BEAM COUPLING IMPEDANCE SIMULATIONS

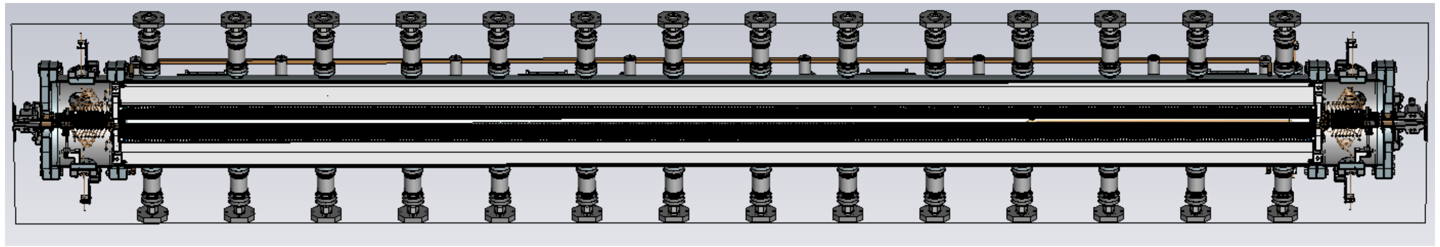
Frederik Quetscher, Erion Gjonaj, Herbert De Gersem

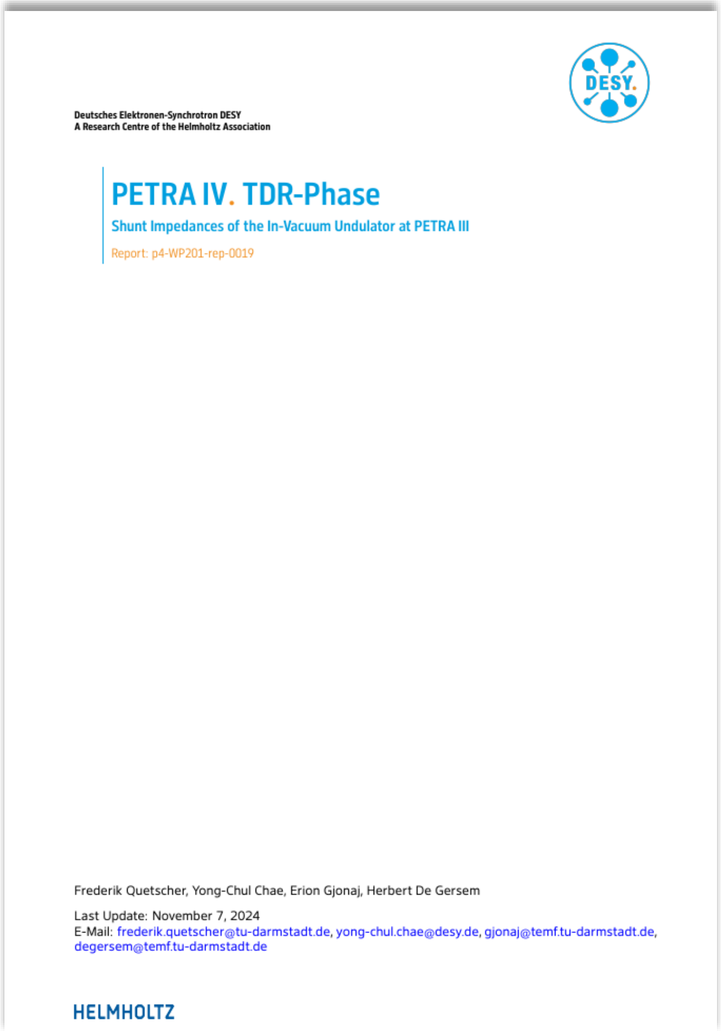
CONTENT

- TDR-phase report
- Progress on domain decomposition for FELIS
  - Schwarz method overview
  - Hybrid transmission condition
  - Application

TDR-PHASE REPORT - IVU

- Investigated shunt impedances of the PETRA III IVU
  - Part of the DESY-TEMF collaboration
- Progress reported in the last DESY-TEMF meetings
- Final key results are compiled in p4-WP201-rep-0019
  - Review phase starting soon





Deutsches Elektronen-Synchrotron DESY
A Research Centre of the Helmholtz Association

PETRA IV. TDR-Phase

Shunt Impedances of the In-Vacuum Undulator at PETRA III

Report: p4-WP201-rep-0019

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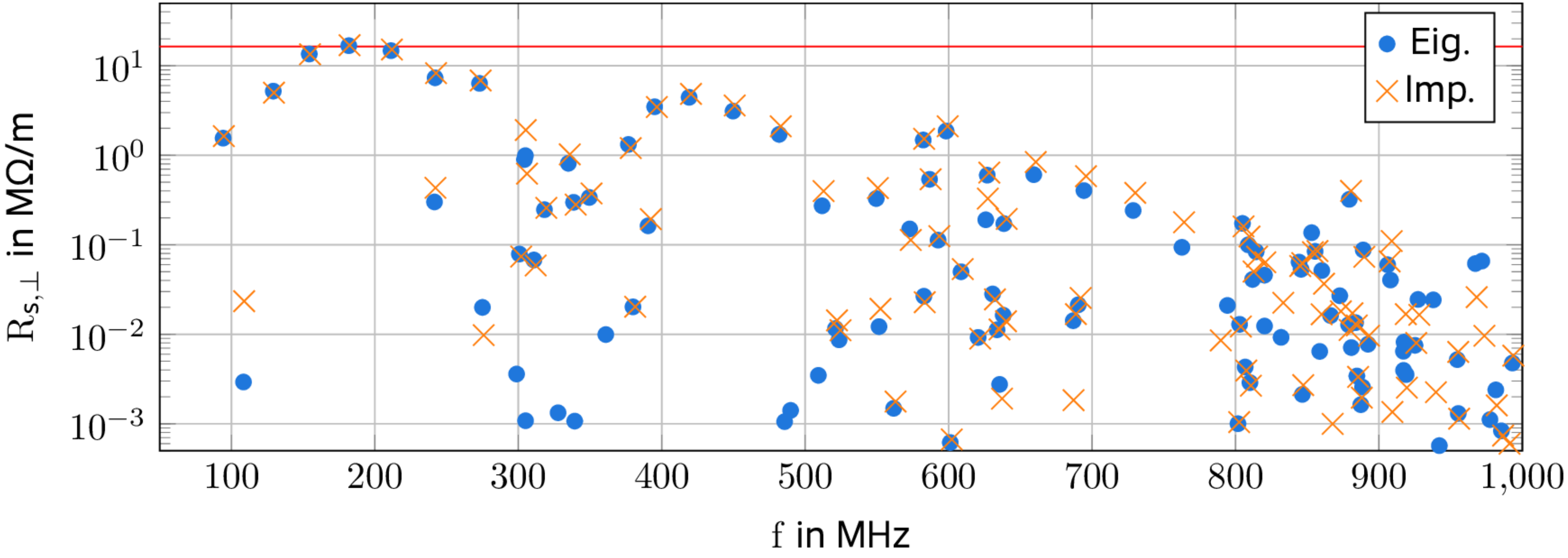
HELMHOLTZ

TDR-PHASE REPORT - IVU

KEY RESULTS

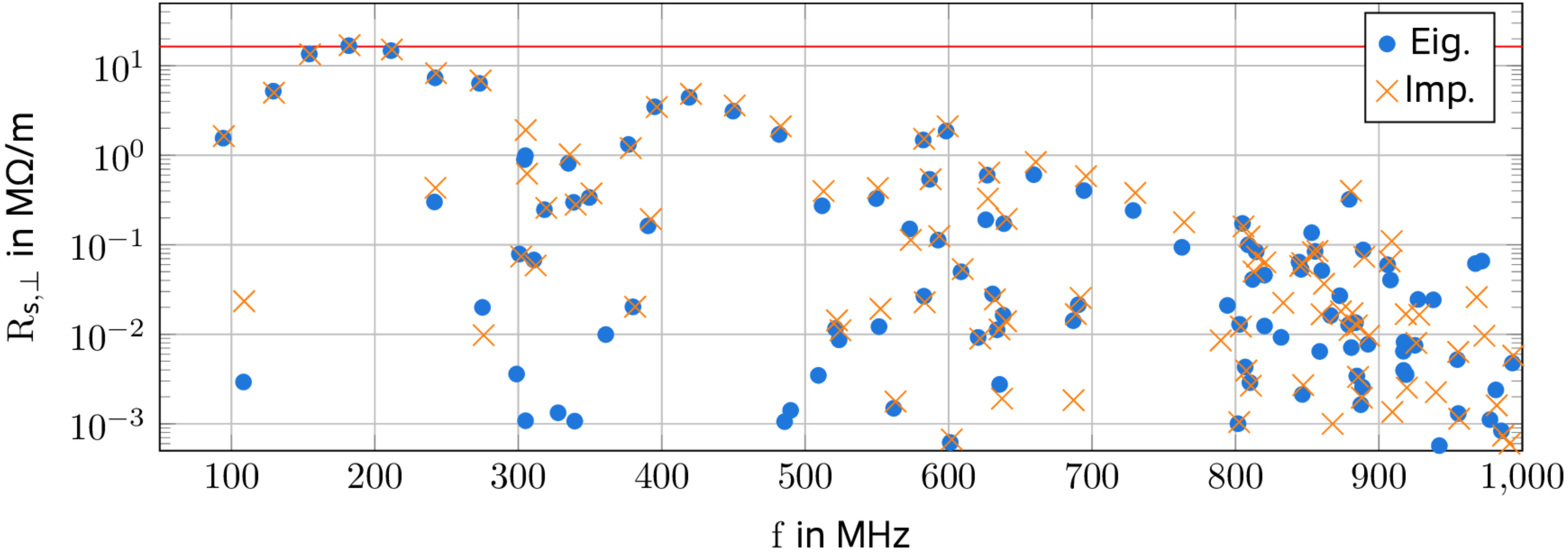
- Transverse shunt impedances
- Excellent agreement for relevant shunt impedances at lower frequencies
- Small differences towards higher frequencies due to discretization
- Mismatch of smaller shunt impedances due to configuration of fast frequency sweep



TDR-PHASE REPORT - IVU

KEY RESULTS

- Originally estimated instability threshold at 16.4MΩ/m
- New BD simulations from Chao Li using our mode map
- Some resonances can lead to instabilities
- Optics modifications or already planned feedback system is necessary



CONTENT

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  - Application

DOMAIN DECOMPOSITION

- Expensive simulations
  - Time consuming
  - Large memory demands
- HPC parallelization necessary for higher frequencies
- Still open challenge for simulations at high frequency
  - Additional challenges with particle beams
- Promising approach: domain decomposition

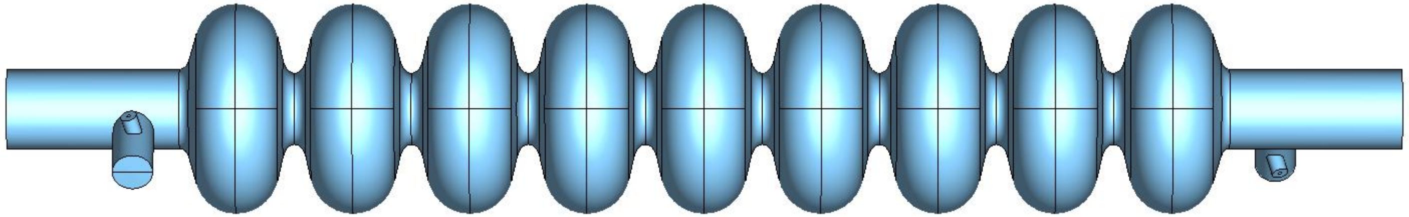
PROBLEM STATEMENT

- Weak formulation: find $E \in H(\mathrm{curl})$ such that $\forall v \in H(\mathrm{curl})$:

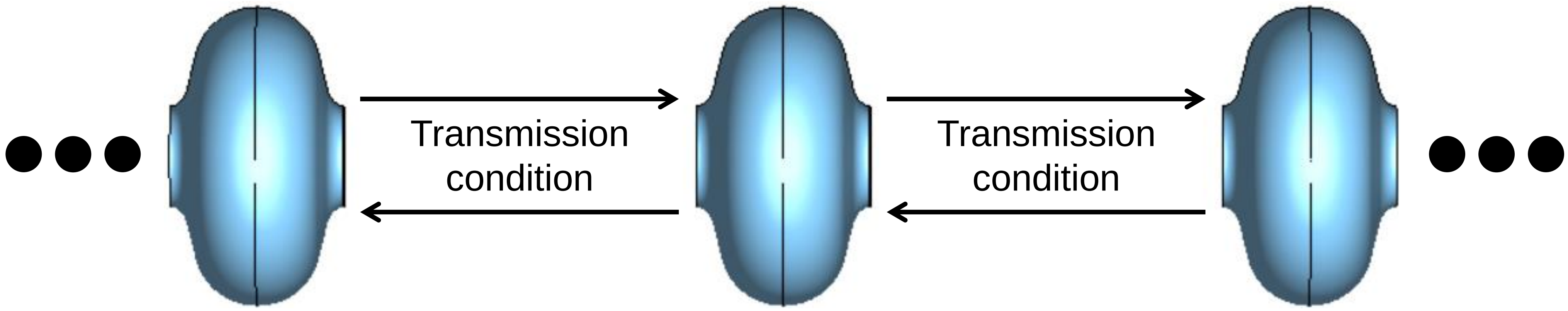
$$\int_\Omega \mu^{-1}\, \nabla \times E \cdot \nabla \times v \,\mathrm{d}V - \omega^2 \int_\Omega \varepsilon\, E \cdot v \,\mathrm{d}V =$$

$$\underbrace{-j\omega \int_\Omega J \cdot v \,\mathrm{d}V}_{\text{Current excitation}} \underbrace{- \int_{\Sigma_{\mathrm{SI}}} \mu^{-1}[n \times \nabla \times E] \cdot v \,\mathrm{d}S}_{\text{Resistive wall}} \underbrace{- \int_{\Sigma_{\mathrm{WG}}} \mu^{-1}[n \times \nabla \times E] \cdot v \,\mathrm{d}S}_{\text{Waveguides}}$$

- Resistive wall BC: $\mu^{-1} n \times \nabla \times E = \mathrm{j}\omega Y(\omega) n \times n \times E$
- Waveguide BC: $\mu^{-1} n \times \nabla \times E = \mu^{-1} n \times \nabla \times E^{\mathrm{Inc}} - j\omega \sum_{m=1}^{\infty} Y_m^{\mathrm{TX}}(\omega) a_m (E - E^{\mathrm{Inc}})\, e_m^{\mathrm{TX}})$



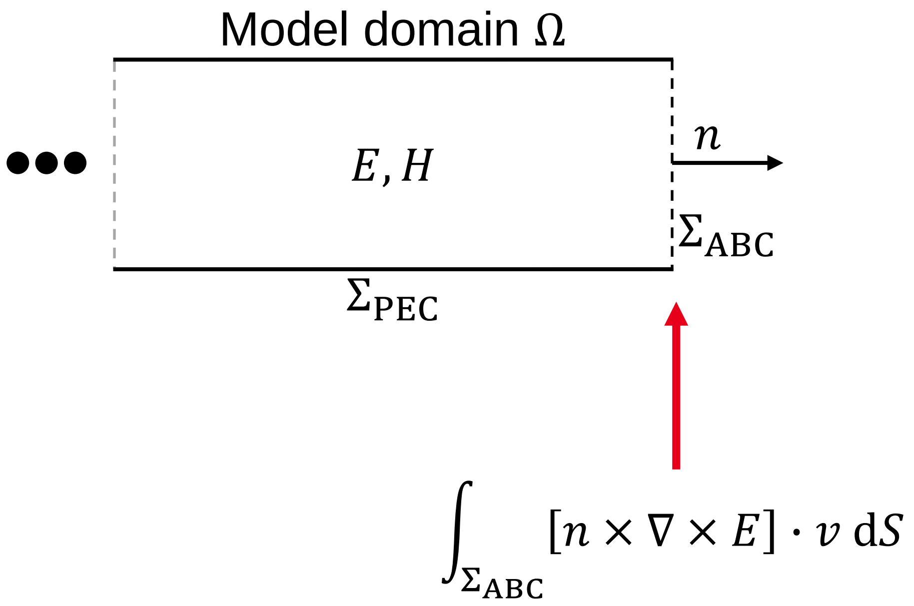
SCHWARZ DOMAIN DECOMPOSITION
OVERVIEW



- Cut model into pieces
- Apply a **transmission condition** at the interfaces
  - Essentially absorbing boundary conditions (ABCs)
  - Additionally exchange outgoing waves as excitations
- Enables to solve each subproblem individually
  - On different nodes of HPC cluster → parallelization

ABSORBING BOUNDARY CONDITIONS
EXACT

- Idea: general wave leaves the domain
- $n \times \nabla \times E = \mathrm{j}\mathbf{k} n \times (n \times E)$
- $\mathbf{k} = (k_0^2 \mathbf{I} + \Delta_\perp)^{-0.5} (k_0^2 \mathbf{I} - \nabla_\perp \times \nabla_\perp \times)$
- Normal wavenumber operator
  - $\mathbf{k}_\mathrm{n} = (k_0^2 \mathbf{I} + \Delta_\perp)^{-0.5}$
  - $\mathcal{F}(\mathbf{k}_\mathrm{n}) = (k_0^2 - \mathrm{k}_\perp^2)^{-0.5}$
- Not directly suitable for numerical scheme
- Non-local

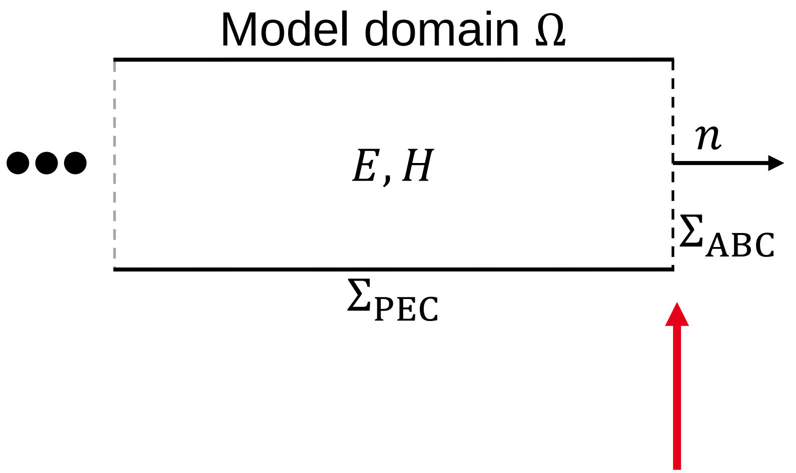


(Trace operators omitted)

[M. El Bouajaji, X. Antoine, C. Geuzaine, Approximate local magnetic-to-electric surface operators for time-harmonic Maxwell's equations, 2014]

ABSORBING BOUNDARY CONDITIONS
FREE SPACE

- Lowest order approximation of exact ABC
  - $\mathbf{k} = (k_0^2 \mathbf{I} + \Delta_\perp)^{-0.5} (k_0^2 \mathbf{I} - \nabla_\perp \times \nabla_\perp \times)$
  - $\mathbf{k} = k_0$
- $n \times \nabla \times E = j k_0 n \times (n \times E)$
  - Silver-Müller radiation condition
  - 0th-order open BC
- Physical meaning: plane wave leaves the domain perpendicularly
- Local boundary condition
- Only exact for perpendicular incidence

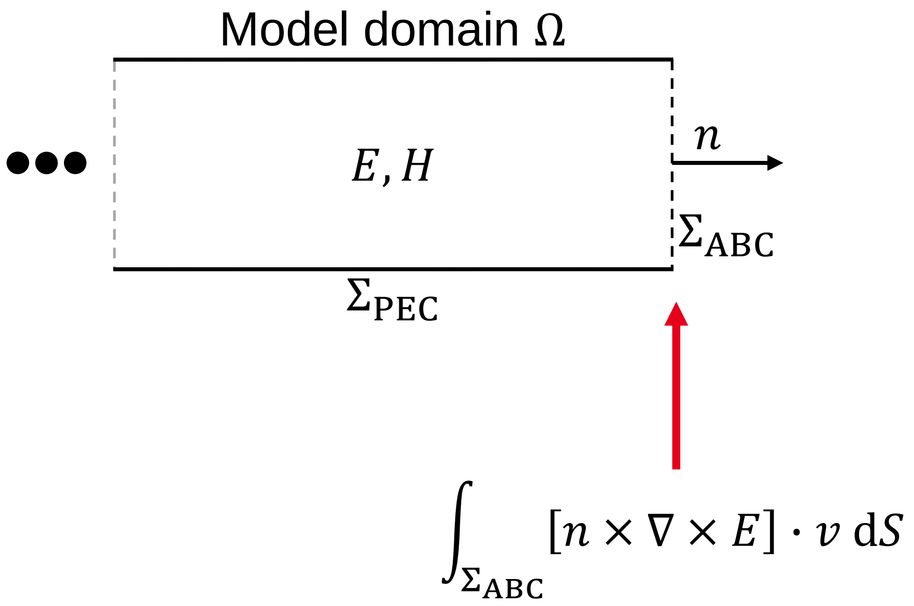


$$\int_{\Sigma_{\mathrm{ABC}}} [n \times \nabla \times E] \cdot v \, \mathrm{d}S$$

(Trace operators omitted)

ABSORBING BOUNDARY CONDITIONS
FREE SPACE - EXTENDED

- Lowest order approximation of normal wavenumber
  - $\mathbf{k} = (k_0^2 \mathbf{I} + \Delta_\perp)^{-0.5}(k_0^2 \mathbf{I} - \nabla_\perp \times \nabla_\perp \times)$
  - $\mathbf{k}_\mathrm{n} = p \in \mathbb{C}$
  - Simple choices: $k \in \{k_0, jk_0, (1+j)k_0/\sqrt{2}\}$
- $n \times \nabla \times E = \frac{j}{p}(k_0^2 \mathbf{I} - \nabla_\perp \times \nabla_\perp \times)(n \times (n \times E))$
  - 0th-order extended open BC
- More accurate but still not exact in general

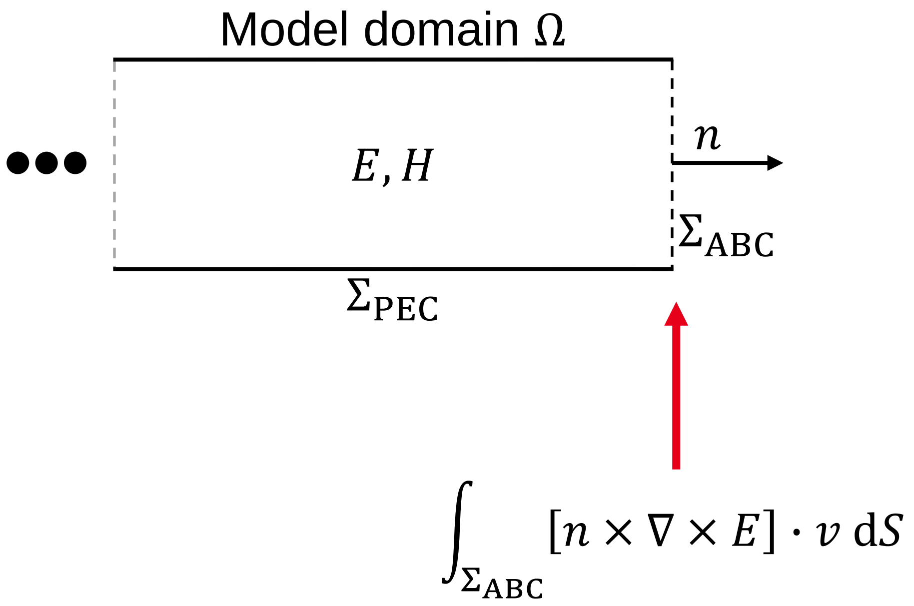


(Trace operators omitted)

TECHNISCHE UNIVERSITÄT DARMSTADT

ABSORBING BOUNDARY CONDITIONS
MODAL

- Idea: waveguide modes leave the domain
- $n \times \nabla \times E = j \sum_{m=1}^{\infty} k_m^{\mathrm{TX}}(\omega) n \times (n \times E_m^{\mathrm{TX}})$
  - $\mathrm{TX} \in \{\mathrm{TE}, \mathrm{TM}, \mathrm{TEM}\}$
  - $k_m^{\mathrm{TX}}(\omega) = \begin{cases} k_n & \text{for TE modes} \\ k_0^2/k_n & \text{for TM modes} \\ k_0 & \text{for TEM modes} \end{cases}$
  - $k_n = \sqrt{k_0^2 - k_{n,\perp}^2}$
- Exact for guided wave structures
- Established BC in numerical applications



(Trace operators omitted)

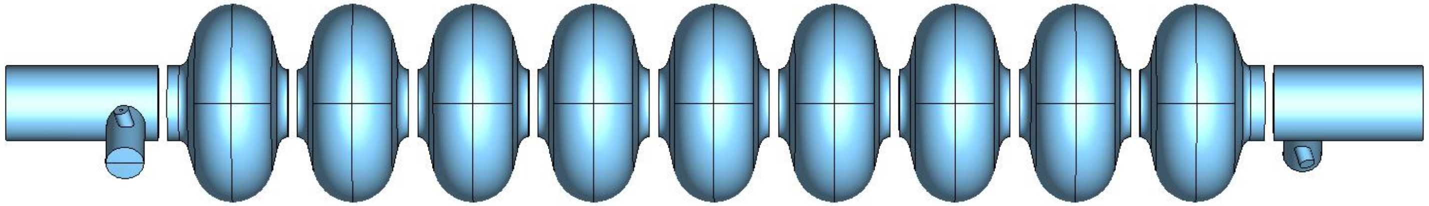
DOMAIN DECOMPOSITION
PROBLEM STATEMENT

- Weak formulation per subdomain: find $E \in H(\text{curl})$ such that $\forall v \in H(\text{curl})$:

$$\int_{\Omega_i} \mu^{-1}\, \nabla \times E \cdot \nabla \times v \,\mathrm{d}V - \omega^2 \int_{\Omega_\mathrm{i}} \varepsilon\, E \cdot v \,\mathrm{d}V =$$

$$\underbrace{-j\omega \int_{\Omega_\mathrm{i}} J \cdot v \,\mathrm{d}V}_{\text{Current excitation}} - \underbrace{\int_{\Sigma_\mathrm{SI}} \mu^{-1} [n \times \nabla \times E] \cdot v \,\mathrm{d}S}_{\text{Resistive wall}} - \underbrace{\int_{\Sigma_\mathrm{WG}} \mu^{-1} [n \times \nabla \times E] \cdot v \,\mathrm{d}S}_{\text{Waveguides}} - \underbrace{\int_{\Sigma_\mathrm{DDM}} \mu^{-1} [n \times \nabla \times E] \cdot v \,\mathrm{d}S}_{\text{DDM interface}}$$

- Resistive wall BC: $\mu^{-1} n \times \nabla \times E = \mathrm{j}\omega Y(\omega) n \times n \times E$
- Waveguide BC: $\mu^{-1} n \times \nabla \times E = \mu^{-1} n \times \nabla \times E^{\mathrm{Inc}} - j\omega \sum_{m=1}^{\infty} Y_m^{\mathrm{TX}}(\omega) a_m (E - E^{\mathrm{Inc}})\, e_m^{\mathrm{TX}})$



TECHNISCHE UNIVERSITÄT DARMSTADT

DOMAIN DECOMPOSITION
DOMAIN COUPLING

- Domain coupling: transmission condition
  - $S_{\Sigma_{12}}(E_{12}) - n_{12} \times \nabla \times E_{12} = S_{\Sigma_{21}}(E_{21}) + n_{21} \times \nabla \times E_{21}$
- Boundary operators
  - OTC0: $S_{\Sigma}(E) \coloneqq jk_0 n \times (n \times E)$
  - OTC0e: $S_{\Sigma}(E) \coloneqq \frac{j}{p}(k_0^2 \mathbf{I} - \nabla_{\perp} \times \nabla_{\perp} \times)(n \times (n \times E))$
  - MTC: $S_{\Sigma}(E) \coloneqq j \sum_{m=1}^{\infty} k_m^{\mathrm{TX}}(\omega) a_m^{\mathrm{TX}}(E) e_{\perp,m}^{\mathrm{TX}}$

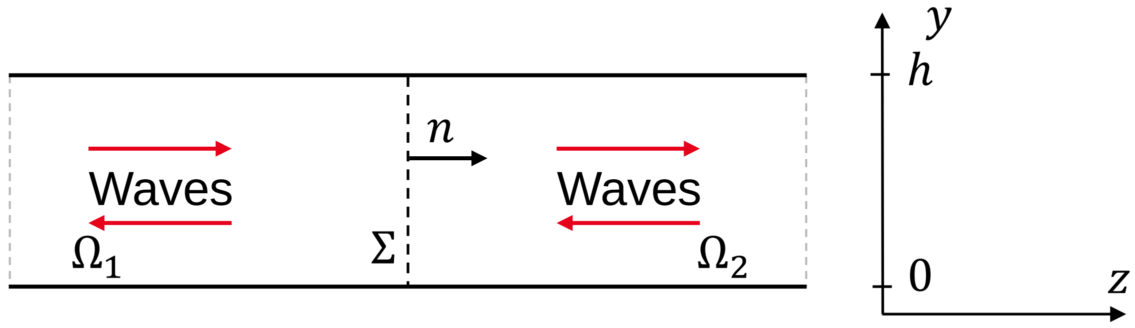
DOMAIN DECOMPOSITION
DOMAIN COUPLING

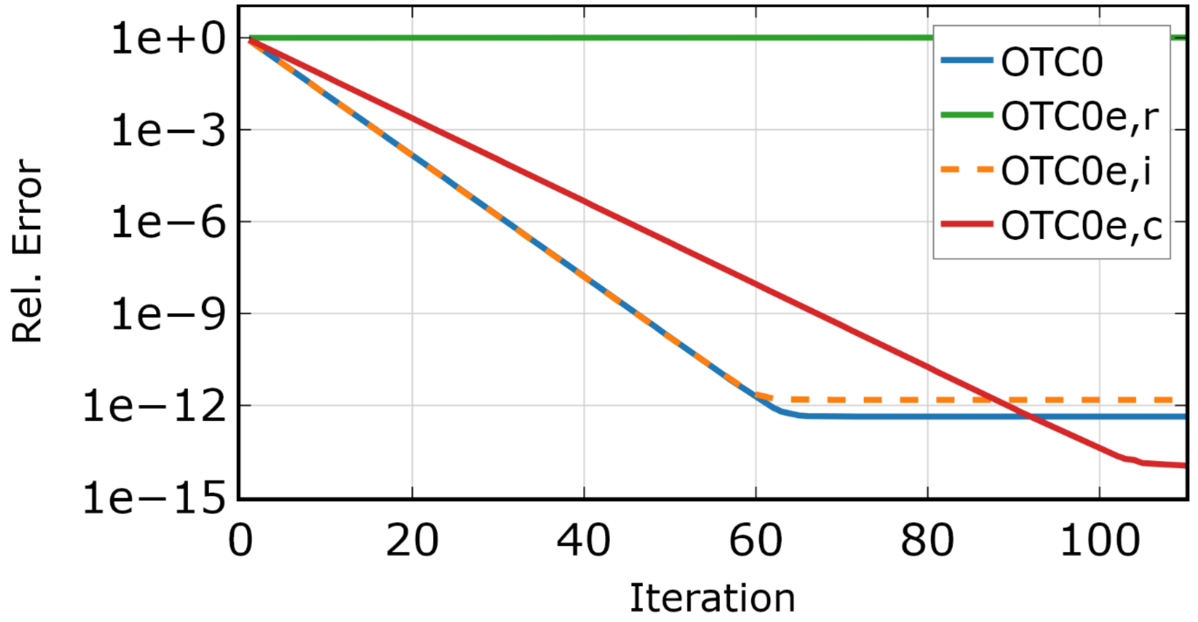
- Domain coupling: transmission condition
  - $S_{\Sigma_{12}}(E_{12}) - n_{12} \times \nabla \times E_{12} = S_{\Sigma_{21}}(E_{21}) + n_{21} \times \nabla \times E_{21} \coloneqq F_{12}$
- Boundary operators
  - OTC0: $S_{\Sigma}(E) \coloneqq jk_0 n \times (n \times E)$
  - OTC0e: $S_{\Sigma}(E) \coloneqq \frac{j}{p}(k_0^2 \mathbf{I} - \nabla_\perp \times \nabla_\perp \times)(n \times (n \times E))$
  - MTC: $S_{\Sigma}(E) \coloneqq j \sum_{m=1}^{\infty} k_m^{\mathrm{TX}}(\omega) a_m^{\mathrm{TX}}(E) e_{\perp,m}^{\mathrm{TX}}$
- Defines an update prescription for an iterative scheme:
  - $F_{12}^{n+1} = -F_{21}^{n} + 2S_{\Sigma_{21}}(E_{21}^{n})$
  - (Fixed point iteration)
- Other iteration schemes like GMRES can be used

CONVERGENCE ON MODEL PROBLEM

PROPAGATING CASE

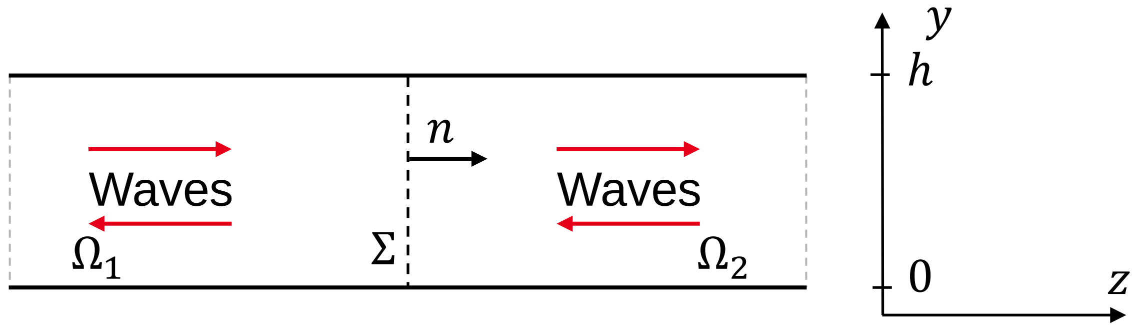
- Excitation
  - Fundamental **TE**
    - Fc=377 MHz
  - F=387 MHz
- OTC0
- OTC0e
  - OTC0e,r: $p = k$
  - OTC0e,i: $p = jk$
  - OTC0e,c: $p = (1 + j)/\sqrt{(2)}\ k$

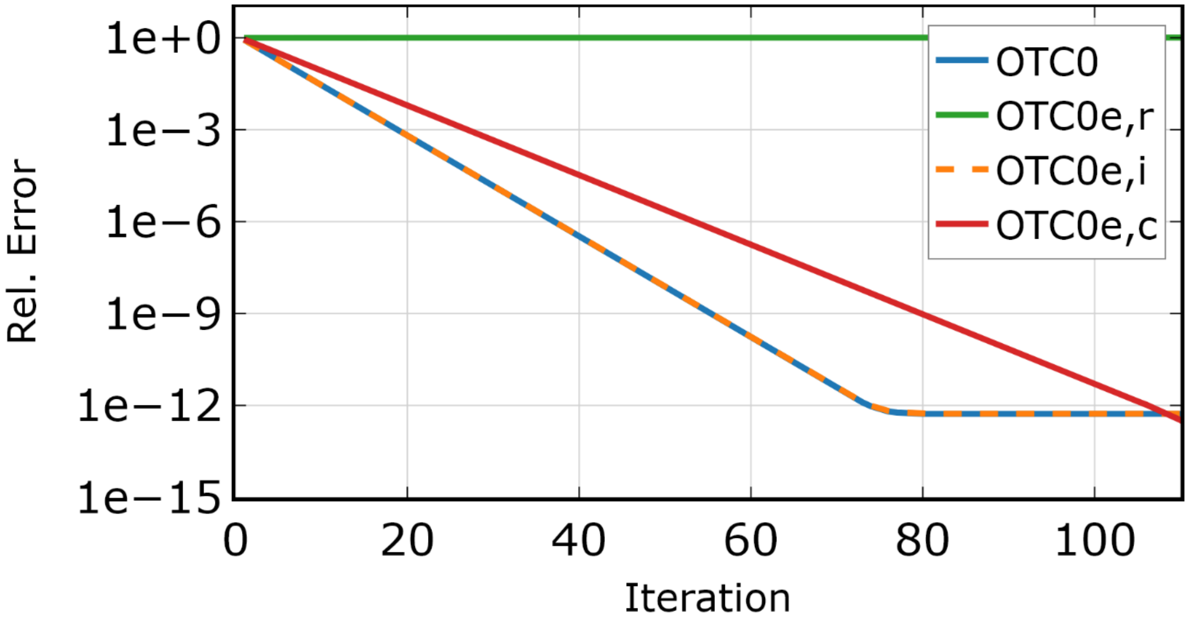




CONVERGENCE ON MODEL PROBLEM
PROPAGATING CASE

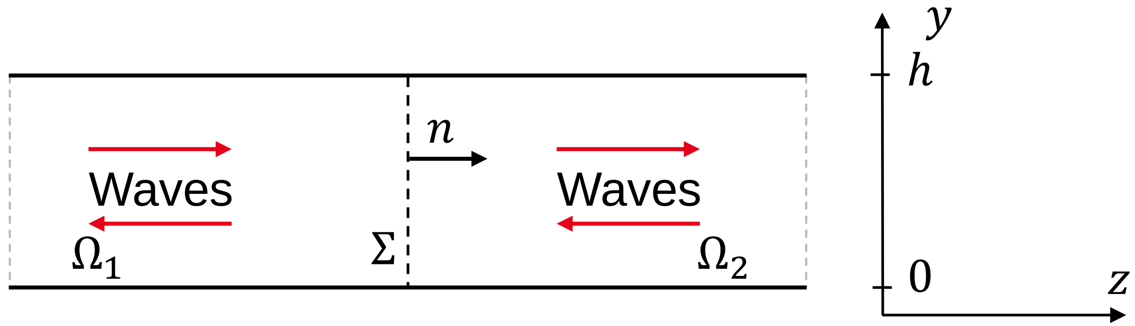
- Excitation
  - Fundamental **TM**
    - Fc=533 MHz
  - F=543 MHz
- OTC0
- OTC0e
  - OTC0e,r: $p = k$
  - OTC0e,i: $p = jk$
  - OTC0e,c: $p = (1 + j)/\sqrt{(2)}\ k$

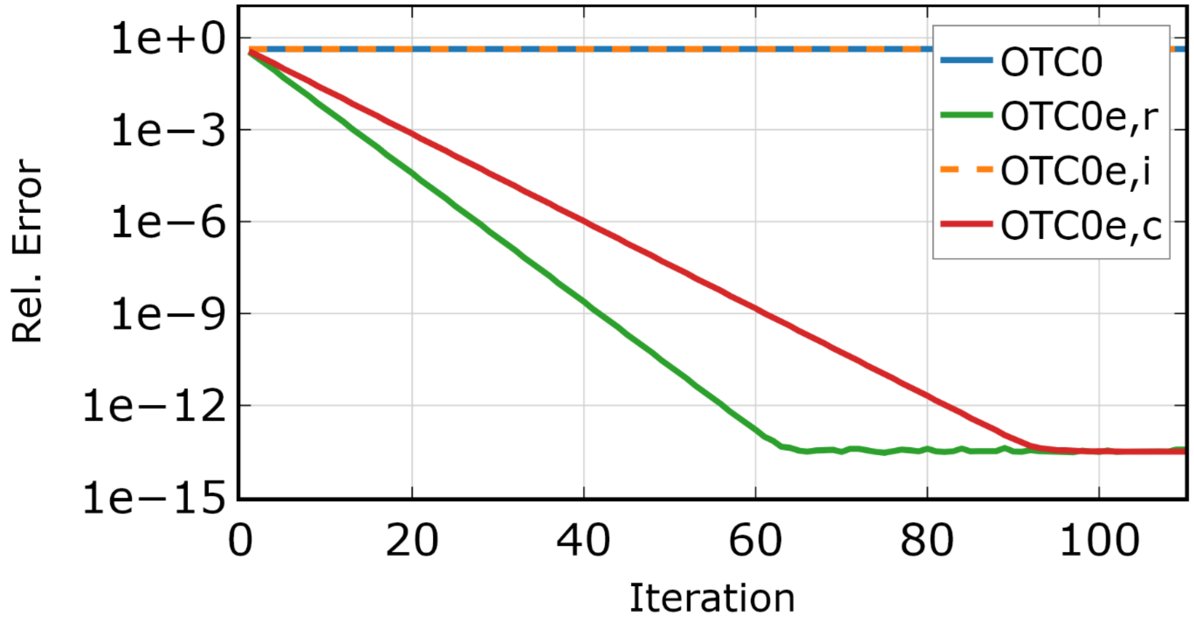




CONVERGENCE ON MODEL PROBLEM
EVANESCENT CASE

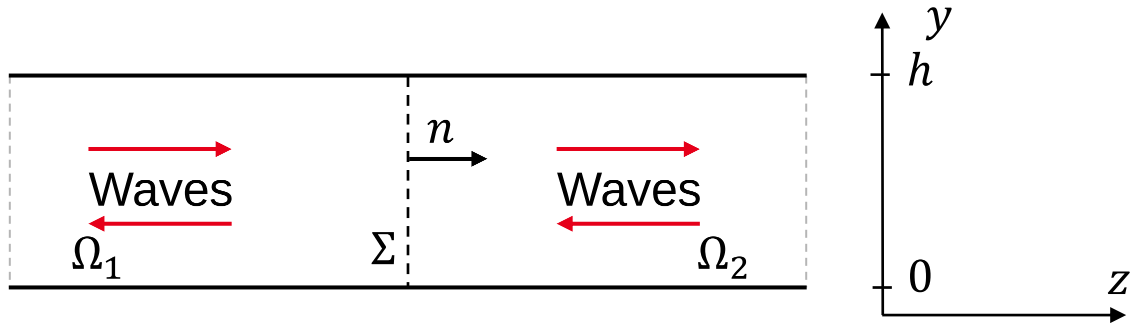
- Excitation
  - Fundamental **TE**
    - Fc=377 MHz
  - F=367 MHz
- OTC0
- OTC0e
  - OTC0e,r: $p = k$
  - OTC0e,i: $p = jk$
  - OTC0e,c: $p = (1 + j)/\sqrt{(2)}\ k$

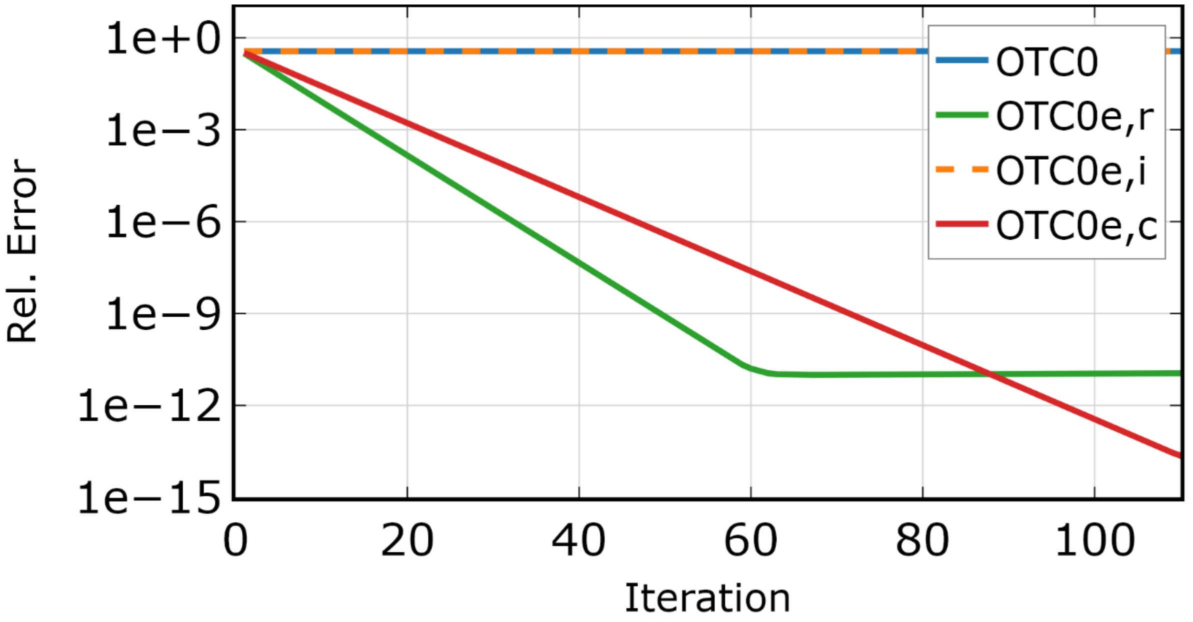




CONVERGENCE ON MODEL PROBLEM
EVANESCENT CASE

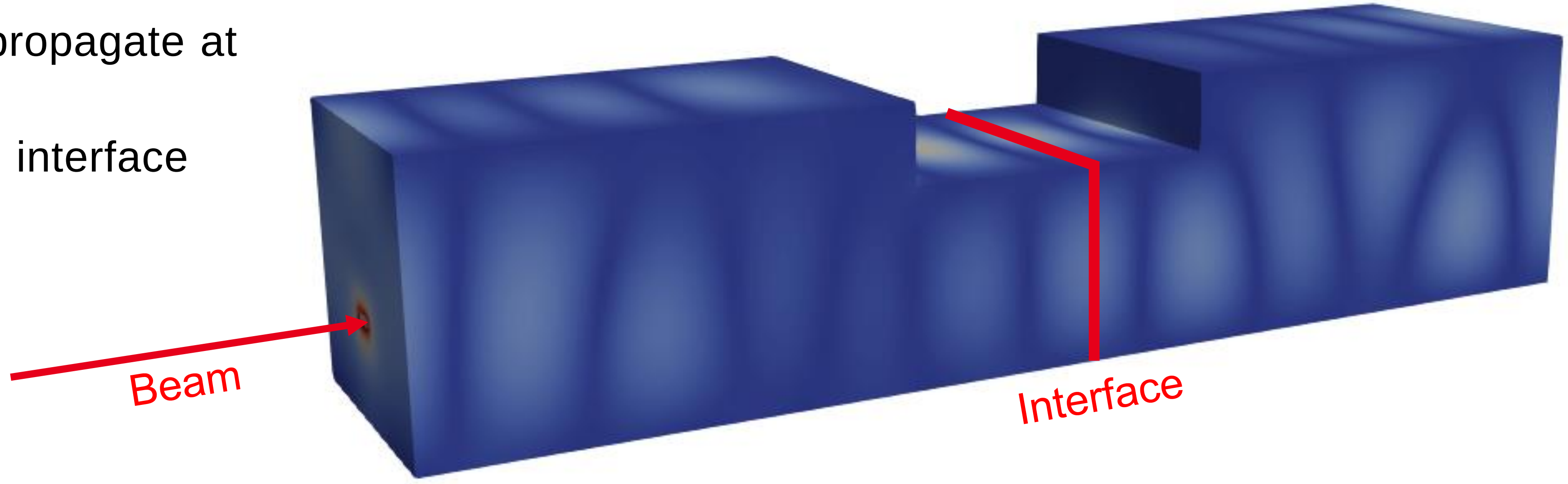
- Excitation
  - Fundamental **TM**
    - Fc=533 MHz
  - F=523 MHz
- OTC0
- OTC0e
  - OTC0e,r: $p = k$
  - OTC0e,i: $p = jk$
  - OTC0e,c: $p = (1 + j)/\sqrt{(2)}\ k$





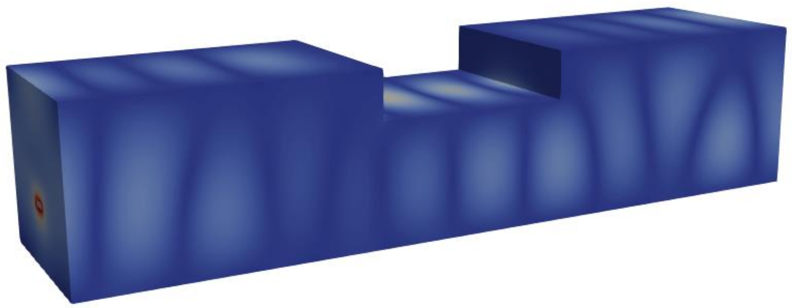
CONVERGENCE RATES

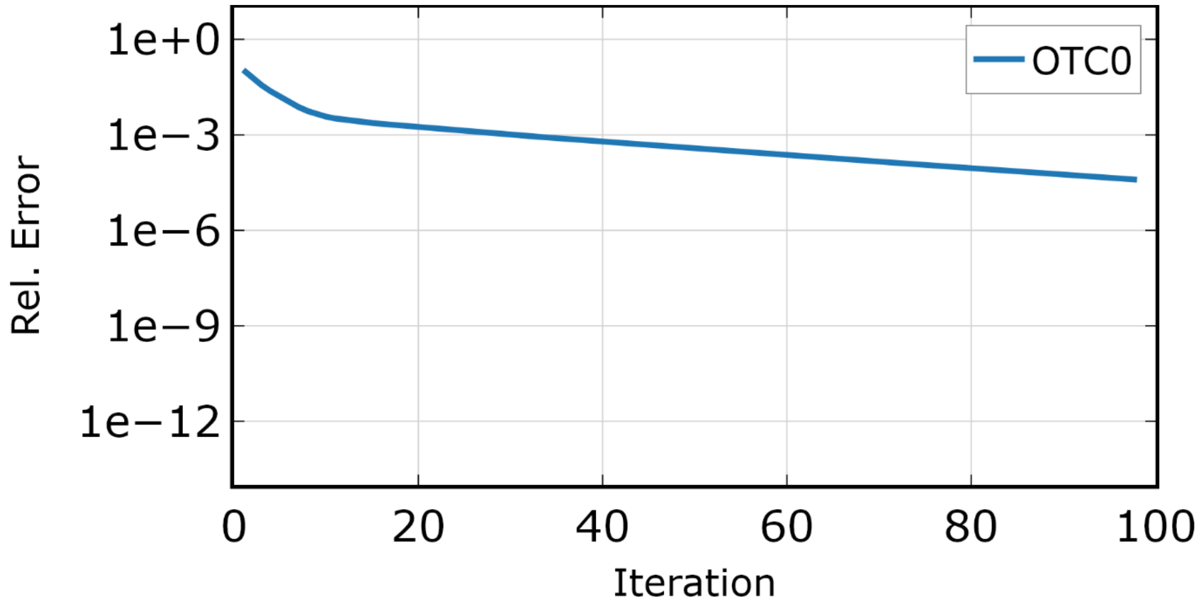
- Waveguide transition
  - 17.5 cm by 20 cm at entry and exit
  - 12.5 cm by 20 cm in the middle
- Beam excitation
  - $f = 2.0\ \mathrm{GHz}$
  - 2 TM and 5 TE modes propagate at interface
  - Many modes excited on interface



CONVERGENCE RATES

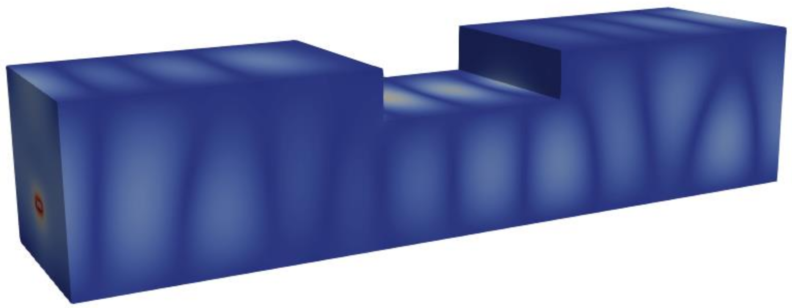
- OTC0
  - No convergence for evanescent modes with ordinary fixed point iteration
  - Relaxed fixed point iteration
  - Slow convergence

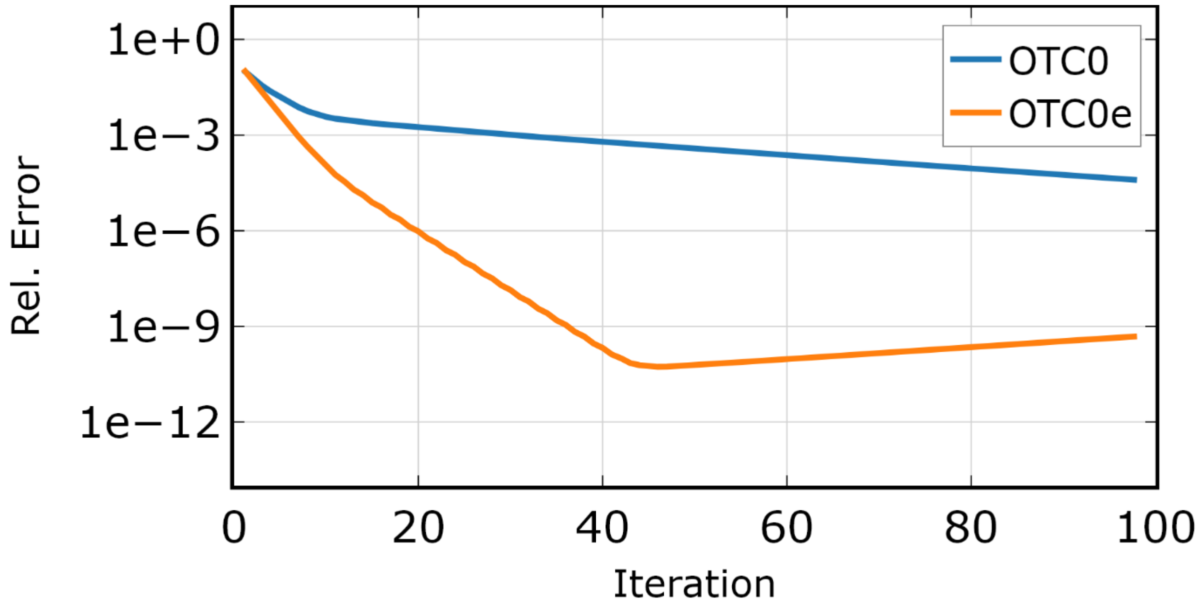




CONVERGENCE RATES

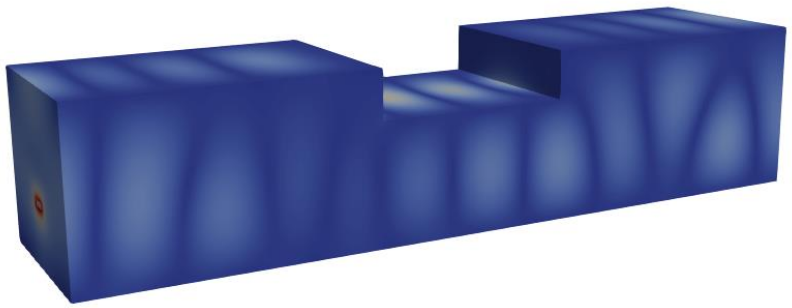
- OTC0e
  - Complex p
  - Much faster initial convergence
  - Leads to convergence for propagating and evanescent modes on model problems without reflection
  - This model has reflections -> divergence for some propagating modes

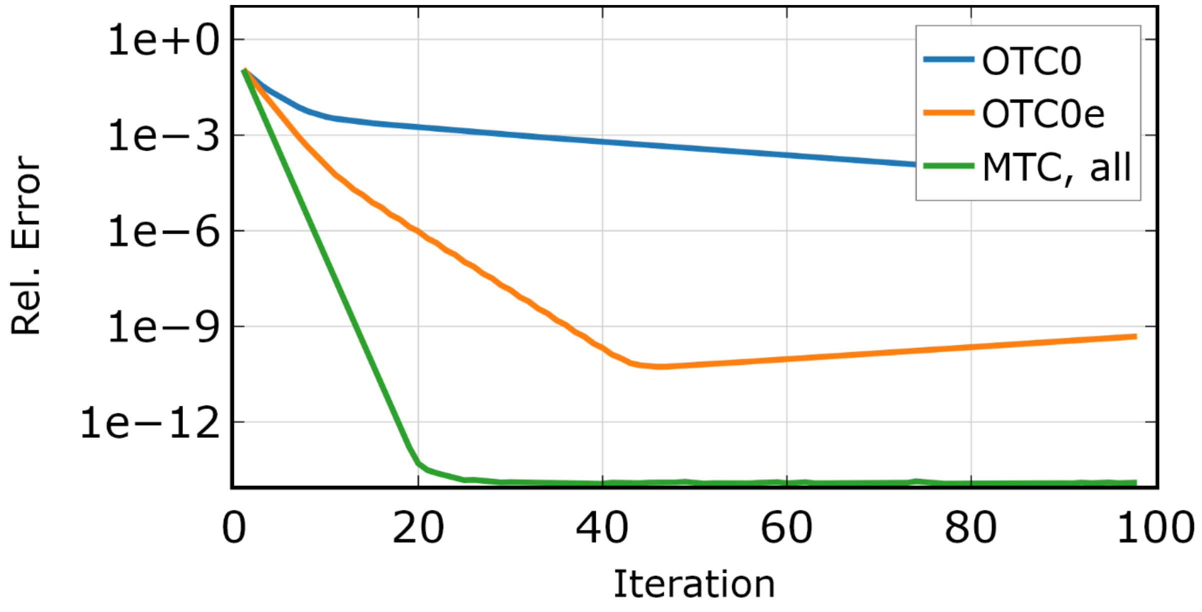




CONVERGENCE RATES

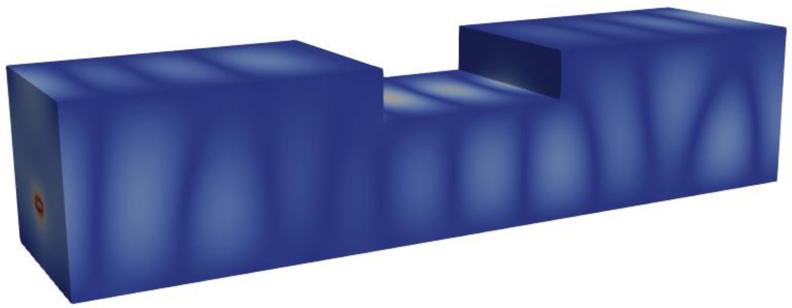
- MTC
  - Rapid (optimal) convergence
    - No artificial reflection
  - $S_\Sigma(E) \coloneqq j\omega\mu \sum_{m=1}^{\infty} Y_m^{\mathrm{TX}}(\omega) a_m^{\mathrm{TX}}(E) e_{\perp,m}^{\mathrm{TX}}$
    - Infinite number of modes in continuous case
  - In discrete case: one mode per DoF on interface
    - Finite but possibly huge number of modes for fine meshes
    - Number of modes increases with mesh refinement

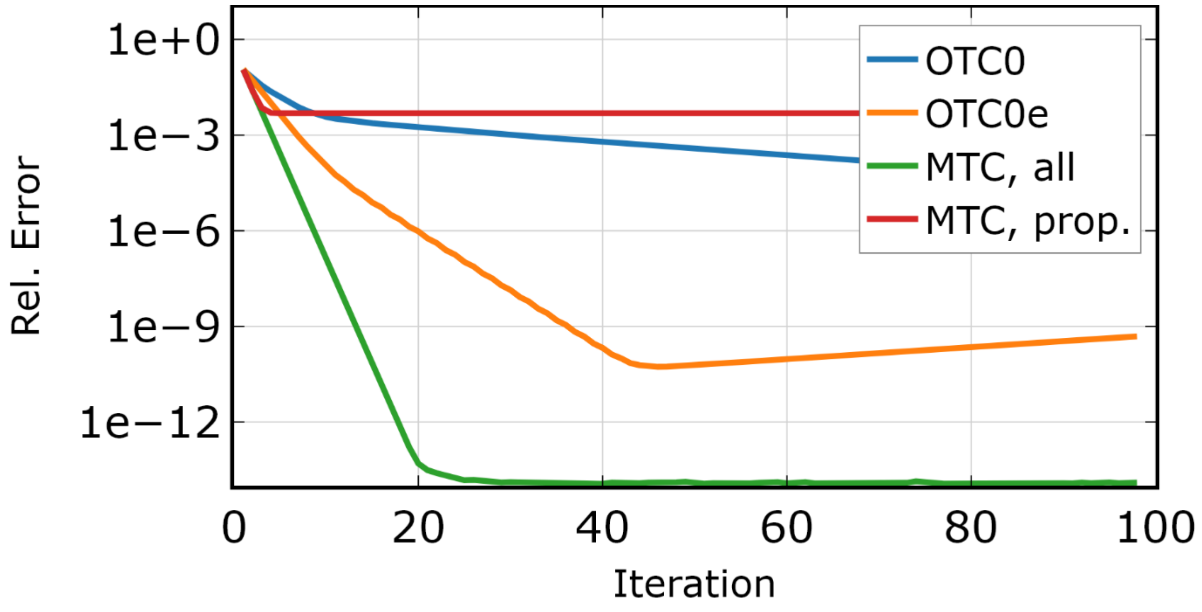




CONVERGENCE RATES

- MTC
  - In practice: truncate sum
  - $S_\Sigma(E) \coloneqq j\omega\mu \sum_{m=1}^{7} Y_m^{\mathrm{TX}}(\omega) a_m^{\mathrm{TX}}(E) e_{\perp,m}^{\mathrm{TX}}$
  - In this case, only propagating modes (2 TM, 5 TE) are considered
  - Initial converge maintained
  - Stagnation at some error



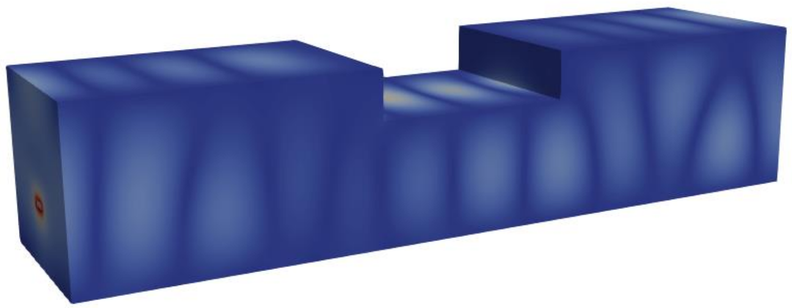


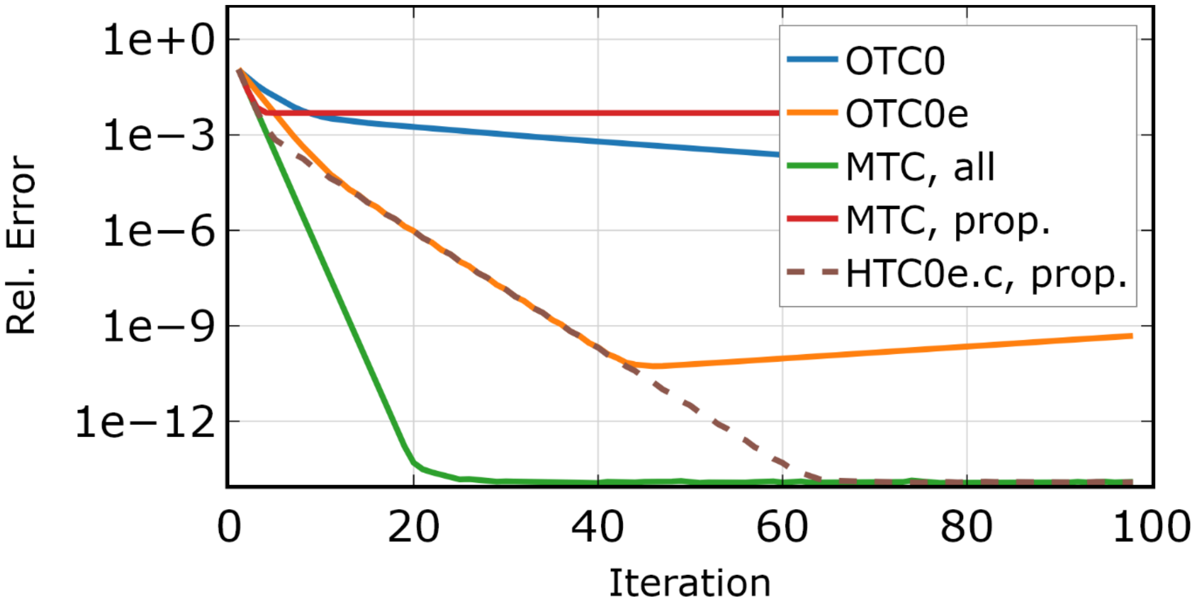
HYBRID TRANSMISSION CONDITION

- Idea: combine OTC and MTC
- OTC: $S_\Gamma(E) \coloneqq j\mathbf{k}n \times (n \times E)$
- MTC: $S_\Gamma(E) \coloneqq j \sum_{m=1}^{M} a_m^{\mathrm{TX}}(E) k_m^{\mathrm{TX}}(\omega)\, e_{\perp,m}^{\mathrm{TX}}$
- HTC: $S_\Gamma(E) \coloneqq j \sum_{m=1}^{M} a_m^{\mathrm{TX}}(E)(k_m^{\mathrm{TX}}(\omega)\mathbf{I} - \mathbf{k})\, e_{\perp,m}^{\mathrm{TX}} + j\mathbf{k}n \times (n \times E)$
  - Initial convergence rate of MTC
  - Asymptotic convergence rate of OTC

CONVERGENCE RATES

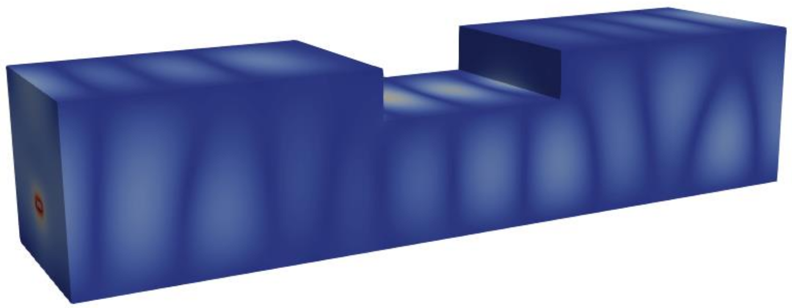
- HTC0e.c
  - MTC (propagating modes) + OTC0e.c
    - $p = (1+j)/\sqrt{(2)}\ k$
  - Error always less than of MTC and OTC0e.c
  - Convergent

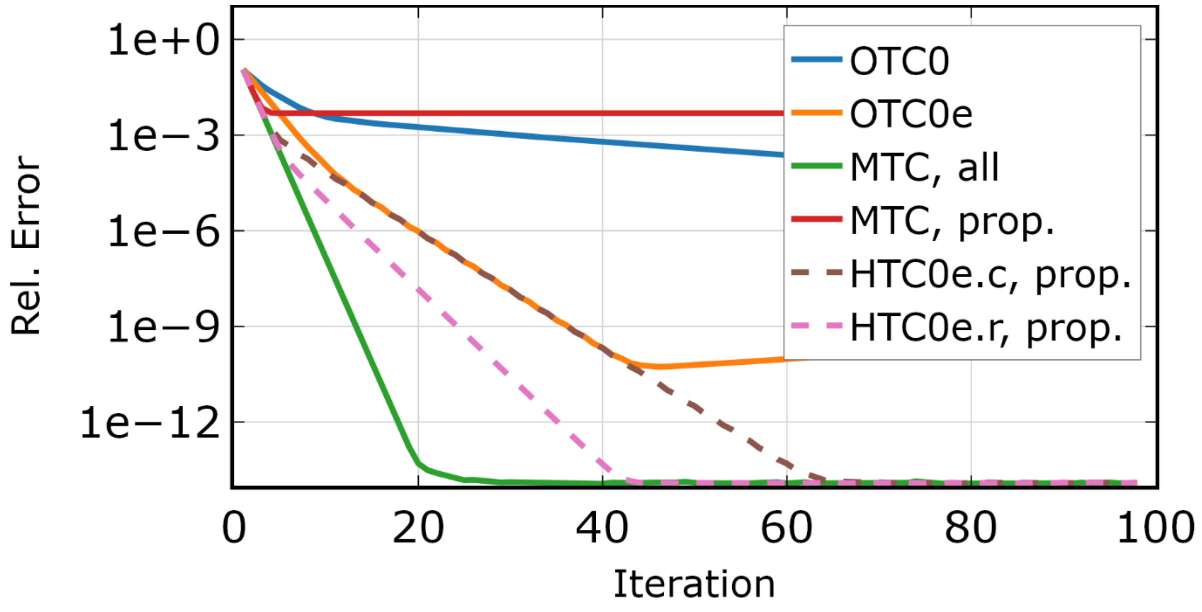




CONVERGENCE RATES

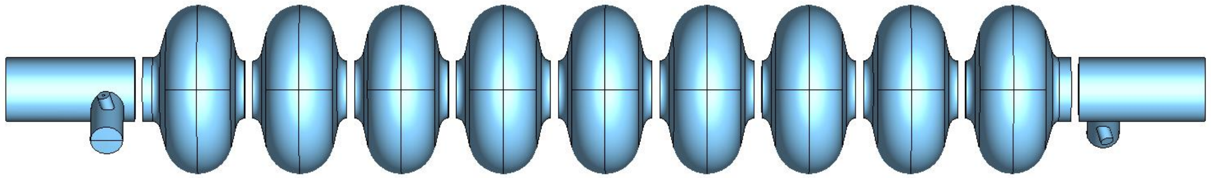
- HTC0e.r
  - MTC (propagating modes) + OTC0e.r
  - Propagating modes are covered by MTC
    - OTC only for evanescent modes
    - Choose $p = k$





APPLICATION
TESLA CAVITY

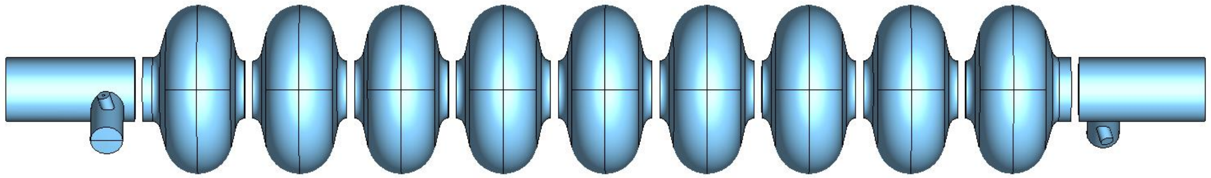
- GMRES iteration scheme
- MTC leads to rapid convergence compared to OTC0
- Large number of modes required (≈ 600)



f=1.3 GHz

APPLICATION
TESLA CAVITY

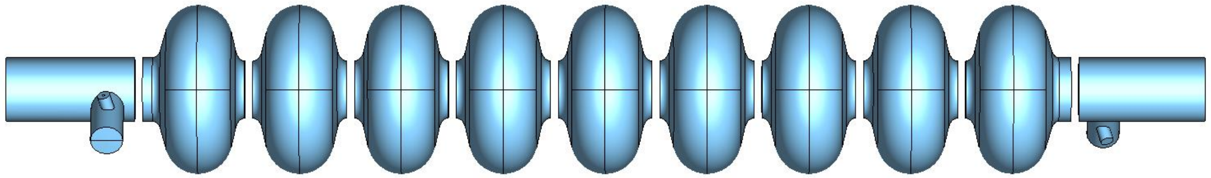
- GMRES iteration scheme
- HTC0 with already 2 modes performs much better than OTC0

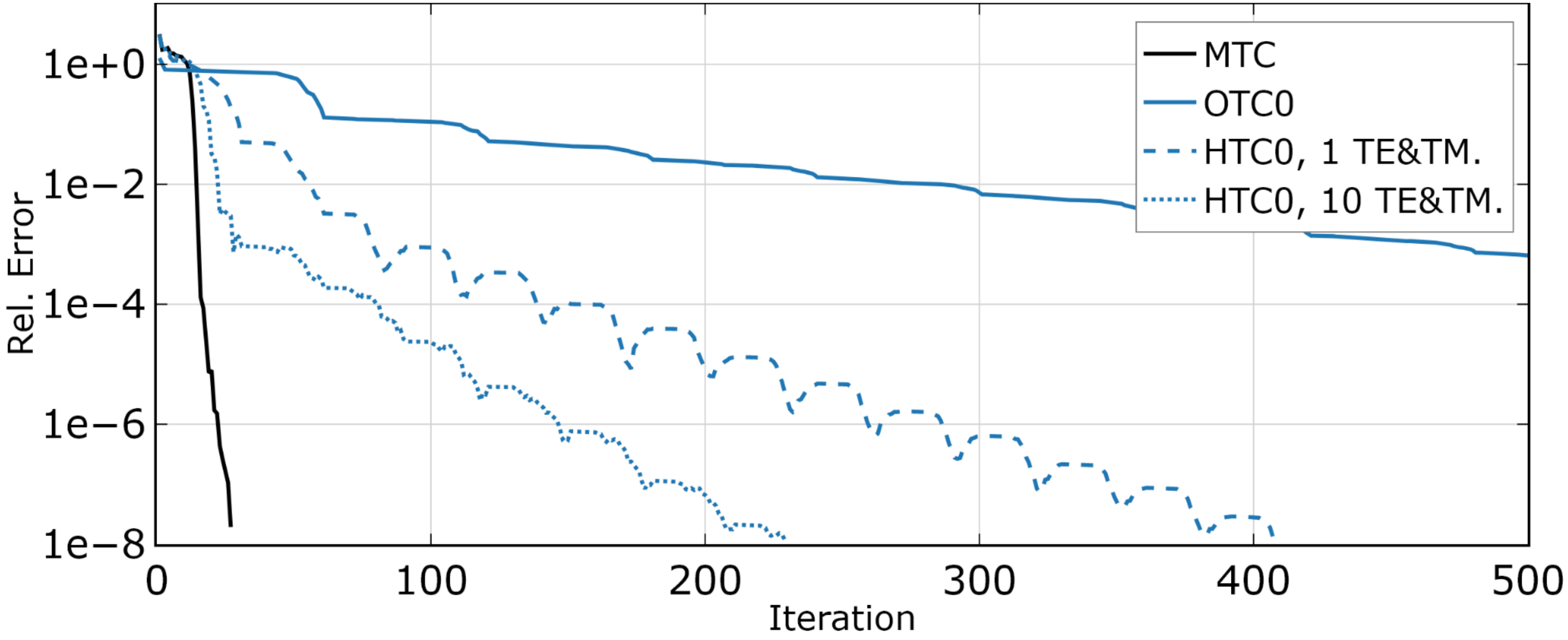


f=1.3 GHz

APPLICATION
TESLA CAVITY

- GMRES iteration scheme
- More modes improve initial and asymptotic convergence rate

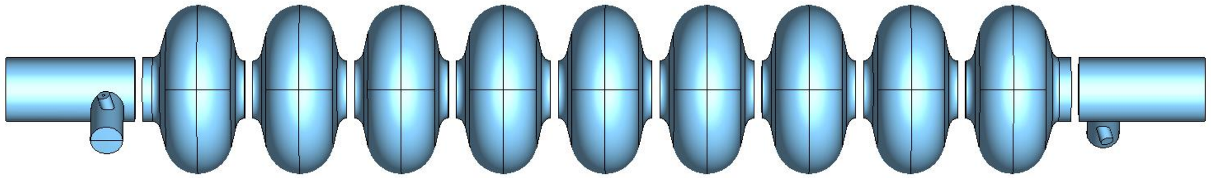


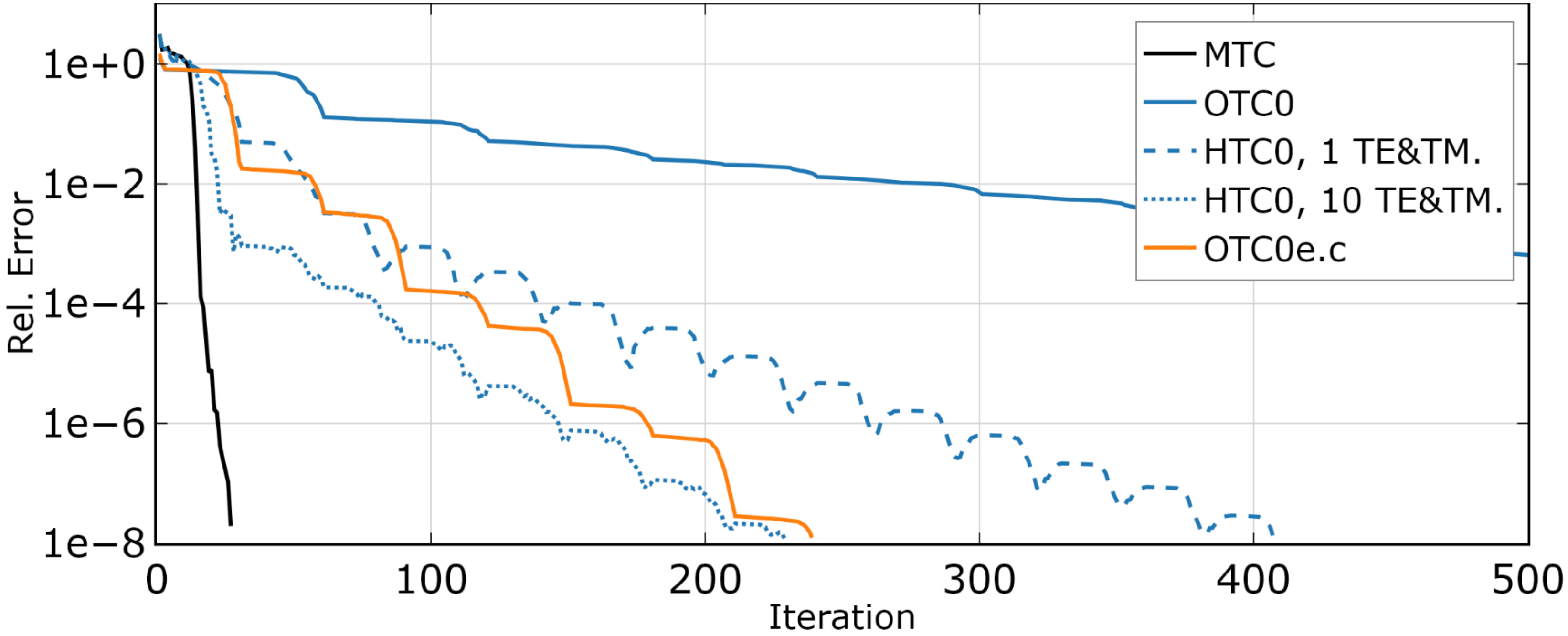


f=1.3 GHz

APPLICATION
TESLA CAVITY

- GMRES iteration scheme
- OTC0e with complex p leads to much faster convergence than OTC0
- Still much slower than MTC

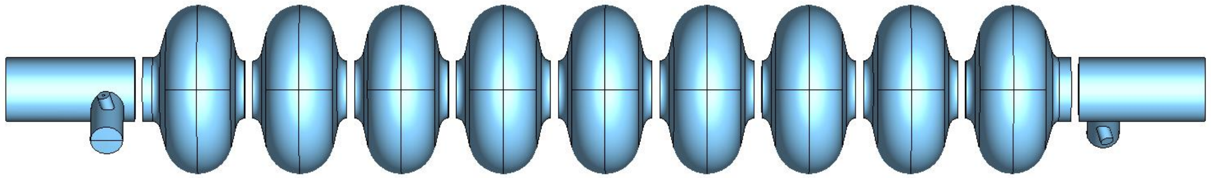


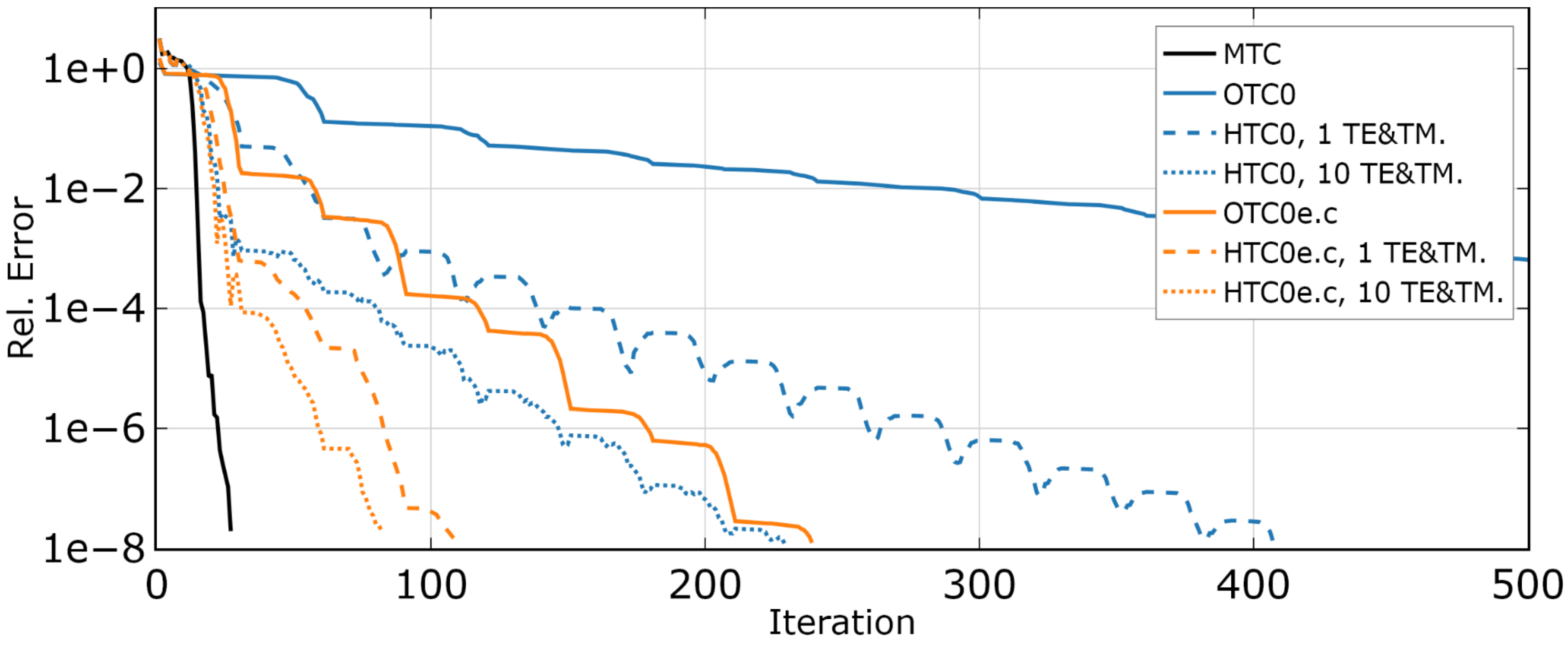


f=1.3 GHz

APPLICATION
TESLA CAVITY

- GMRES iteration scheme
- Hybrid variant improves convergence significantly

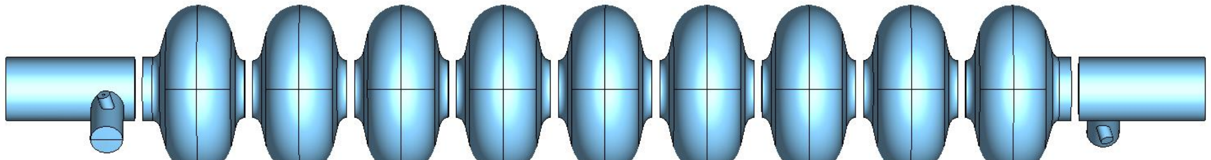




f=1.3 GHz

APPLICATION
TESLA CAVITY

- GMRES iteration scheme
- Further improvement by choosing $p = k$
  - Just 3 times the iterations of MTC
  - Small number of modes (mesh independent)



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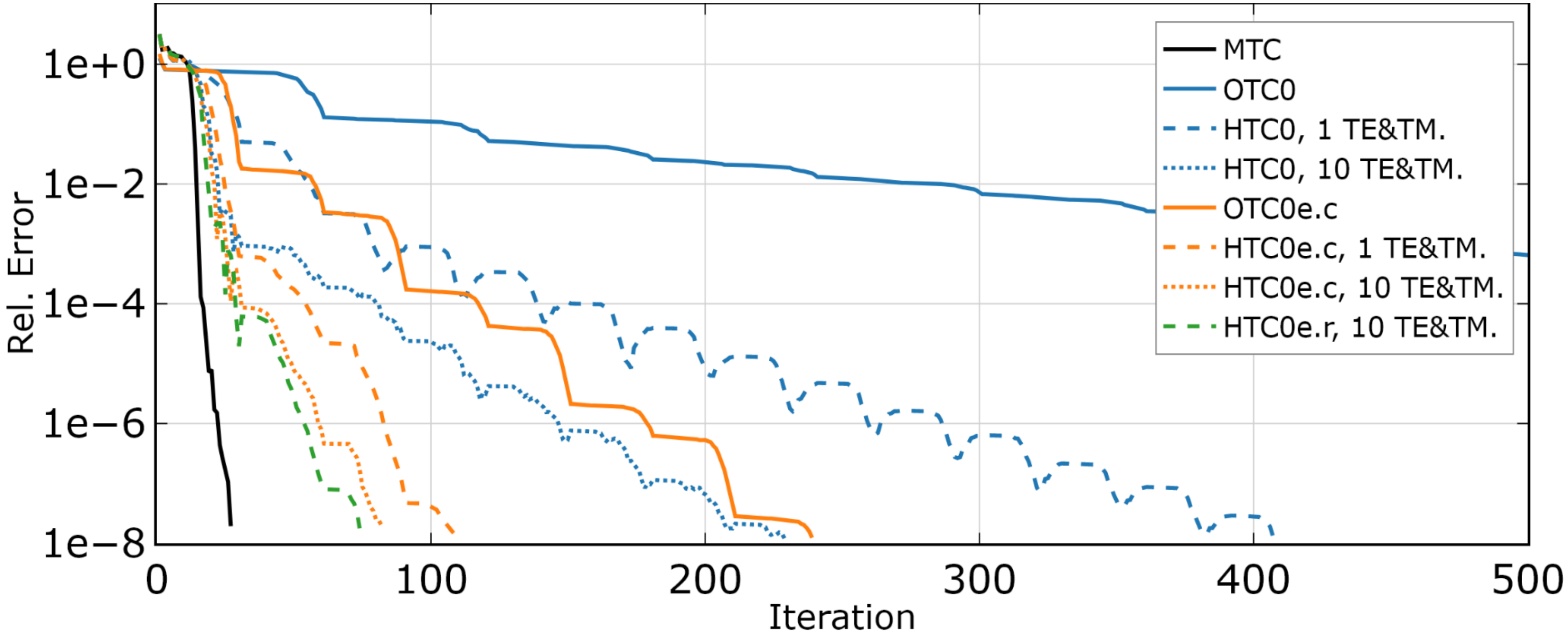

CONCLUSION

- Introduced hybrid transmission conditions (HTC) for DDM
  - Combines MTC with mode truncation and OTCs
  - Rapid initial convergence of MTC
  - Asymptotic convergence of OTC
  - More specific choice of free parameter improves convergence
  - With or without particle beam
- Application to Tesla cavity
  - HTC improves convergence rate significantly compared to OTCs
  - Only small number of modes required