: $\nabla \times (\nu(\vec{r}\,)\nabla \times \underline{\vec{A}}\,(\vec{r})) + j\omega\sigma(\vec{r})\underline{\vec{A}}(\vec{r}) = \underline{\vec{J}}_s(\vec{r})$
- Replace reluctivity $\nu(\vec{r})$ and conductivity $\sigma(\vec{r})$ in the laminated yoke with spatially constant tensors

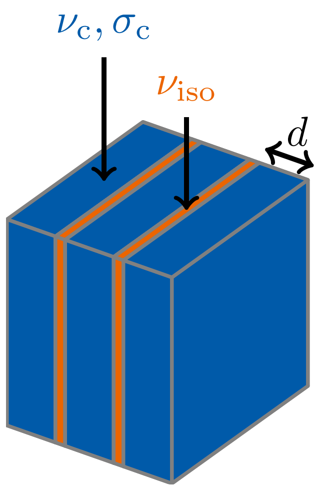
$$\nu(\vec{r}) \rightarrow \underline{\bar{\bar{\nu}}} = \frac{1}{8}\sigma_c d\delta\omega(1+j)\frac{\sinh((1+j)\delta^{-1}d)}{\sinh^2\big((1+j)\delta^{-1}\,d/2\big)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix} + \nu_c\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

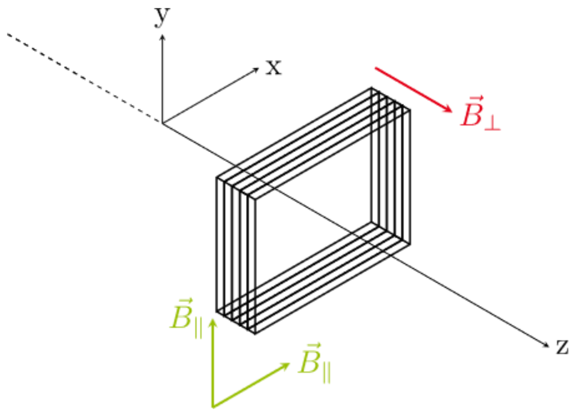
$$\sigma(\vec{r}) \rightarrow \bar{\bar{\sigma}} = \gamma\sigma_c\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$$

Skin depth $\delta = \sqrt{2/\omega\sigma_c\mu_c}$

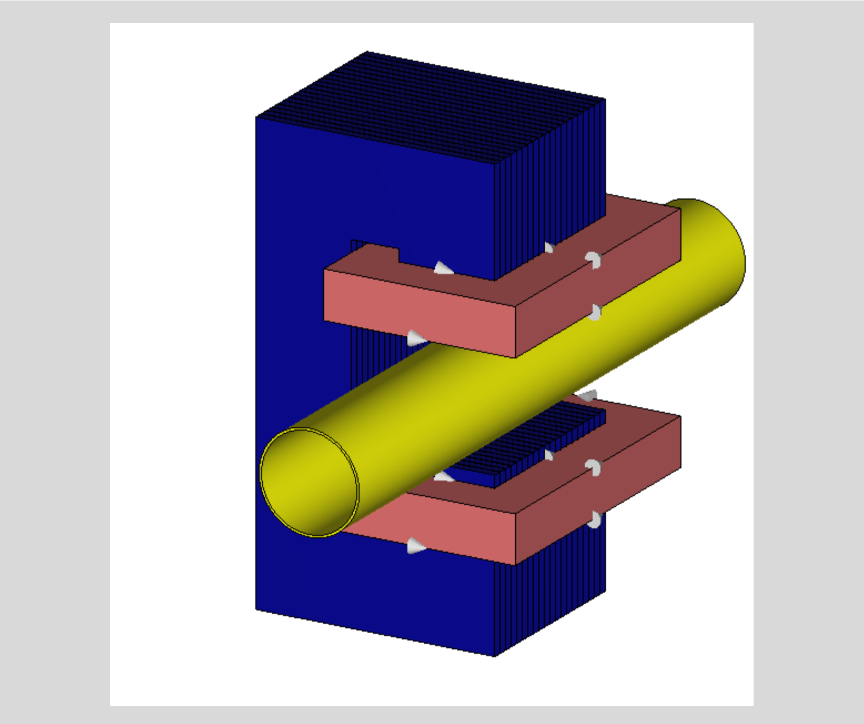
Stacking factor $\gamma = \dfrac{V_c}{V_{\text{Yoke}}}$

P. Dular et al., 2003
L. Krählenbühl et al., 2004
H. De Gersem et al., 2012

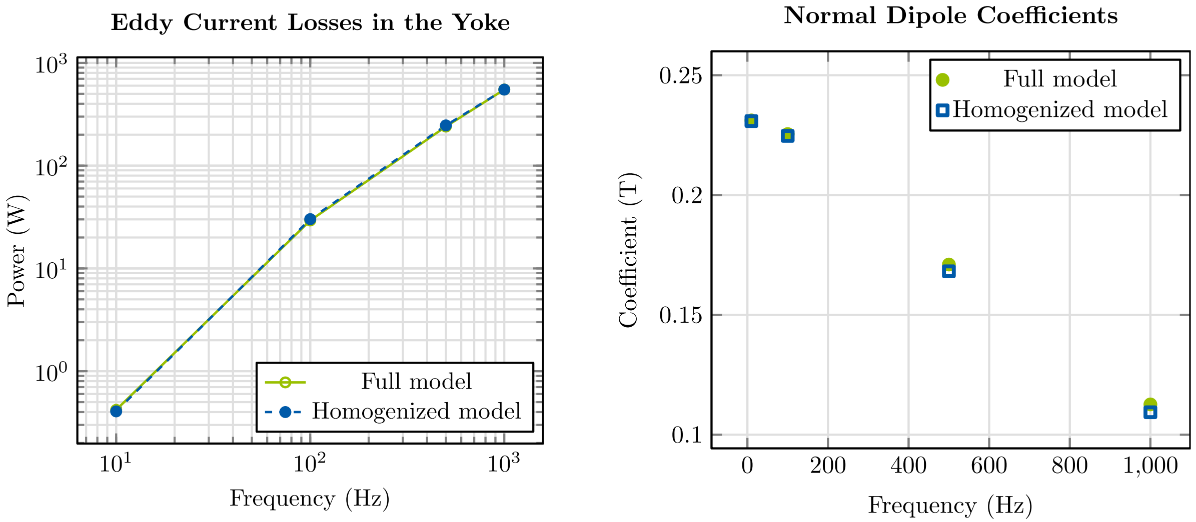




HOMOGENIZATION



VERIFICATION STUDY

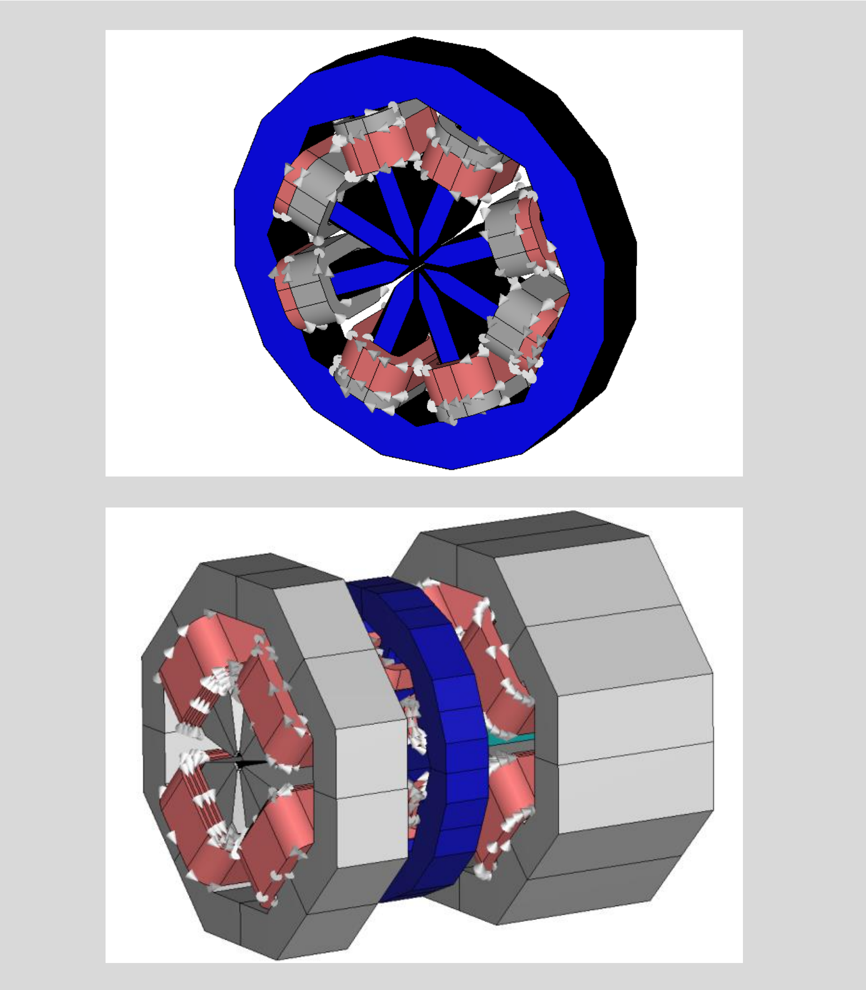


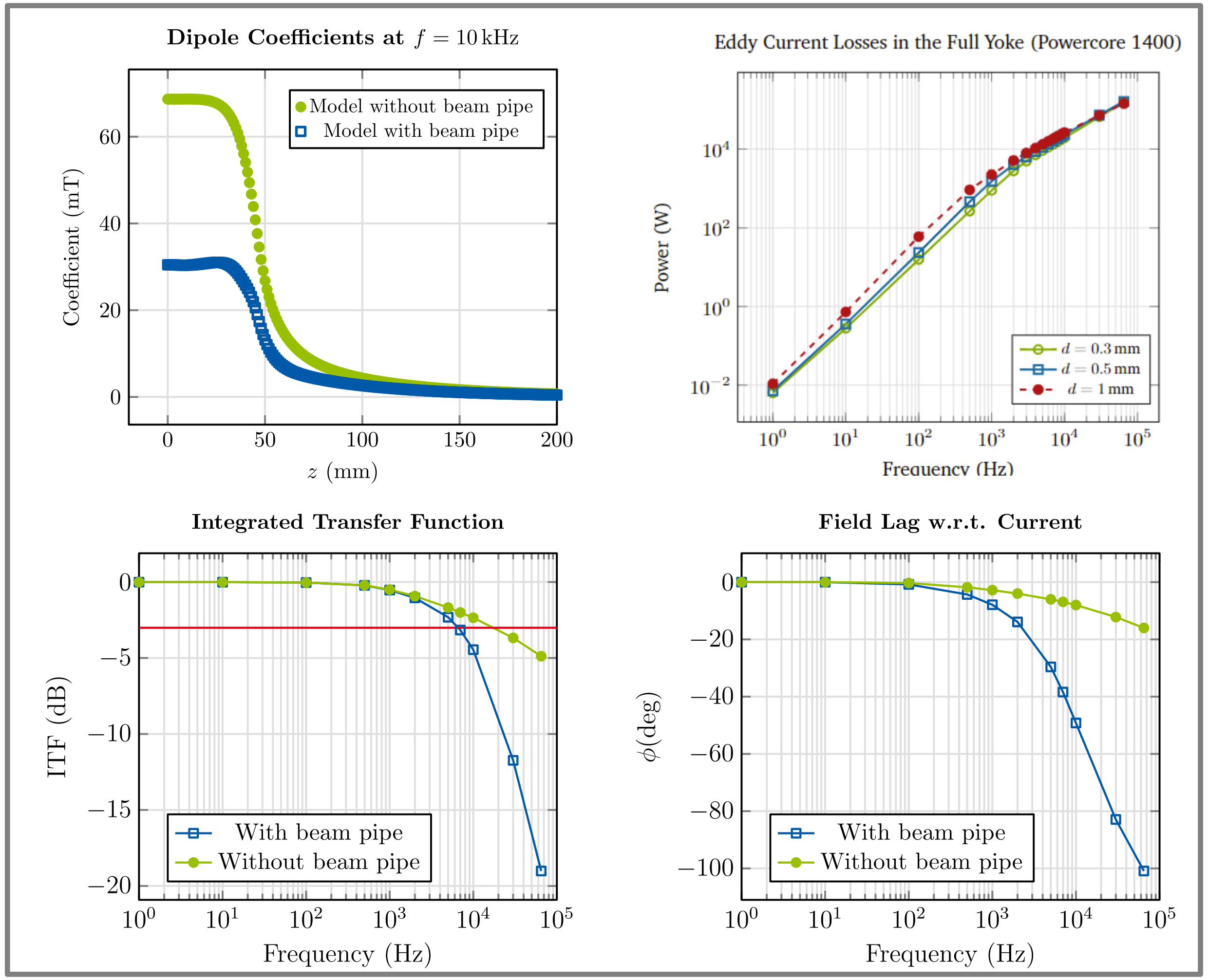
- Eddy current losses and aperture field well approximated

- **Simulation time reduced from several hours to just a few minutes!**

➔ After comparing to other techniques, we decided to **use this technique to simulate the corrector magnets**

LINEAR SIMULATION STUDIES





CONTENTS

WITHOUT DC BIAS

- To incorporate **non-linear $\boldsymbol{BH}$-curves** into simulations: **combine homogenization technique and harmonic balance FEM (HBFEM)**
- HBFEM is a technique to approximate periodic solutions of nonlinear transient PDEs in frequency domain
- Example: excitation current with 1st and 3rd harmonic, include field quantities up to 3rd harmonic

$$\nabla \times (\nu(t)\nabla \times \vec{A}(t)) + \sigma \frac{\partial \vec{A}(t)}{\partial t} = \vec{J}_s(t)$$

$$\nabla \times (\underline{\nu}(\omega) \circledast \nabla \times \underline{\vec{A}}(\omega)) + j\omega\sigma\underline{\vec{A}}(\omega) = \underline{\vec{J}}_s(\omega)$$

$$\xrightarrow{\text{+ Homogenization}}$$

$$\begin{bmatrix} K_{\underline{\overline{\overline{\nu}}}_0(3\omega_\mathrm{f})} + 3j\omega_\mathrm{f}M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 & 0 \\ K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_\mathrm{f})} + j\omega_\mathrm{f}M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 \\ 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_\mathrm{f})} - j\omega_\mathrm{f}M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} \\ 0 & 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(3\omega_\mathrm{f})} - 3j\omega_\mathrm{f}M_{\overline{\overline{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 \\ \underline{j}_1 \\ \underline{j}_{-1} \\ \underline{j}_{-3} \end{bmatrix}$$

S. Yamada and K. Bessho (1988)
H. De Gersem, H. Vande Sande, K. Hameyer (2001)

WITHOUT DC BIAS

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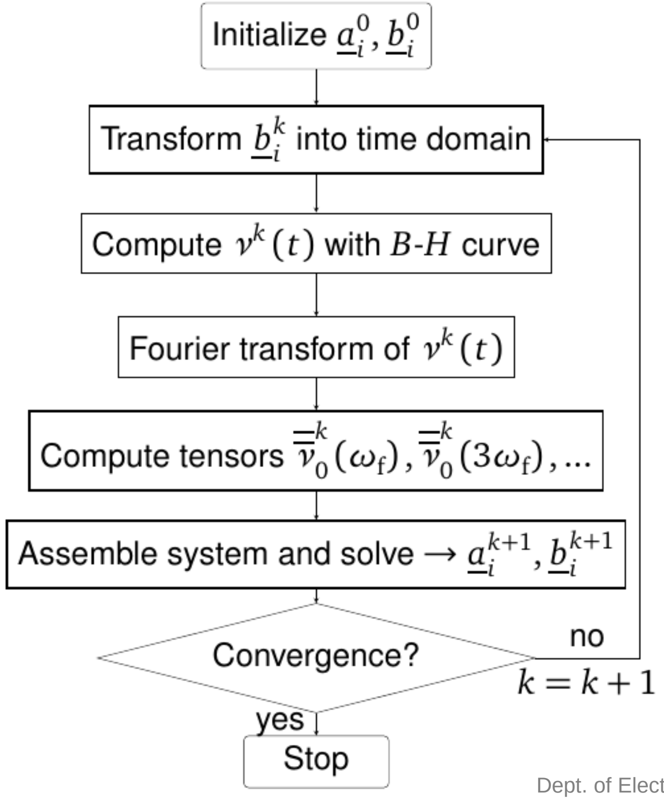
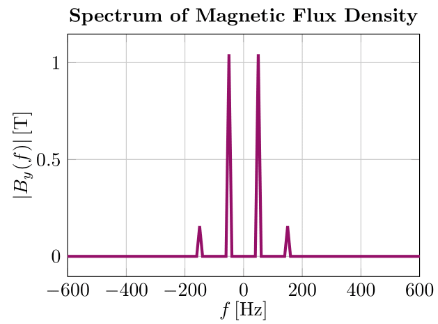
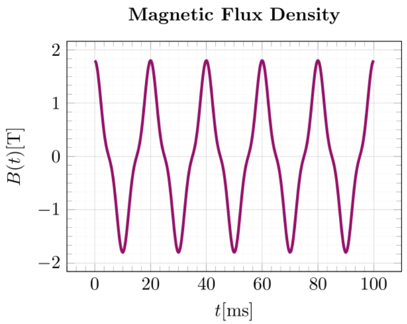
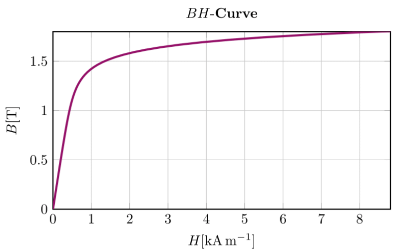
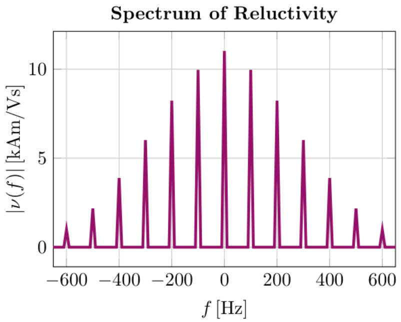
$$\begin{bmatrix} K_{\underline{\overline{\overline{\nu}}}_0(3\omega_{\mathrm{f}})} + 3j\omega_{\mathrm{f}}M_{\overline{\overline{\sigma}}} & 0 & 0 & 0 \\ 0 & K_{\underline{\overline{\overline{\nu}}}_0(\omega_{\mathrm{f}})} + j\omega_{\mathrm{f}}M_{\overline{\overline{\sigma}}} & 0 & 0 \\ 0 & 0 & K_{\underline{\overline{\overline{\nu}}}_0(\omega_{\mathrm{f}})} - j\omega_{\mathrm{f}}M_{\overline{\overline{\sigma}}} & 0 \\ 0 & 0 & 0 & K_{\underline{\overline{\overline{\nu}}}_0(3\omega_{\mathrm{f}})} - 3j\omega_{\mathrm{f}}M_{\overline{\overline{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 - K_{\underline{\nu}_2}\underline{a}_1 \\ \underline{j}_1 - K_{\underline{\nu}_{-2}}\underline{a}_3 - K_{\underline{\nu}_2}\underline{a}_{-1} \\ \underline{j}_{-1} - K_{\underline{\nu}_{-2}}\underline{a}_1 - K_{\underline{\nu}_2}\underline{a}_{-3} \\ \underline{j}_{-3} - K_{\underline{\nu}_{-2}}\underline{a}_{-1} \end{bmatrix}$$

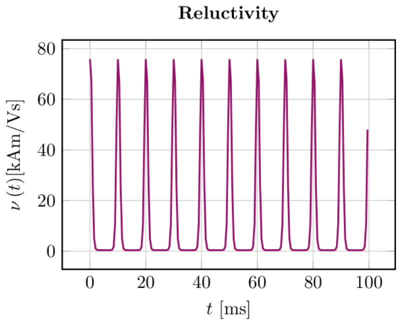












THEORY

WITH DC BIAS

- Current signal of corrector magnet: DC current + oscillations ➔ modify HBFEM method to include DC bias
- Again, we combine HBFEM with a homogenization technique

$$\nabla \times \left(\underline{\nu}(\omega) \circledast \nabla \times \underline{\vec{A}}\,(\omega) \right) + j\omega\sigma \underline{\vec{A}}(\omega) = \underline{\vec{J}_s}(\omega) \;\; \Rightarrow \nabla \times \left(\underline{\nu_d}(\omega) \circledast \nabla \times \underline{\vec{A}}\,(\omega) \right) + j\omega\sigma \underline{\vec{A}}(\omega) = \underline{\vec{J}_s}(\omega) - \nabla \times \underline{\vec{H}_c}\,(\omega)$$

chord reluctivity

differential reluctivity

magnetizing field strength

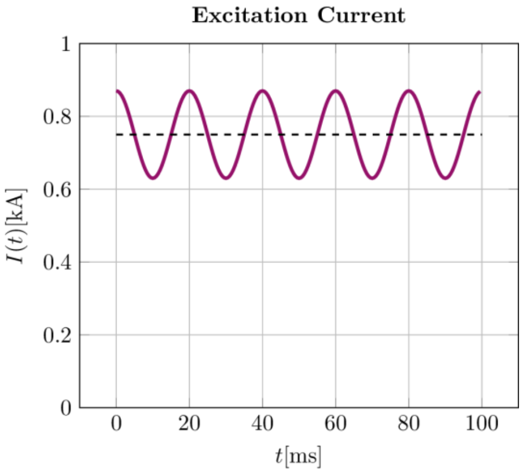
HOMOGENIZATION

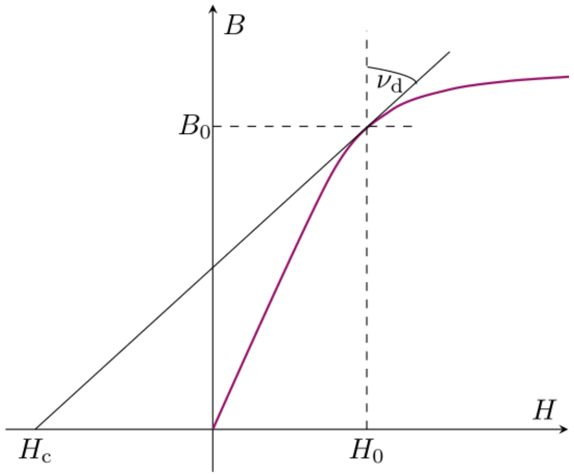
$$\nabla \times (\bar{\bar{\nu}} \nabla \times \vec{A}) + \nabla \times \left(\bar{\bar{\xi}} \nabla \times \frac{\partial \vec{A}}{\partial t} \right) + \bar{\bar{\sigma}} \frac{\partial \vec{A}}{\partial t} = \vec{J}_s$$

$$\bar{\bar{\nu}} = \frac{1}{\dfrac{\gamma}{\nu_{\mathrm{Fe}}} + \dfrac{1-\gamma}{\nu_{\mathrm{Iso}}}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (\nu_{\mathrm{Fe}}\gamma + \nu_{\mathrm{Iso}}(1-\gamma)) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{\xi}} = \frac{1}{12} \sigma_{\mathrm{Fe}} d^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

J. Gyselinck et al., 1999





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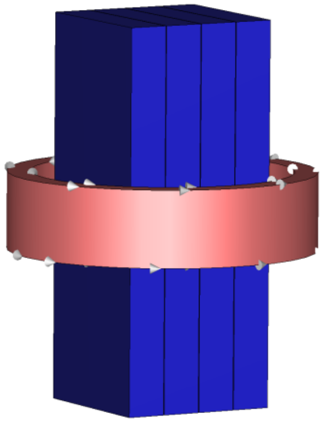
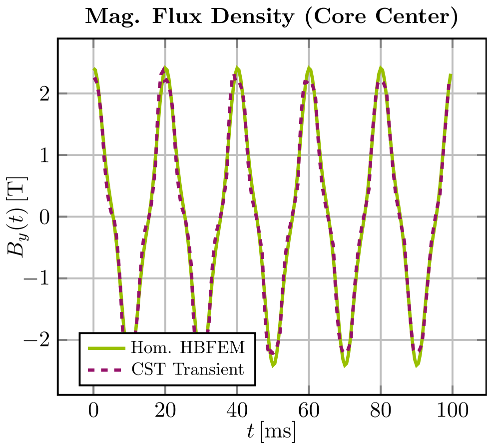
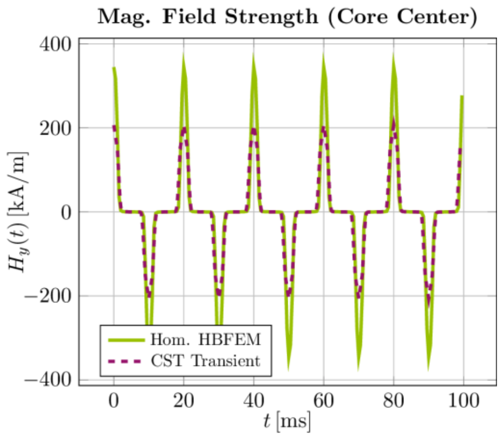
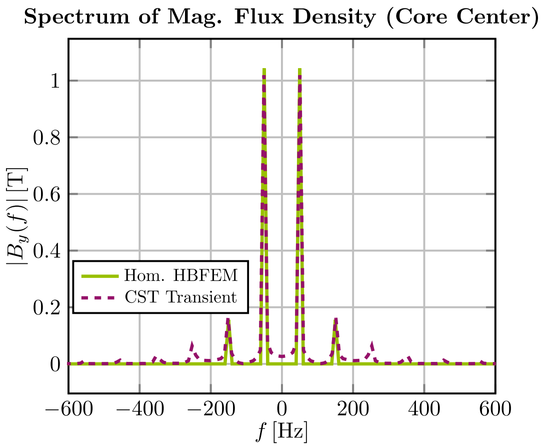
TOY MODEL WITHOUT DC BIAS





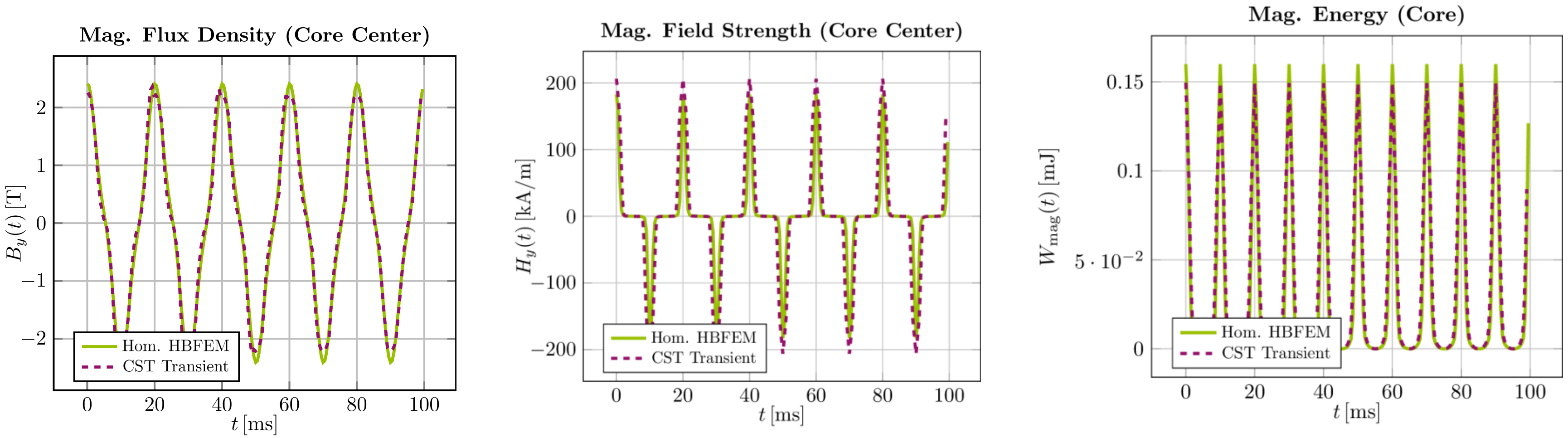




- Simple inductor with laminated core, excitation current: $I_\mathrm{s}(t) = 1.5\ \mathrm{kA}\cos(2\pi 50\mathrm{Hz}\ t) + 0.24\ \mathrm{kA}\cos(2\pi 150\mathrm{Hz}\ t)$
- Compare results of HBFEM + homogenization (GetDP + Python) to transient CST simulation with individually resolved laminations

➔ Good agreement in magnetic flux density
➔ Larger differences in magnetic field strength
➔ **Suspicion: differences in magnetic field strength are due to not having included enough harmonics**

TOY MODEL WITHOUT DC BIAS



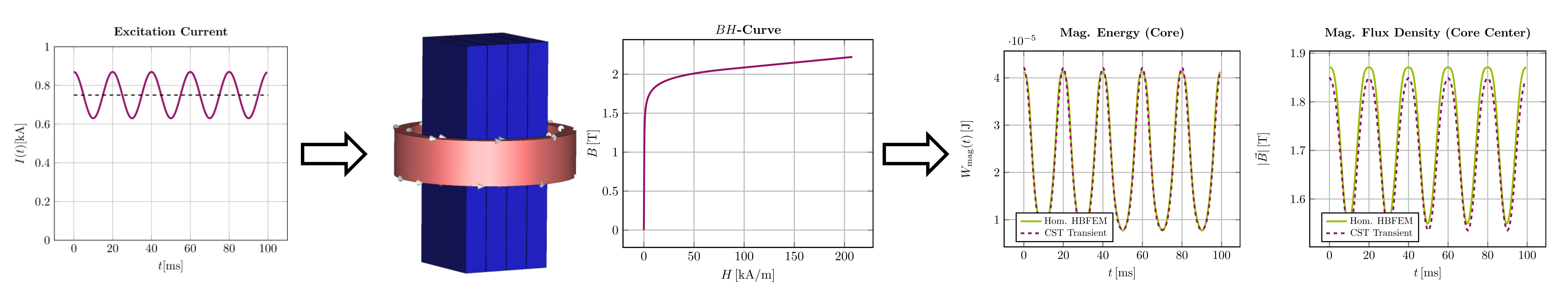
- Include 5th harmonic in the analysis

➔ Still good agreement in magnetic flux density, large differences in magnetic field strength vanish

➔ Decent agreement in magnetic energy

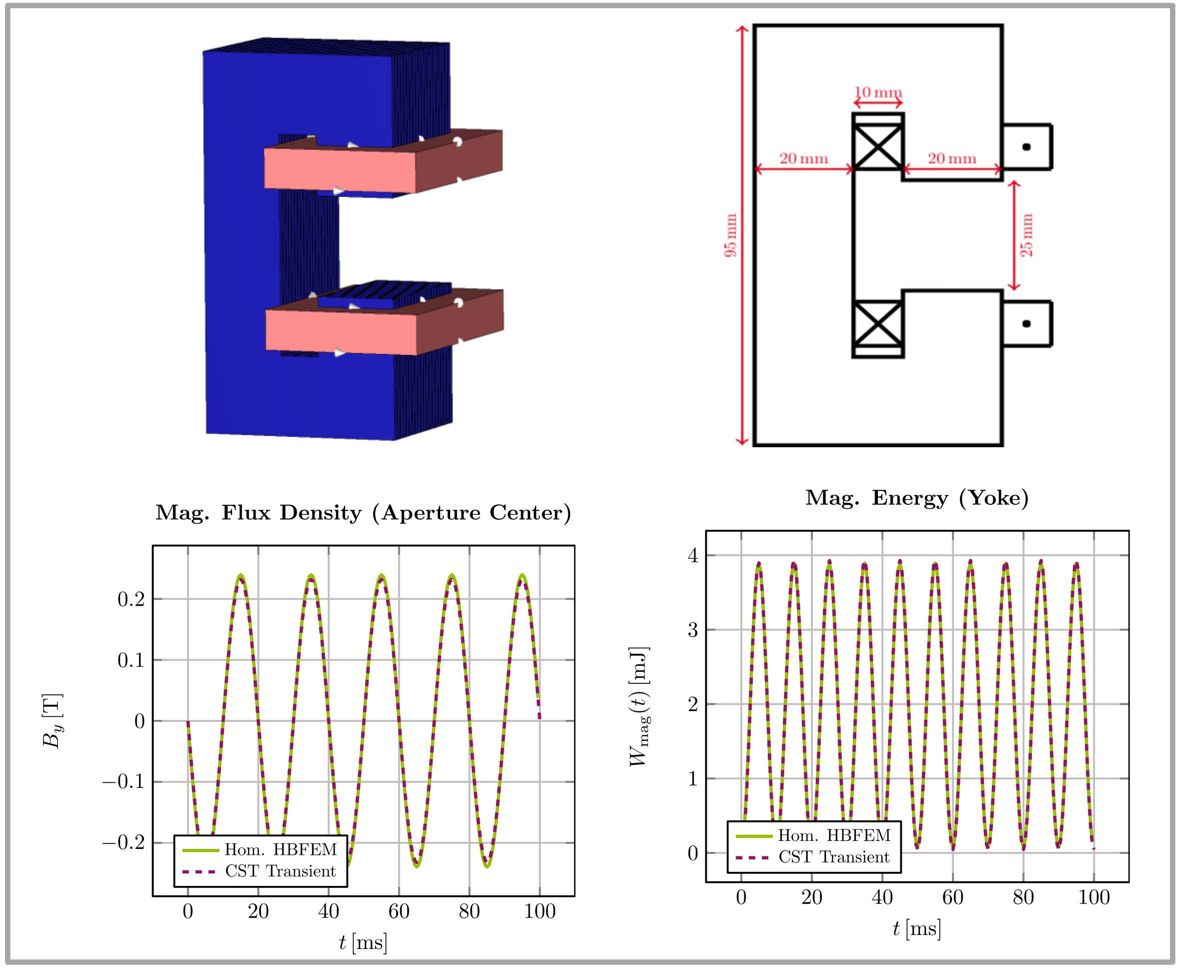
TOY MODEL WITH DC BIAS

- Excitation current: $I_s(t) = 750\text{A} + 120\text{A}\cos(2\pi 50\text{Hz}\; t)$
- Comparison to transient CST simulation of toy model:
  - Very good **agreement in magnetic energy** in the core
  - Decent **agreement in magnetic flux densities** at individual points inside the core (average rel. error 3.7 %)



C-DIPOLE WITHOUT DC BIAS

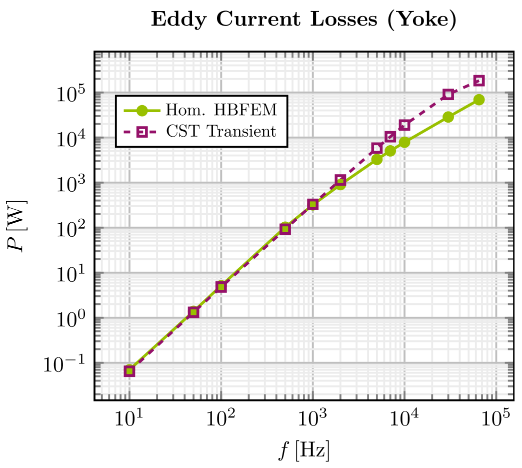
- Same magnet as before, lam. thickness $d = 0.5$ mm
- Excitation current for both coils:
  $$I_{\mathrm{s}}(t) = 2.5\ \mathrm{kA}\cos(2\pi 50\mathrm{Hz}\ t)$$
- Agreement in **aperture field** and **magnetic energy** in the yoke
- **Eddy current losses** well approximated:
  1.36 W with Hom. HBFEM vs. 1.32 W with CST
  ➔ 3 % relative error
- **Higher order finite elements*** to achieve good approximation of losses and energy

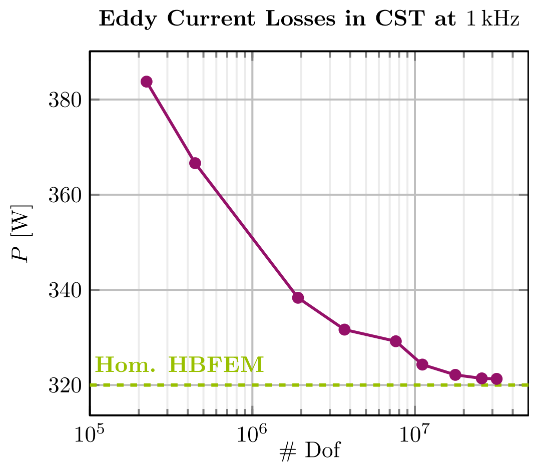


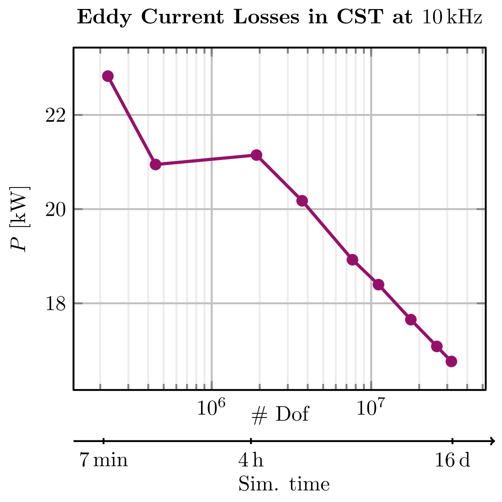
*J.P. Webb and B. Forghani, “Hierarchal Scalar and Vector Tetrahedra“, 1993

C-DIPOLE WITHOUT DC BIAS

- Compute eddy current losses up to $f = 65$ kHz
  ➔ Scaling behavior as expected from theory*
  ➔ Good agreement with CST results up to $f \approx 1$ kHz
- Reason for differences between Hom. HBFEM and CST: At higher frequencies, CST results are mesh-dependent, Hom. HBFEM results have converged
- Hom. HBFEM reduces simulation time for nonlinear simulations in kilohertz range from days to hours







Hom. Hbfem Eddy Current Losses at 10 kHz	
# Dof	*P* (kW)
$2.4 \cdot 10^4$	7.38
$5.6 \cdot 10^4$	7.31
$1.1 \cdot 10^5$	7.38

* R. L. Stoll, *The Analysis of Eddy Currents.* 1974.
J. Lammeraner and M. Štafl, *Eddy Currents.* 1966.

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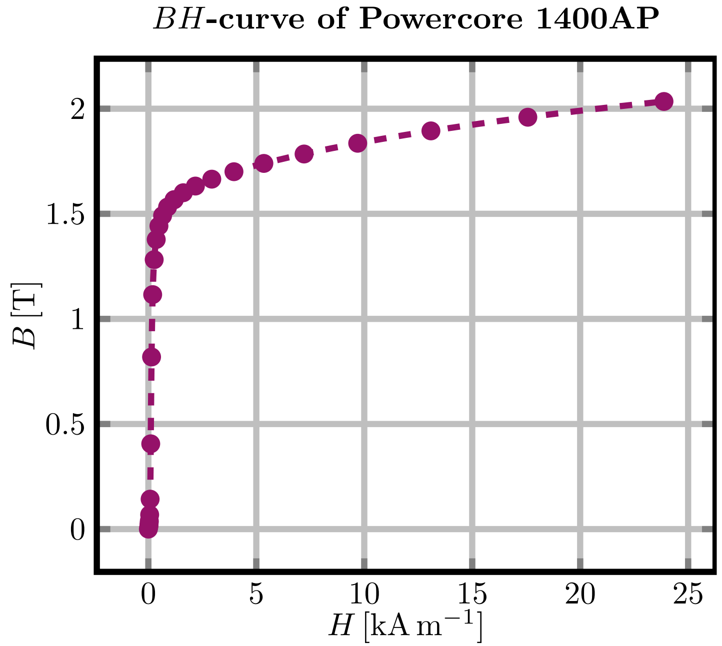
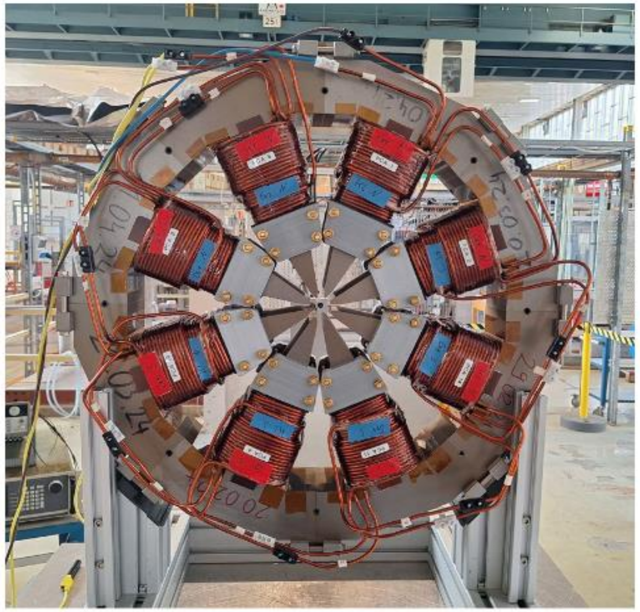
NONLINEAR SIMULATION OF THE CORRECTOR MAGNET

- Simulate model of prototype with Hom. HBFEM
- Yoke material:
  - Powercore 1400AP
  - $\sigma = 5.814\,\frac{\mathrm{MS}}{\mathrm{m}}$
  - 1 mm laminations, stacking factor $\gamma \approx 0.985$
- Main coils 975 At, auxiliary coils 405 At



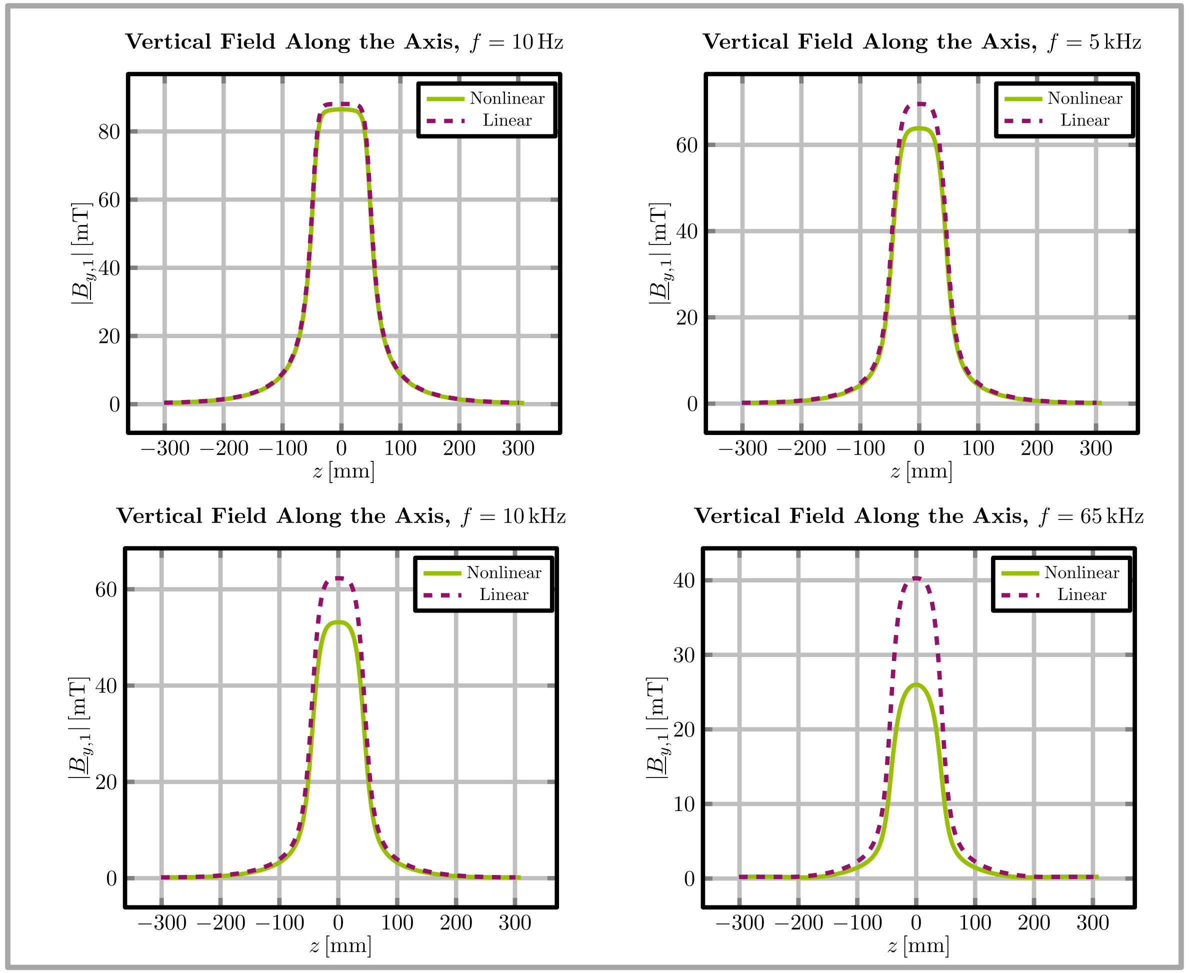


courtesy of Matthias Thede

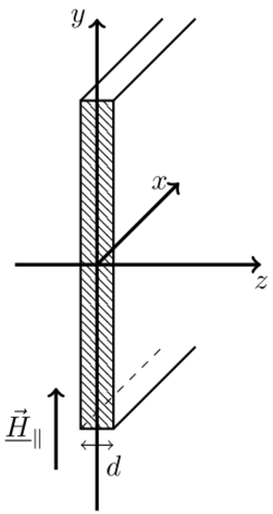
NONLINEAR SIMULATION OF THE CORRECTOR MAGNET

f (Hz)	linear		nonlinear	
	B_{int} (mTm)	$\varphi_{\mathrm{center}}$ (deg.)	B_{int} (mTm)	$\varphi_{\mathrm{center}}$ (deg.)
10	11.7	−0.1	11.4	0.0
1000	10.0	−6.8	10.0	−8.0
5000	7.9	−13.0	7.2	−16.1
10000	6.9	−16.3	5.8	−20.2
65000	4.2	−25.4	2.7	−31.8

- Effect of nonlinearity more significant at higher frequency ➔ smaller integrated flux density, greater phase shift
- This is due to interplay of eddy currents/skin effect and nonlinearity



NONLINEAR SIMULATION OF THE CORRECTOR MAGNET

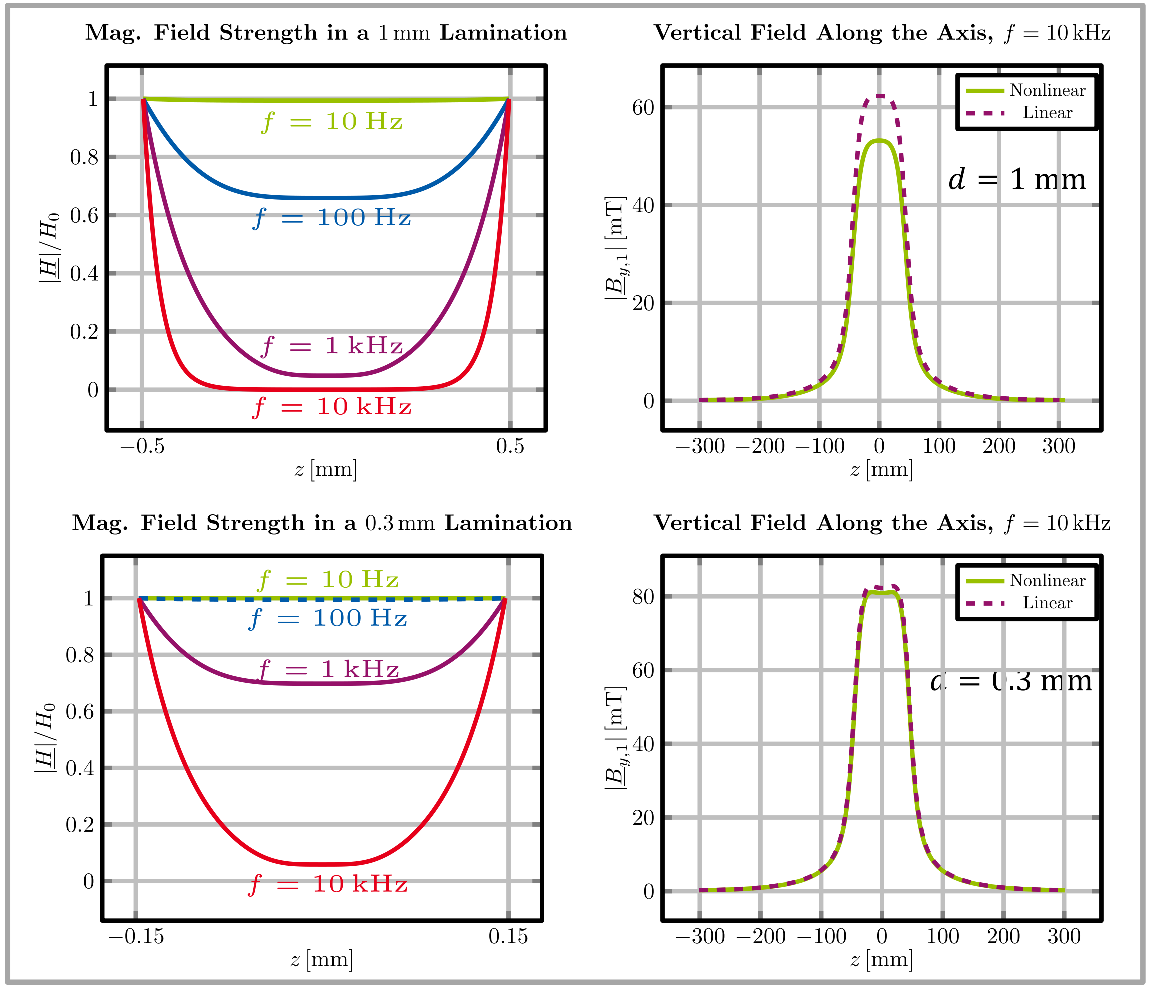


MAGNETIC FIELD IN ONE LAMINATION

$$\frac{d^2\underline{H}\,(z)}{dz^2} = j\omega\sigma\mu\,\underline{H}(z) \Rightarrow \underline{H}(z) = H_0\,\frac{\cosh((1+j)\frac{z}{\delta})}{\cosh((1+j)\frac{d}{2\delta})}$$

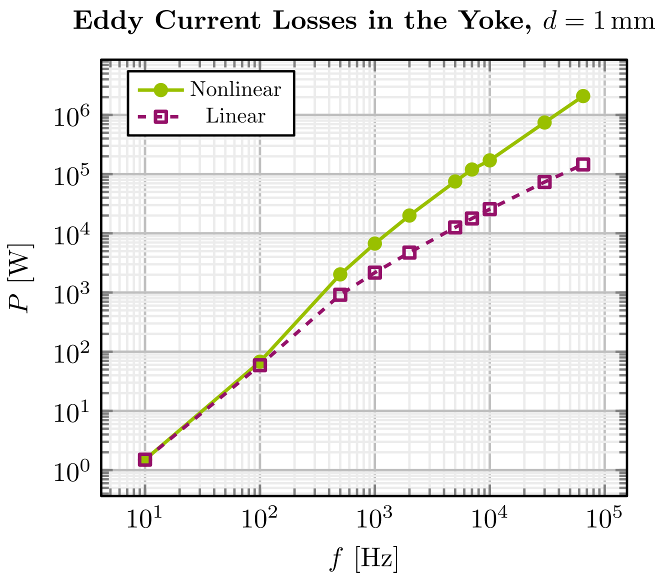
Skin depth $\delta = \sqrt{2/\omega\sigma_c\mu_c}$

- Skin effect results in non-uniform field across laminations
  - ➔ Non-uniform reluctivity
  - ➔ Considering nonlinearity becomes more important
- Importance of skin effect depends on ratio $\frac{d}{\delta}$
  - ➔ **Thinner lamination decreases impact of nonlinearity**



APPLICATION

NONLINEAR SIMULATION OF THE CORRECTOR MAGNET



- At low frequencies, losses are quite similar to the linear case
- With increasing frequency, differences increase
- Keep in mind: in reality currents will decrease at higher frequencies, here they are kept constant

f (Hz)	Eddy Current Losses (W)	
	linear	nonlinear
10	1.5	1.5
100	$5.9 \cdot 10^1$	$6.8 \cdot 10^1$
500	$9.2 \cdot 10^2$	$2.0 \cdot 10^3$
1000	$2.2 \cdot 10^3$	$6.7 \cdot 10^3$
2000	$4.7 \cdot 10^3$	$2.0 \cdot 10^4$
5000	$1.3 \cdot 10^4$	$7.5 \cdot 10^4$
7000	$1.8 \cdot 10^4$	$1.2 \cdot 10^5$
10000	$2.6 \cdot 10^4$	$1.7 \cdot 10^5$
30000	$7.4 \cdot 10^4$	$7.5 \cdot 10^5$
65000	$1.5 \cdot 10^5$	$2.1 \cdot 10^6$

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CONCLUSION

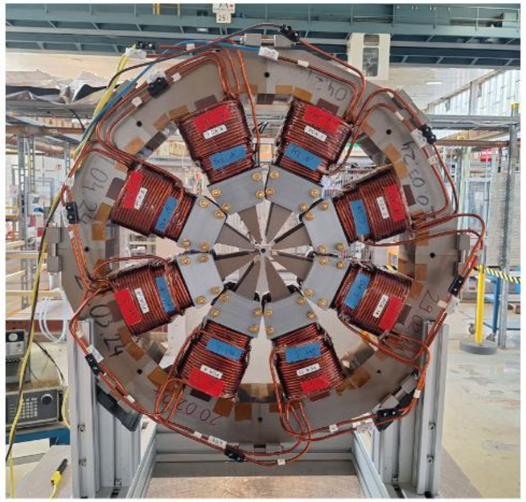
METHOD

- Dedicated method to enable nonlinear simulation of fast corrector magnets
  ➔ Implemented in getDP and python
  ➔ Combines homogenization techniques with HBFEM
  ➔ Several examples tested for verification

RESULTS

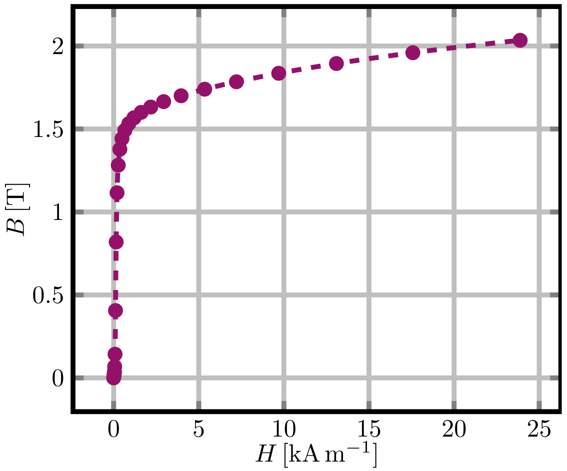
- First simulations of prototype magnet
  ➔ effect of nonlinearity significant at higher frequencies
  ➔ smaller integrated fields, greater phase shift, increased losses
  ➔ smaller lamination thickness would decrease effect of nonlinearity

- Interplay between eddy currents, skin effect, and nonlinearity
  ➔ importance of nonlinearity depends not just on applied field
  ➔ further investigation necessary



courtesy of Matthias Thede

BH-curve of Powercore 1400AP



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