The "1D CSR" Model

Introduction

It is successful, simple, effective, empirical

The Model

From classical wake ($\mathbf{v} = c\mathbf{e}_z$)
to "1d synchrotron radiation" model

How to Choose E_{SR}?

Handling the singularity
Point particles
Coherent Effects
References

Application 1: BCs for FELs

A benchmark case: Comparison with CC

Application 2 (questionable): Undulators

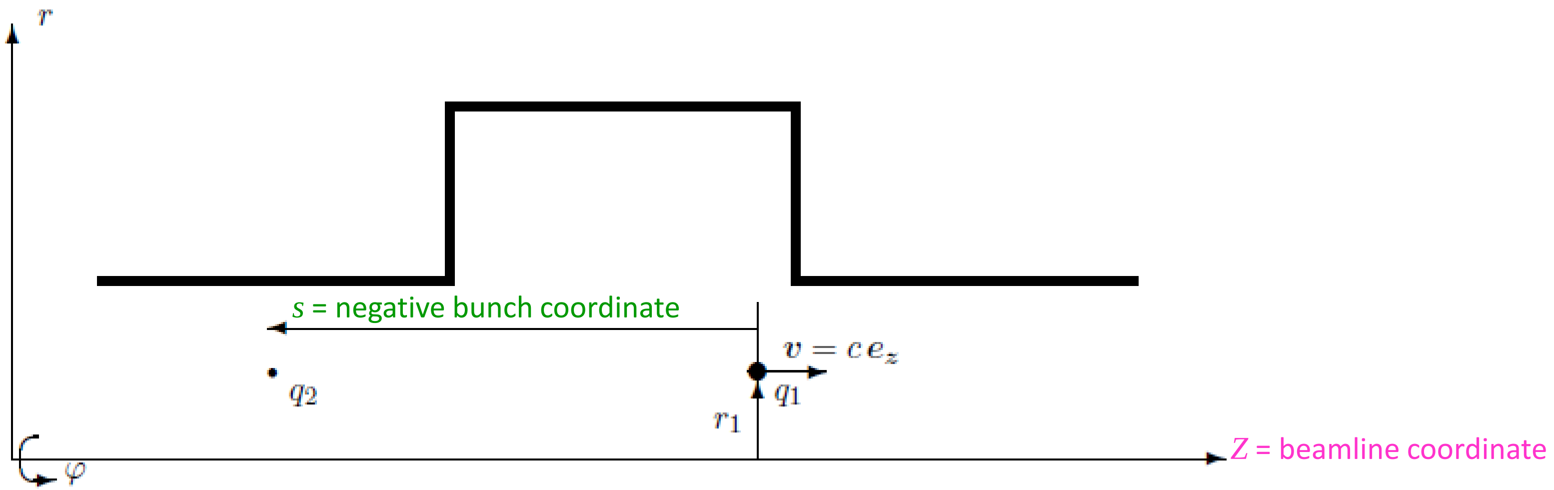
Some simplified FEL sources (point/gaussian disc/infinite plane)

Appendix: 11 Diagrams by 13 Methods

(homework for the interested reader)

The Model

The classical wake ($\boldsymbol{v} = c\boldsymbol{e}_z$)



$$\Delta\mathbf{p} = \frac{q_1 q_2}{c}\mathbf{w}\left(x_1, y_1, x_2, y_2, s\right)$$

sometimes per length

(adiabatic variation versus Z)

$$\Delta\mathbf{p}' = \frac{q_1 q_2}{c}\mathbf{w}'\left(x_1, y_1, x_2, y_2, s, Z\right)$$

effect of all particles

$$\Delta\mathbf{p}'_\Sigma = \frac{q_2}{c}\sum_\nu q_\nu \mathbf{w}'\left(x_1, y_1, x_\nu, y_\nu, s = z_\nu - z_2, Z\right)$$

Equation of motion

either discrete update in a certain reference plane, or continuous

$$\frac{d\mathbf{p}}{dZ} = \mathbf{F}_{\text{ext}}\frac{dt}{dZ} + \Delta\mathbf{p}'_\Sigma$$

Monopole wakes

in structures with symmetry of revolution !!!

$$\mathbf{w}'_{\mathrm{m}}\left(x_1, y_1, x_2, y_2, s, Z\right) = w'_m\left(s, Z\right)\mathbf{e}_z$$

lowest order contribution to longitudinal dynamic!

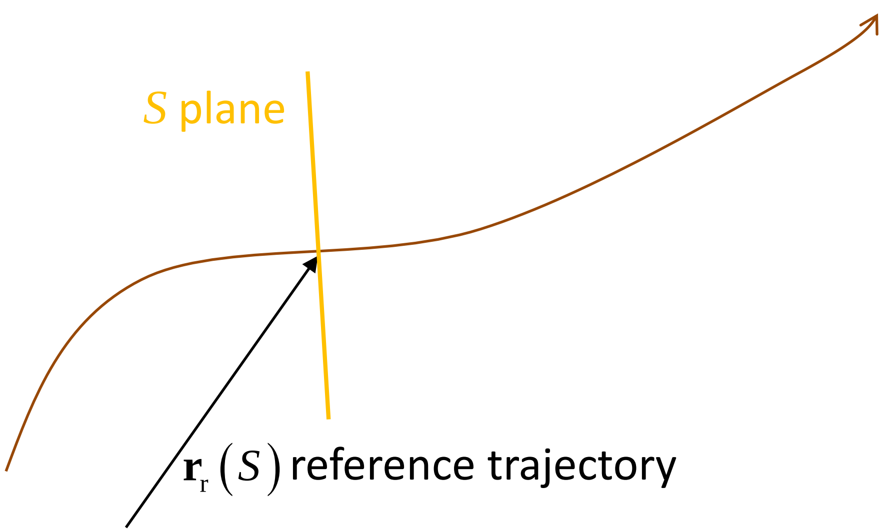
$$\frac{d\mathcal{E}_\Sigma}{dZ} = q_2 \sum_\nu w'_m\left(s_2 - s_\nu, Z\right)$$

energy part of EoM in a magnetic lattice
(transverse part is not changed)

monopole wakes are the scheme for …

The “1D Synchrotron Radiation” Model

$$\boxed{\frac{d\mathcal{E}_\Sigma}{dS} = q_2 \sum_\nu E_{\mathrm{SR}}\left(s_2 - s_\nu, S\right)}$$



accelerator coordinates

S = path length coordinate

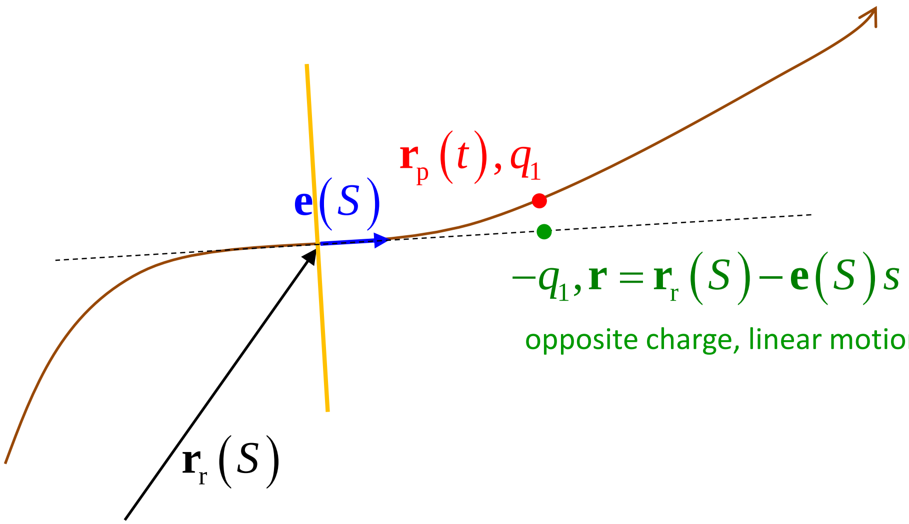
s = positive bunch coordinate

time $t = \dfrac{S - s}{v_{\mathrm{r}}}$

How to choose E_{SR}?

one-particle-model $\mathbf{r}_{\mathrm{r}}(v_{\mathrm{r}}t) = \mathbf{r}_{\mathrm{r}}(S-s)$ reference trajectory, constant velocity

$\rightarrow$ Lienert Wiechert field $\mathbf{E}_{\mathrm{LW}}(\mathbf{r},t)$



$$E_{\mathrm{SR}}(s,S) =?\ \mathbf{e}(S)\cdot\mathbf{E}_{\mathrm{LW}}\left(\mathbf{r}_{\mathrm{r}}(S), \frac{S-s}{v_{\mathrm{r}}}\right)$$

almost! except for the singularity for $s \rightarrow 0$

remedy: extraction of the singularity $\rightarrow$ add the field of $-q_1$

$$E_{\mathrm{SR}}(s,S) = \mathbf{e}(S)\cdot\mathbf{E}_{\mathrm{LW}}\left(\mathbf{r}_{\mathrm{r}}(S), \frac{S-s}{v_{\mathrm{r}}}\right) + \frac{q_1}{4\pi\varepsilon_0}\frac{1}{\gamma_{\mathrm{r}}^2 s|s|}$$

this is all!
the rest is implementation

About E_{SR}

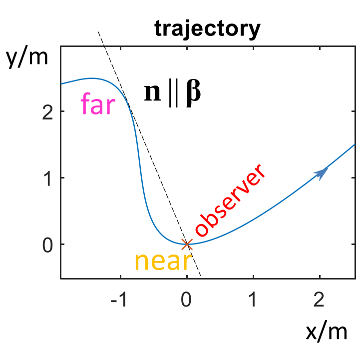
$$E_{\mathrm{SR}}(s,S) = \frac{q_1}{4\pi\varepsilon_0}\mathbf{e}(S)\cdot\left(\frac{1}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\left\{\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2 R^2}+\frac{\mathbf{n}\times\left((\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{cR}\right\}\right)_{S'} + \frac{q_1}{4\pi\varepsilon_0}\frac{1}{\gamma_{\mathrm{r}}^2 s|s|}$$

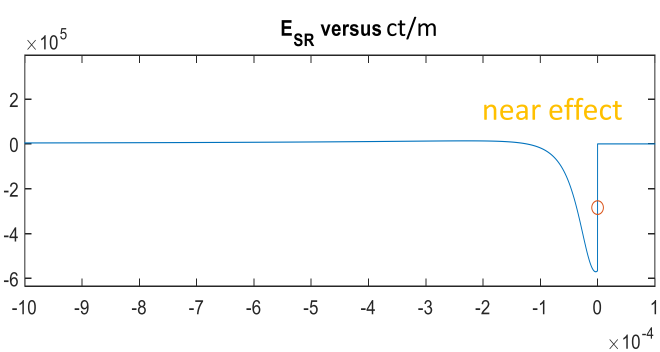
$$s = S - S' - \beta_{\mathrm{r}} R$$

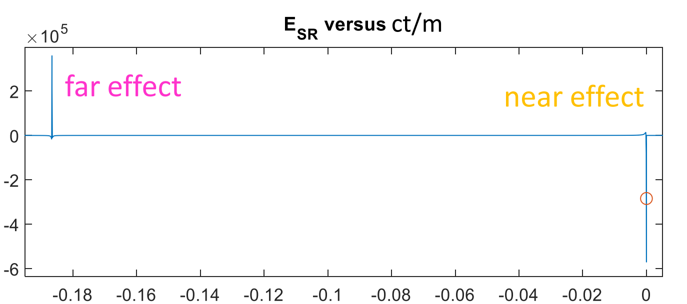
$$R = \left\|\mathbf{r}_{\mathrm{r}}(S) - \mathbf{r}_{\mathrm{r}}(S')\right\| \qquad \mathbf{n} = \frac{\mathbf{r}_{\mathrm{r}}(S) - \mathbf{r}_{\mathrm{r}}(S')}{R} \qquad \boldsymbol{\beta} = \beta_{\mathrm{r}}\mathbf{e}(S') \qquad \mathbf{e}(S) = \partial_S \mathbf{r}_{\mathrm{r}}(S)$$

minor numerical problems:

- E_{SR} is a bit implicit, but this is not a numerical disadvantage
- singularity extraction needs some care ($\rightarrow$ Taylor expansion)
- $(E_{\mathrm{SR}}(0-,S) + E_{\mathrm{SR}}(0+,S))/2$ is point particle loss; compare beam loading theorem; same result as far field radiation
- $E_{\mathrm{SR}}(s > 0,S)$ negligible (tail $\rightarrow$ head interaction)
- case $\mathbf{n} \parallel \boldsymbol{\beta}$ might require some care







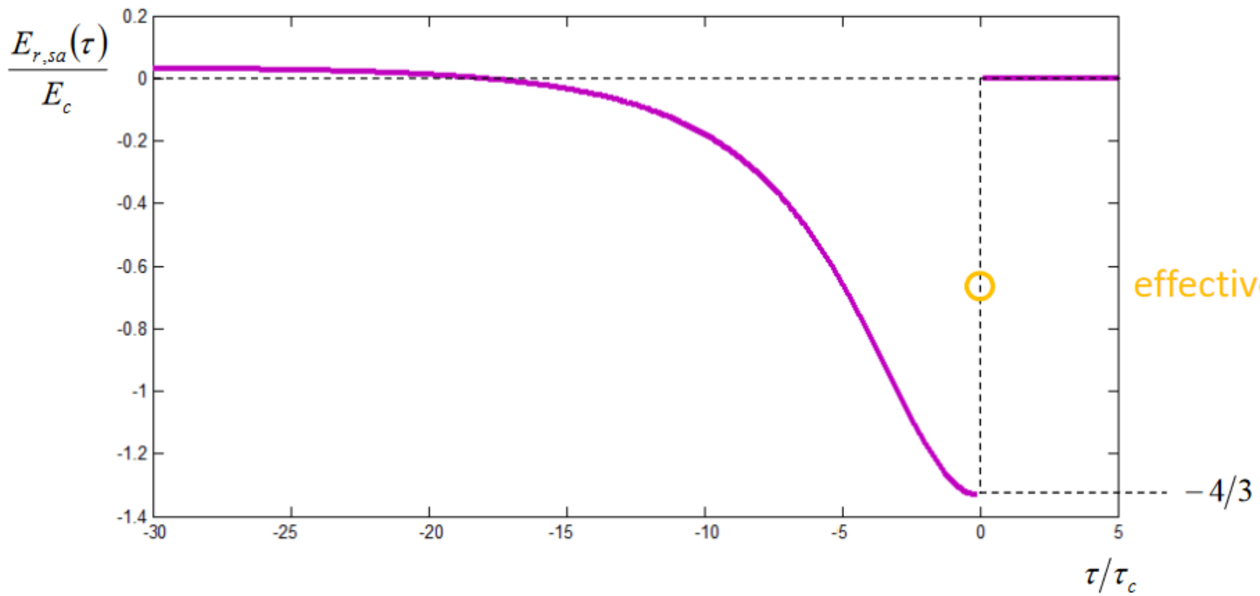
from https://www.desy.de/fel-beam/s2e/publications/CSR_tutorial_with_L.pdf

near field

small angle approximation of residual part

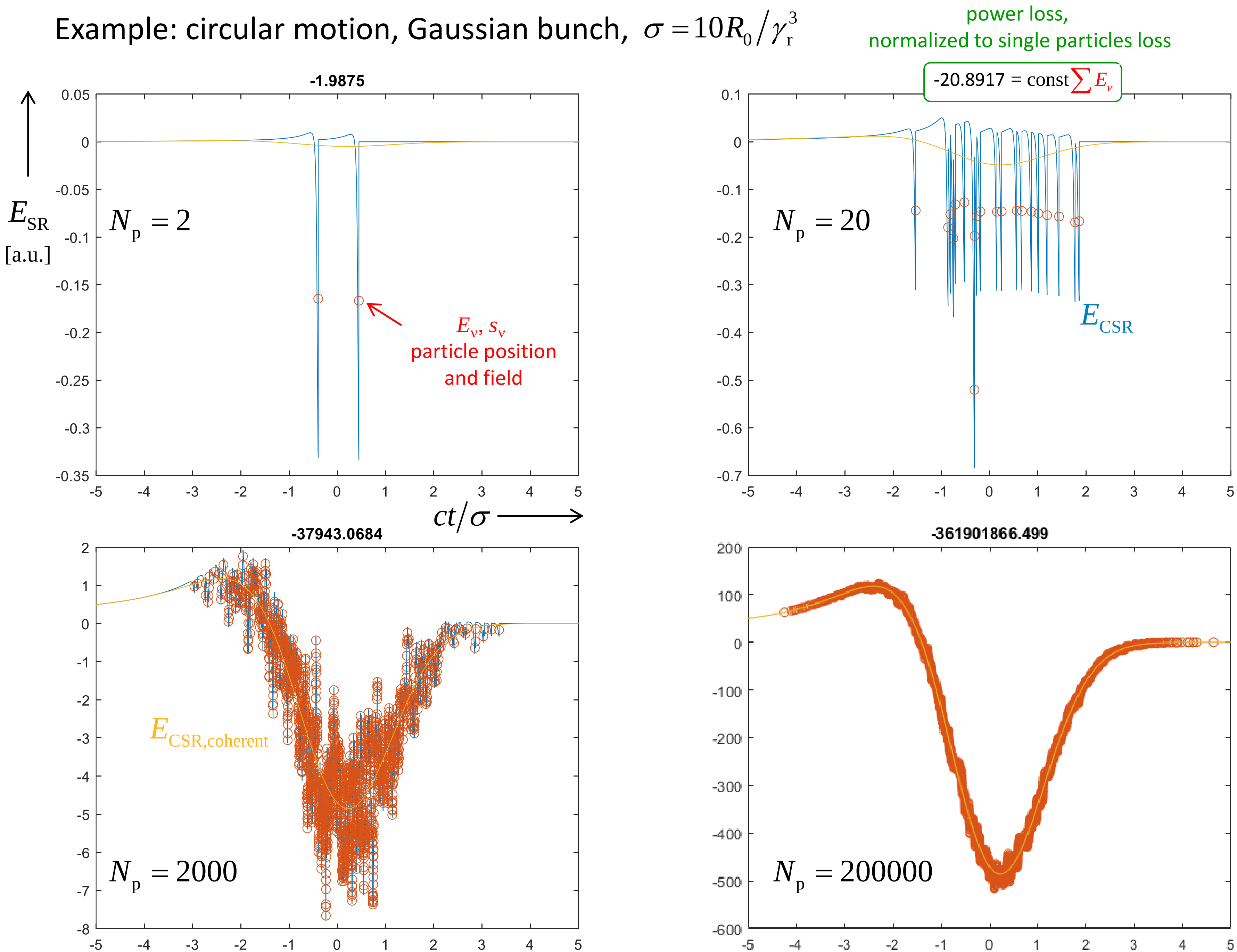
$$E_{r,sa}(\tau < 0) = \frac{q_0 \gamma^4}{4\pi\varepsilon R_0^2} \frac{-32}{4+\phi^2} \frac{\partial}{\partial \phi}\left(\frac{\phi(8+\phi^2)}{(4+\phi^2)(12+\phi^2)} \right)$$

with $\tau = \frac{R_0}{c\gamma^3}\left(\phi/2 + \phi^3/24\right)$

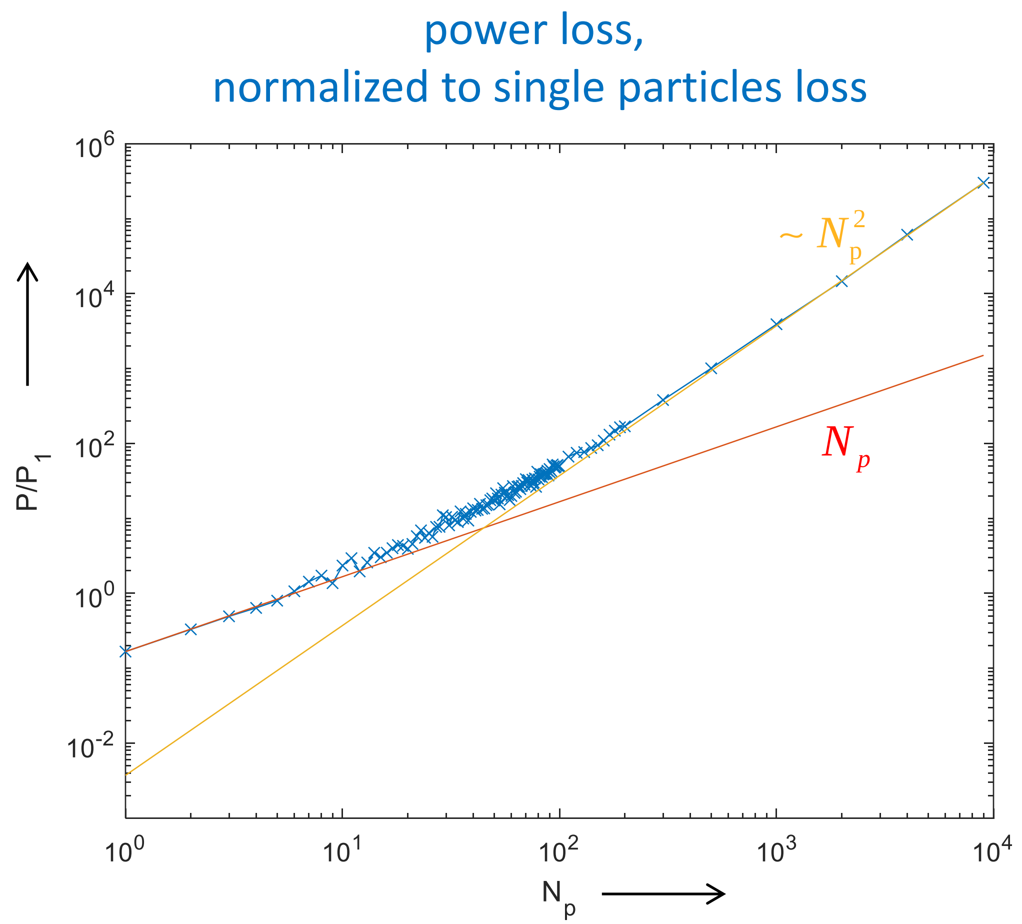


power loss of one particle: $q c_0 \frac{E_{r,sa}(0-) + E_{r,sa}(0+)}{2} = P_{rad}$ far field radiation

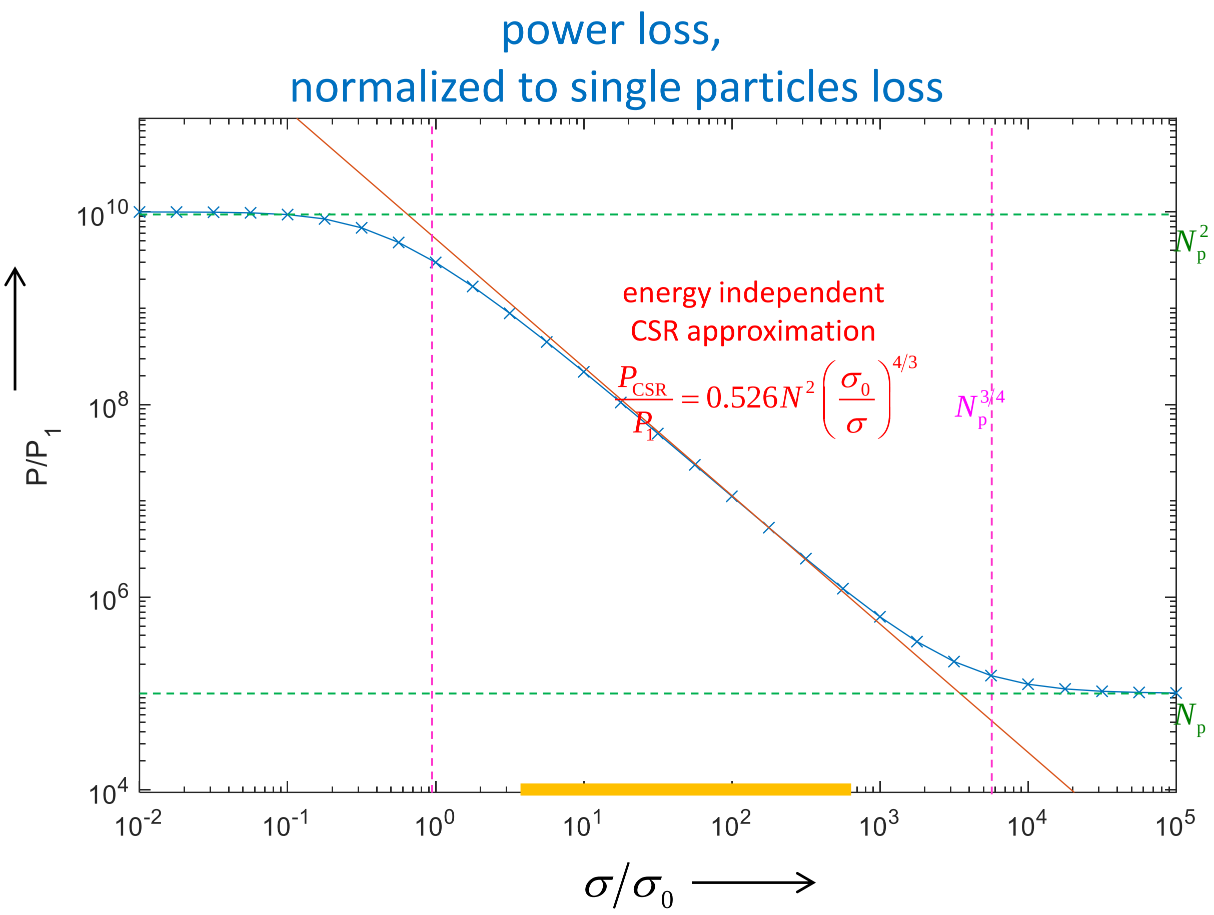
Example: circular motion, Gaussian bunch, $\sigma = 10R_0/\gamma_r^3$



Example: circular motion, Gaussian bunch, $\sigma = 10R_0/\gamma_r^2$



Example: circular motion, Gaussian bunch, $N_p = 10^5$, $\sigma = var\ \sigma_0$ with $\sigma_0 = R_0 / \gamma_r^3$

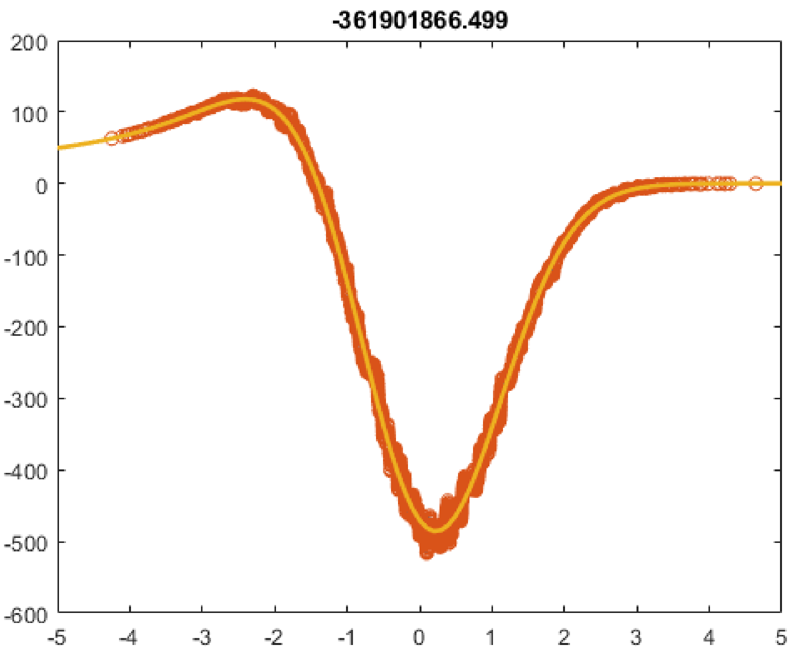


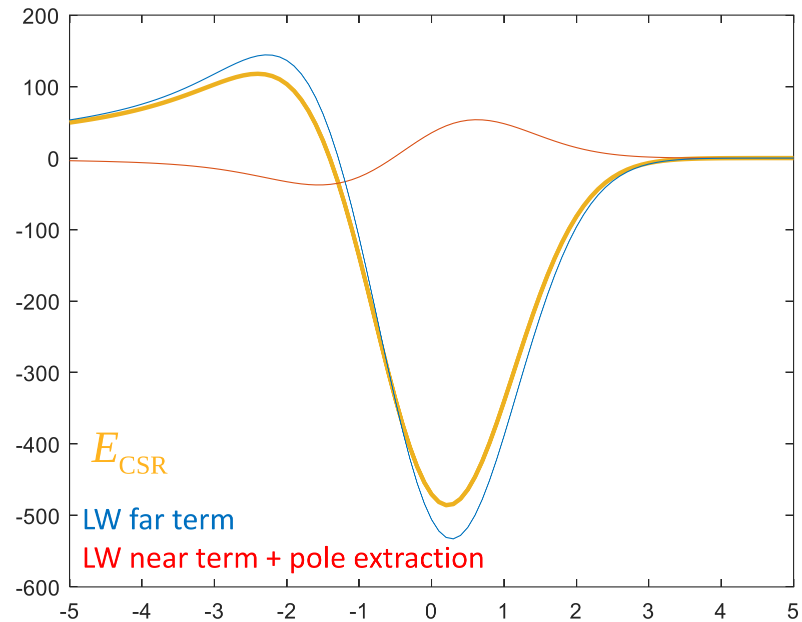
$$P_1 = \frac{e^2 c}{\varepsilon_0} \frac{1}{6\pi} \frac{\gamma_r^4}{R_0^2}$$

$$P_{CSR} = \frac{e^2 c}{\varepsilon_0} \frac{\Gamma(5/6)}{4\pi^{3/2} 6^{1/3}} \frac{1}{R_0^{2/3} \sigma^{4/3}}$$

super-coherent energy independent incoherent

Coherent Synchrotron Radiation





$$\lambda(s) = q\sum_{\{s_\nu\}} \delta(s - s_\nu) \quad \text{binning and smoothing} \rightarrow \quad \lambda(s) = \sum \lambda_k b(s - k\Delta)$$

basis function

$$\mathcal{E}'(s) = q\sum_{\nu} E_{\mathrm{SR}}(s - s_\nu, S)$$

$$\mathcal{E}'(s) = \int E_{\mathrm{SR}}(s - \tilde{s}, S)\lambda(\tilde{s})d\tilde{s} = E_{\mathrm{CSR}}(s)$$

$\downarrow$

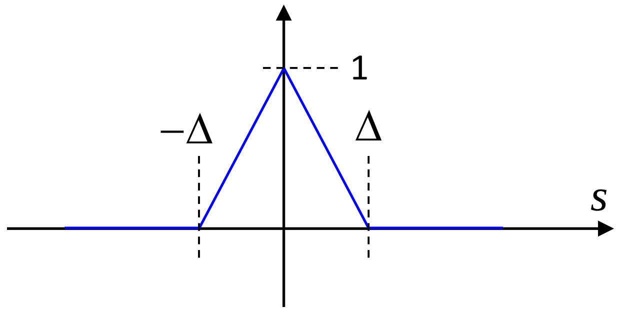
$$E_{\mathrm{CSR}}(m\Delta) = \Delta\sum K_{m-k}^{(S)}\lambda_k$$

numerical kernel

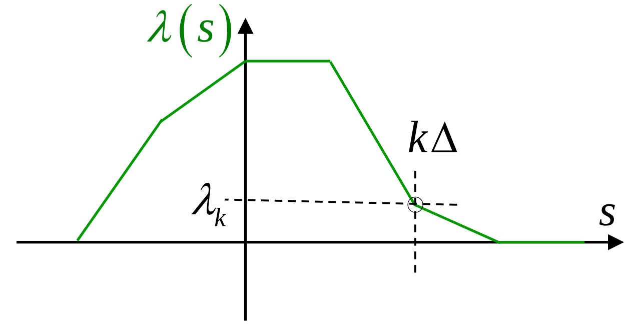
$$K_{m-k}^{(S)} = \frac{1}{\Delta}\int E_{\mathrm{SR}}(-\tilde{s}, S)b(\tilde{s} + (m-k)\Delta)d\tilde{s}$$

Numerical Kernel $b(s) = \max\{1 - |s/\Delta|, 0\}$

linear interpolation basis



line charge density



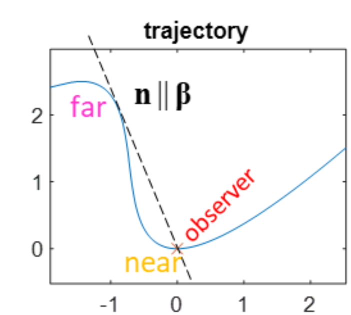
$$K_m^{(S)} = \frac{1}{\Delta} \int_{(m-1)\Delta}^{(m+1)\Delta} E_{\text{SR}}(\tilde{s}, S)\left(1 - |\tilde{s}/\Delta - m|\right) d\tilde{s}$$

$m > 0$ $\quad K_m^{(S)} = 0$

$m = 0$ $\quad K_0^{(S)} = \frac{1}{\Delta} \int_{-\Delta}^{0} E_{\text{SR}}(\tilde{s}, S)(1 + \tilde{s}/\Delta) d\tilde{s}$

m $\quad K_m^{(S)} = \frac{1}{\Delta} \int_{(m-1)\Delta}^{m\Delta} E_{\text{SR}}(\tilde{s}, S)(1 - m + \tilde{s}/\Delta) d\tilde{s} + \frac{1}{\Delta} \int_{m\Delta}^{(m+1)\Delta} E_{\text{SR}}(\tilde{s}, S)(1 + m - \tilde{s}/\Delta) d\tilde{s}$

$m <\approx -1$ $\quad K_m^{(S)} \approx E_{\text{SR}}(m\Delta, S)$ sampling!



but: use exact integral if $\boldsymbol{\beta} \parallel \mathbf{n}$ happens in the integration range

Some references

Saldin, Schneidmiller, Yurkov: Radiative Interaction of Electrons in a Bunch Moving in an Undulator, TESLA-FEL-1997-08.
later: Nucl. Instrum. Methods Phys. Res., Sect. A 417, 158 (1998)

$$\frac{d\mathscr{E}}{c\,dt} = e^2 \int_{-\infty}^{s} ds'\, \Phi(s-s',S) \frac{d\lambda(s')}{ds'},$$

implemented in **Xtrack** & **Ocelot**
= projected "PRJ" method

Sagan, Hoffstaetter, Mayes, Udom Sae-Ueng: Extended one-dimensional method for coherent synchrotron radiation including shielding, Phys. Rev., ST – Accelerators and Beams, 12, 040703 (2009)

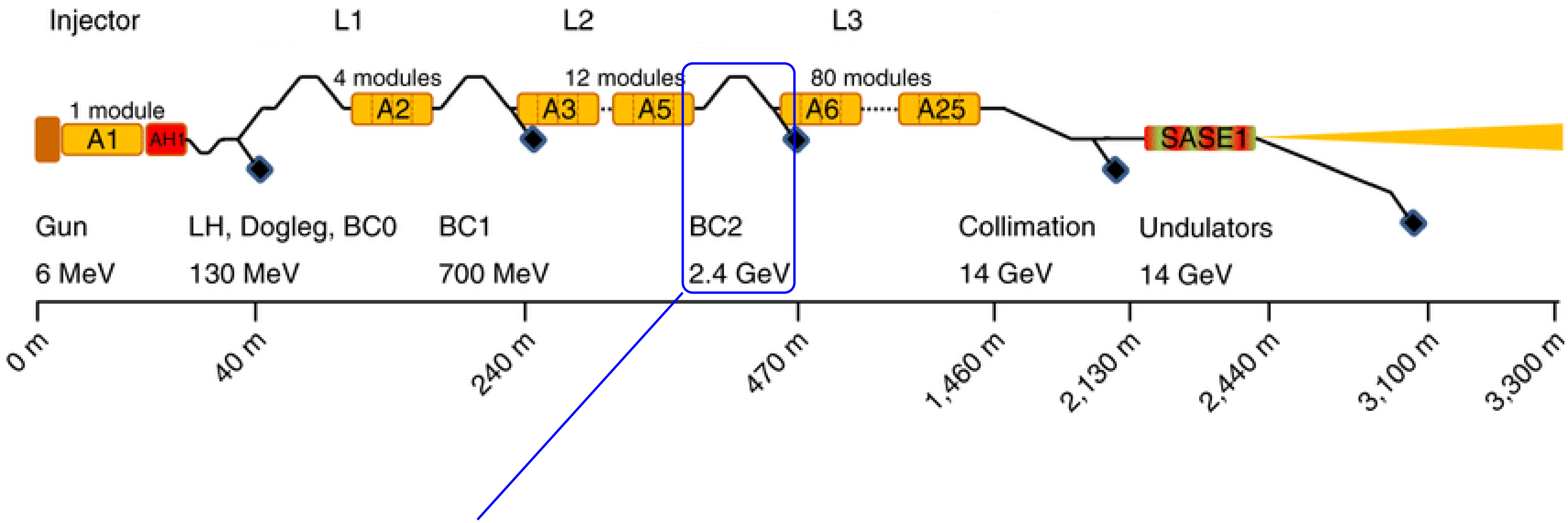
$$\mathbf{E}_{SC} = \frac{e}{4\pi\epsilon_0} \frac{\text{sign}(\zeta)\mathbf{n}}{\gamma^2 \zeta^2}, \qquad \mathbf{E}_{CSR} = \mathbf{E} - \mathbf{E}_{SC}, \qquad (3)$$

where sign(ζ) is 1 for positive and -1 for negative ζ. The rate $K \equiv d\mathcal{E}/ds$ at which the kicked particle is changing energy due to the field of the source particle is

$$K = K_{CSR} + K_{SC} = e\mathbf{n} \cdot \mathbf{E}_{CSR} + e\mathbf{n} \cdot \mathbf{E}_{SC}. \qquad (4)$$

Application 1: BCs for FELs

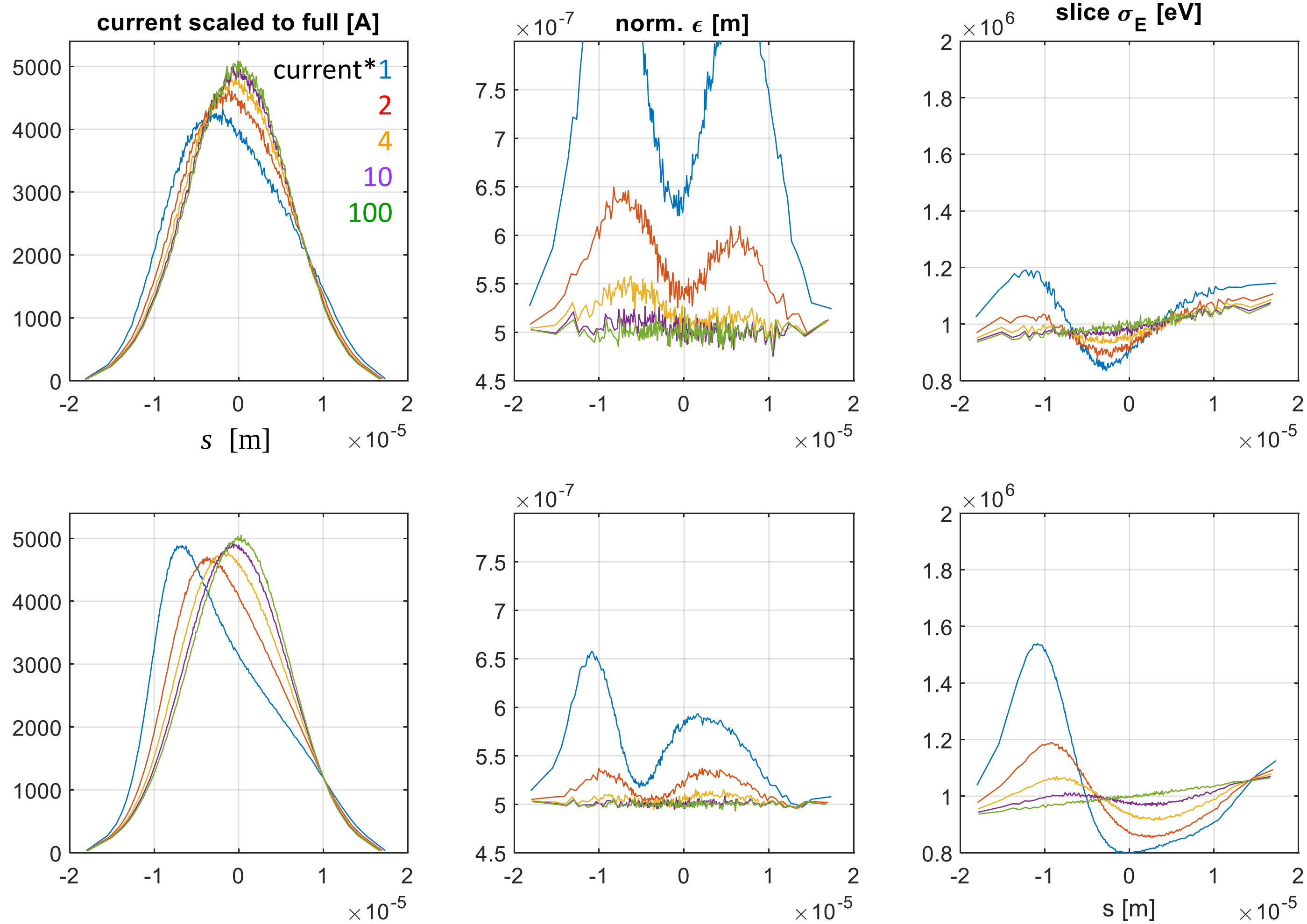
A benchmark case: BC2



ideal gaussian bunch, linear chirp
compression of 250 pC, 750 A to 5000 kA
with slice energy spread at exit 1 MeV
optics as for BC2
emittance = 0.5 μm/(γβ)
BC2 deflection angle 2.36 deg (r56 = 30.03 mm)

reduce charge:
q/250 pC = 1, 0.5, 0.25, 0.1, 0.01

figures in row 1 = ImpactZ{CUM+CSR}
figures in row 2 = Xtrack{PRJ}+NWTN



q/250 pC = 1, 0.5, 0.25, 0.1, 0.01

	methods	**plots** (slice analysis)
2d perturbation method full dynamic with fields of unperturbed source	CC{MA}+NWTN	norm. emittance [m]
	CC{MA}+HMLT	slice σ_E [eV]
	CC{CUM}+NWTN	slice σ_E/C [eV]
	CC{CUM}+HMLT	current scaled to full [A]
	CC{CUM+PRJ}+NWTN	C=local compression
	CC{CUM+PRJ}+HMLT	av(Δ_E [eV])
	CC{PRJ}+NWTN	av(Δ _E [eV])-linear correlation
3d, parallel dynamic with self effects	ImpactZ{CUM+CSR}	x offset [m]
	ImpactZ{CUM}	x'' offset [rad]
3d dynamic with self effects same models as in Ocelot	Xtrack{PRJ}+NWTN	Twiss σ
	Xtrack{CUM+PRJ}+NWTN	Twiss β [m]
	Xtrack{CUM2+PRJ}+NWTN	
	Xtrack{CUM}+NWTN	

NWTN = Newtonian equation of motion
HMLT = Hamiltonian equation of motion

MA = full Maxwell-EM field
PRJ = “1d” CSR model as described above
CSR = ImpactZ “1d” CSR model
CUM = collective uniform motion
CUM2 = collective uniform motion, modified force

the complete comparison is in the appendix

	methods (reference method: CC{MA}+NWTN)	**plots** (slice analysis)
2d perturbation method full dynamic with fields of unperturbed source	CC{MA}+NWTN	norm. emittance [m]
	CC{MA}+HMLT	slice σ_E [eV]
	CC{CUM}+NWTN	slice σ_E/C [eV]
	CC{CUM}+HMLT	current scaled to full [A]
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	Xtrack{CUM}+NWTN	

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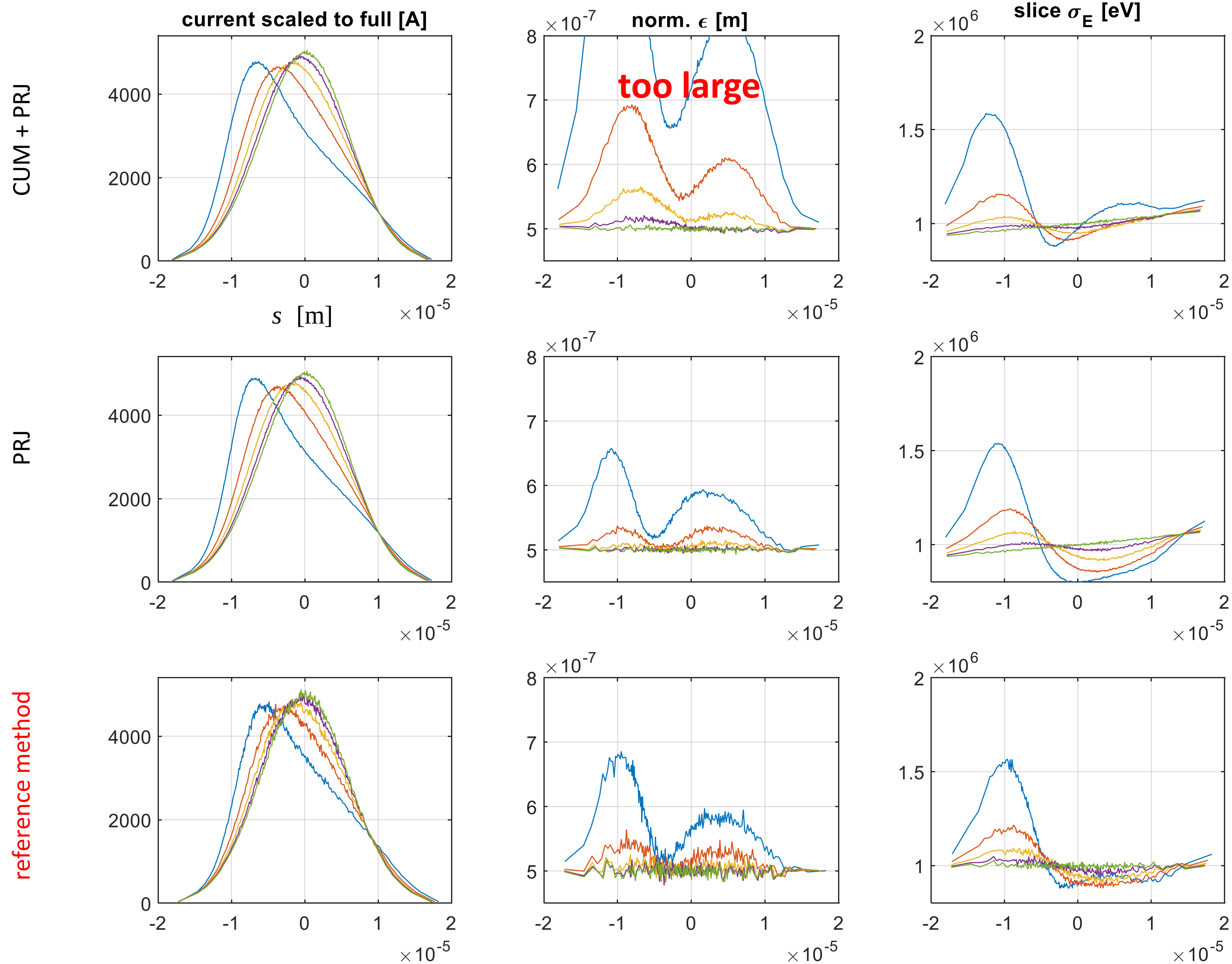
figures in row 1 = Xtrack{CUM+PRJ}+NWTN
figures in row 2 = Xtrack{PRJ}+NWTN
figures in row 3 = CC{MA}+NWTN



Application 2 (questionable): Undulators

three types of **infinitely thin sources** in undulator motion
motion into z direction, oscillation in x direction

(1) point source $\rightarrow \approx$ 1d CSR model $\rightarrow$ Hertzian dipole
(2) round gaussian disc $\rightarrow$ Gaussian beam
(3) infinite disc $\rightarrow$ = 1d FEL model $\rightarrow$ plane waves

simplifications:

- very small transverse oscillation (undulator parameter K $\rightarrow$ 0)
- $\rightarrow$ constant longitudinal velocity
- Lorentz transformation to rest frame $\rightarrow$ DC + time harmonic fields
- only **fundamental time harmonic** part

source terms:

$$\mathbf{J}_1 = \mathbf{e}_x \delta(z) \operatorname{Re}\left\{ \exp(j\omega t) A \delta(x) \delta(y) \right\}$$

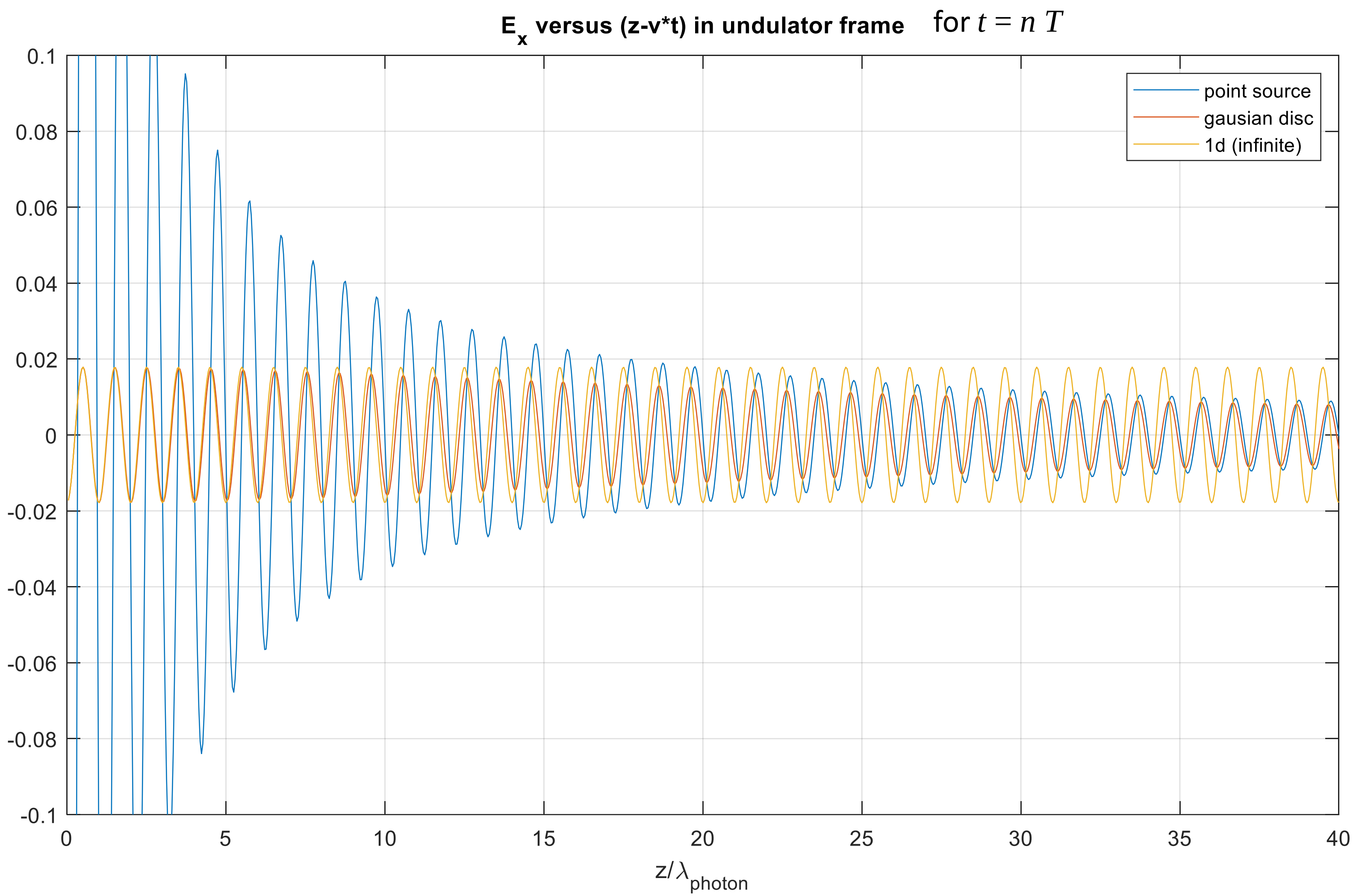
$$\mathbf{J}_2 = \mathbf{e}_x \delta(z) \operatorname{Re}\left\{ \exp(j\omega t) \frac{A}{\sigma_r^2} g\left(\frac{x}{\sigma_r}\right) g\left(\frac{y}{\sigma_r}\right) \right\}$$

round gaussian disc $\int \mathbf{J}_2 dxdy = \int \mathbf{J}_1 dxdy$

$$\mathbf{J}_3 = \mathbf{e}_x \delta(z) \operatorname{Re}\left\{ \exp(j\omega t) \frac{A}{\sigma_r^2} g(0) g(0) \right\}$$

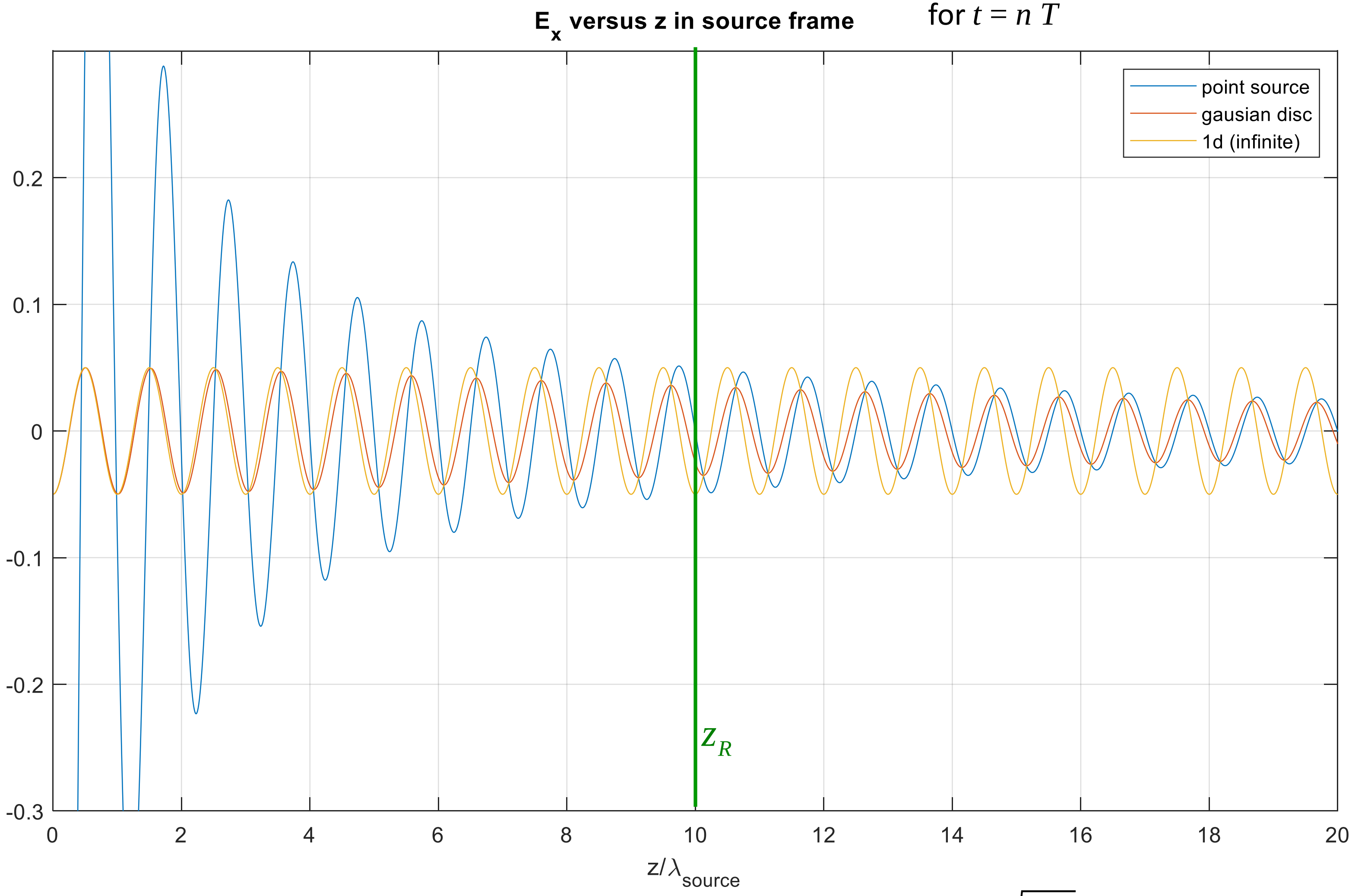
infinite disc $\mathbf{J}_3(0,0,z,t) = \mathbf{J}_2(0,0,z,t)$

transverse E-field in **undulator** frame



transverse E-field in **source** frame

for $t = n\,T$



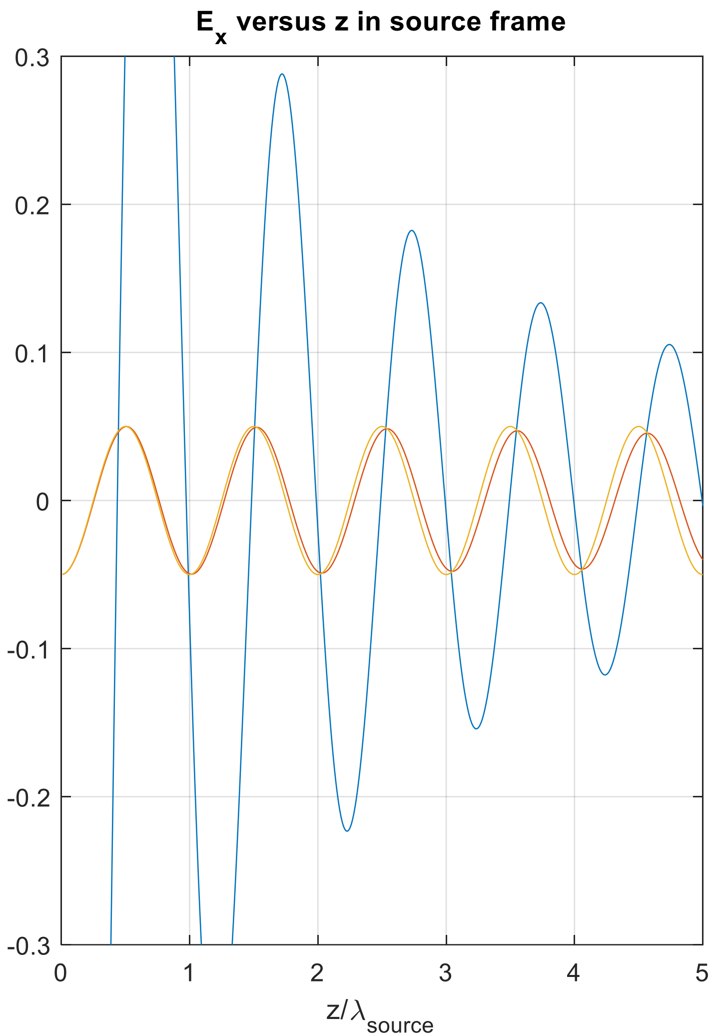
there is a new length scale with $z_R = k\sigma_r^2$

Rayleigh length

here: $\sigma_r = \sqrt{\frac{10}{2\pi}}\lambda_{\text{source}}$ $\quad k = 2\pi/\lambda_{\text{source}}$

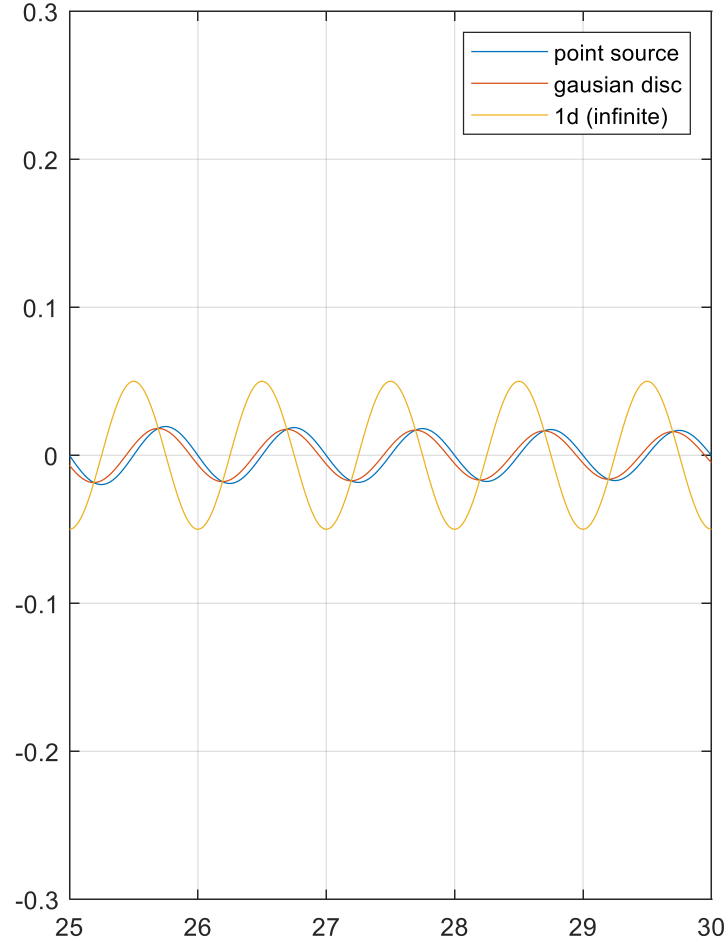
$z_R = 10\lambda_{\text{source}}$

$z << z_R$



1d (infinite) is similar to gaussian disc
(amplitude and phase)

$z >> z_R$



point source is similar to gaussian disc
(amplitude and phase)
g.d. is 90 deg shifted compared to infinite

$$E_{1x}\left(x=0,y=0,z>0\right)=\mathrm{Re}\left\{-Z_0Ak^2\frac{\exp\left(jk\left(ct-z\right)\right)}{4\pi kz}\left(j+\frac{1}{kz}+\frac{1}{j\left(kz\right)^2}\right)\right\}$$

$$E_{2x}\left(x=0,y=0,z>0\right)=\mathrm{Re}\left\{-Z_0Ak^2\frac{\exp\left(jk\left(ct-z\right)\right)}{4\pi k\left(z_R-jz\right)}\right\}=\mathrm{Re}\left\{-Z_0Ak^2\frac{\exp\left(jk\left(ct-z\right)+j\psi\left(z\right)\right)}{4\pi k\sqrt{z_R^2+z^2}}\right\}$$

$$E_{3x}\left(x,y,z>0\right)=\mathrm{Re}\left\{-Z_0Ak^2\frac{\exp\left(jk\left(ct-z\right)\right)}{4\pi kz_R}\right\}$$

Gouy phase → π/2 phase shift for large z

conclusion

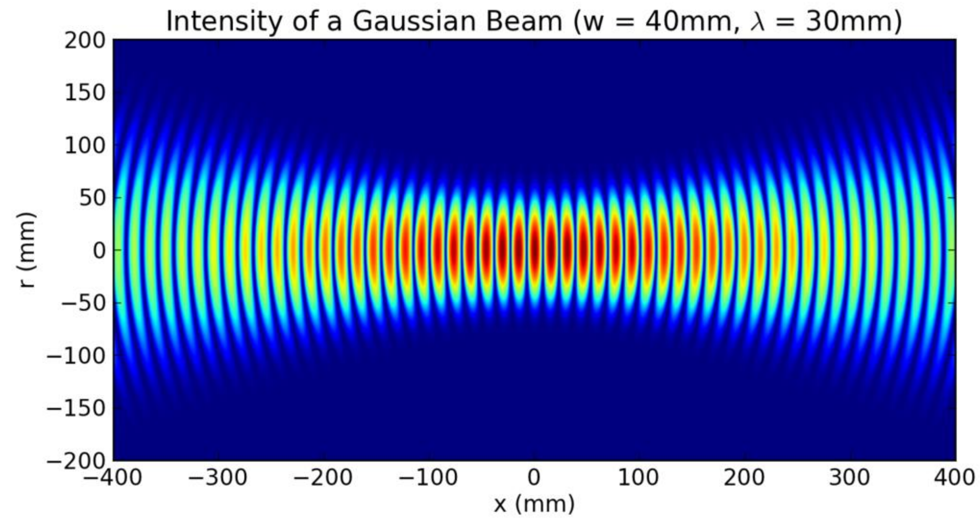
finite model is not in agreement with point model nor infinite 1d model for the full range

$z \ll z_r$: infinite 1d model ≈ finite disc

$z \gg z_r$: point model ≈ finite disc

round gaussian laser

see text books or Wikipedia



$$\mathbf{E}_{\mathrm{L}} = \mathbf{e}_x \,\mathrm{Re}\left\{ \frac{Z_0 A}{4\pi\sigma_r^2} \exp\left(j\omega\left(t - z/c\right) + j\psi\left(z\right)\right) \frac{w_0}{w\left(z\right)} \exp\left(-\left(\frac{\rho}{w\left(z\right)}\right)^2 - j\frac{k\rho^2}{2R\left(z\right)}\right)\right\}$$

$z_R = k\sigma_r^2$ Rayleigh length

$\psi\left(z\right) = \arctan\left(z/z_R\right)$ Gouy phase → π/2 phase shift for large z

$R\left(z\right) = z\left(1 + \left(z_R/z\right)^2\right)$ curvature of wave front

$w_0 = \sqrt{2}\sigma_r \qquad w = w_0\sqrt{1 + \left(z/z_R\right)^2}$

equivalent source

a PEC mirror at $z = 0$ shields the laser for $z > 0$;

therefore the sources on the mirror create the field $\mathbf{E}_3\left(z > 0\right) = -\mathbf{E}_{\mathrm{L}}\left(z > 0\right)$

the current density on the mirror is $\mathbf{J}_3 = -\mathbf{J}_{\mathrm{L}}$

Appendix: 11 Diagrams by 13 Methods

11 slides with diagrams are organized as:

Newtonian equation of motion (spans columns 2–5)

Maxwell fields	CC{MA} +HMLT	CC{MA} +NWTN				
PRJ (projected method)		CC{PRJ} +NWTN		Xtrack{PRJ} +NWTN		
CUM+PRJ/CSR	CC{CUM+PRJ} +HMLT	CC{CUM+PRJ} +NWTN	ImpactZ{CUM +CSR}	Xtrack{CUM +CSR}+NWTN	Xtrack{CUM2 +CSR}+NWTN	CUM2+…
CUM (collective uniform motion)	CC{CUM} +HMLT	CC{CUM} +NWTN	ImpactZ{CUM}	Xtrack{CUM} +NWTN		

modified transverse force

$F_\perp = qE_\perp / \gamma_r^2$

instead of

$q\mathbf{e}_\perp \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$

CC = 2d

perturbation method

full dynamic with fields of unperturbed source

ImpactZ = 3d

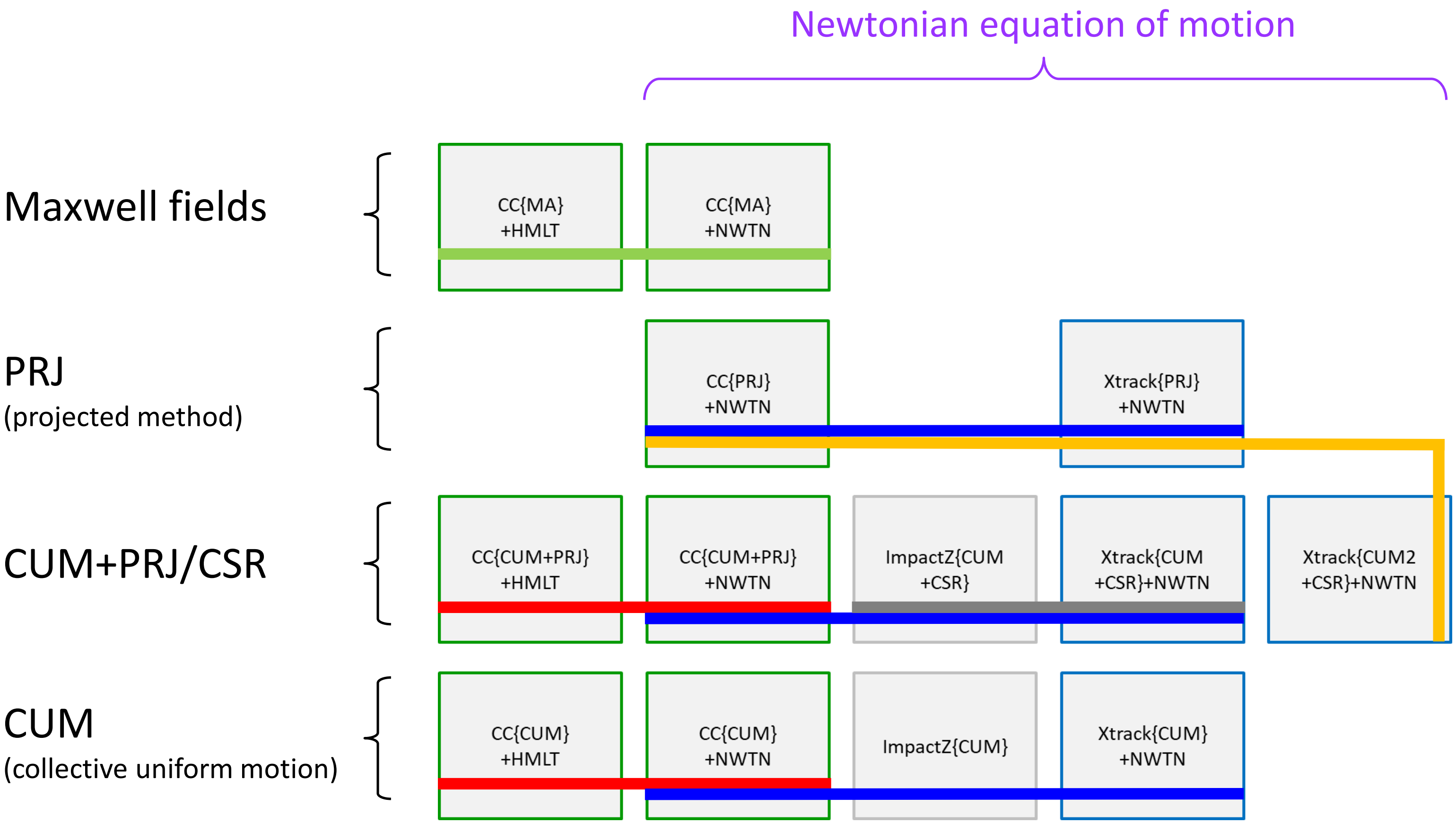
self effects

CSR model with less effects

Xtrack = 3d

dynamic with **self effects**
same models as in Ocelot

some conclusions (before the diagram slides):



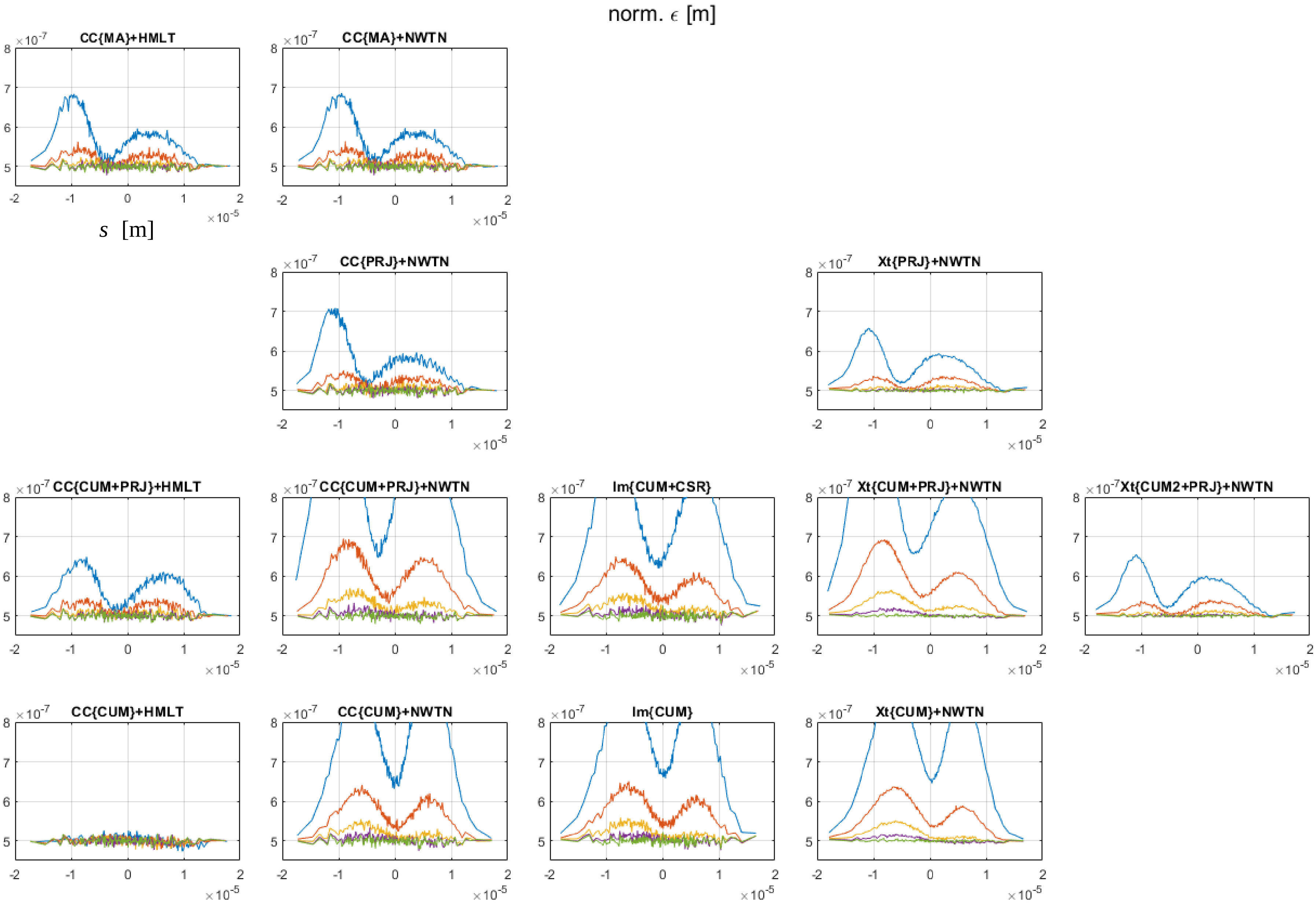
very good agreement, **reference method**

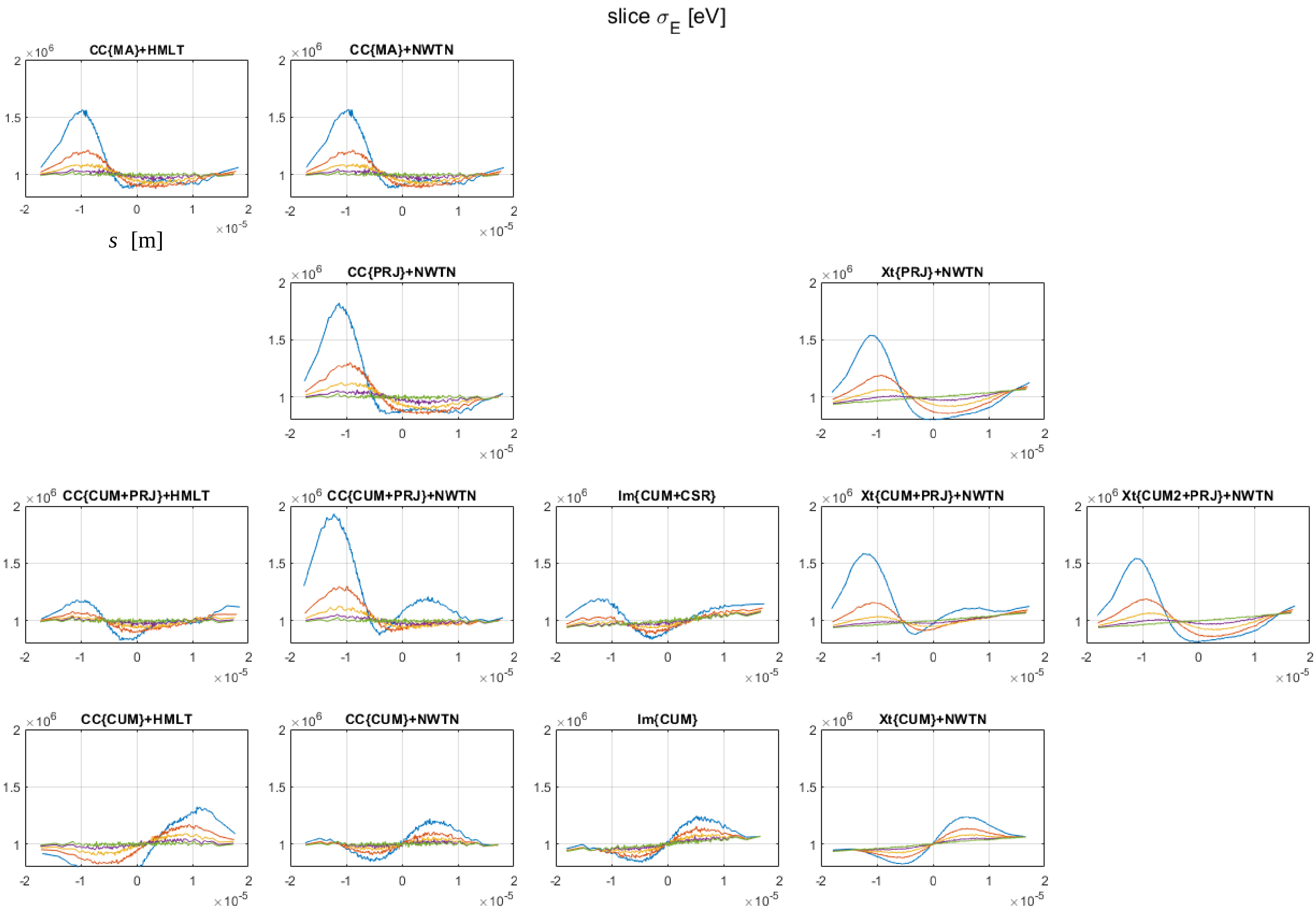
quite good agreement

not quite the same

significant disagreement

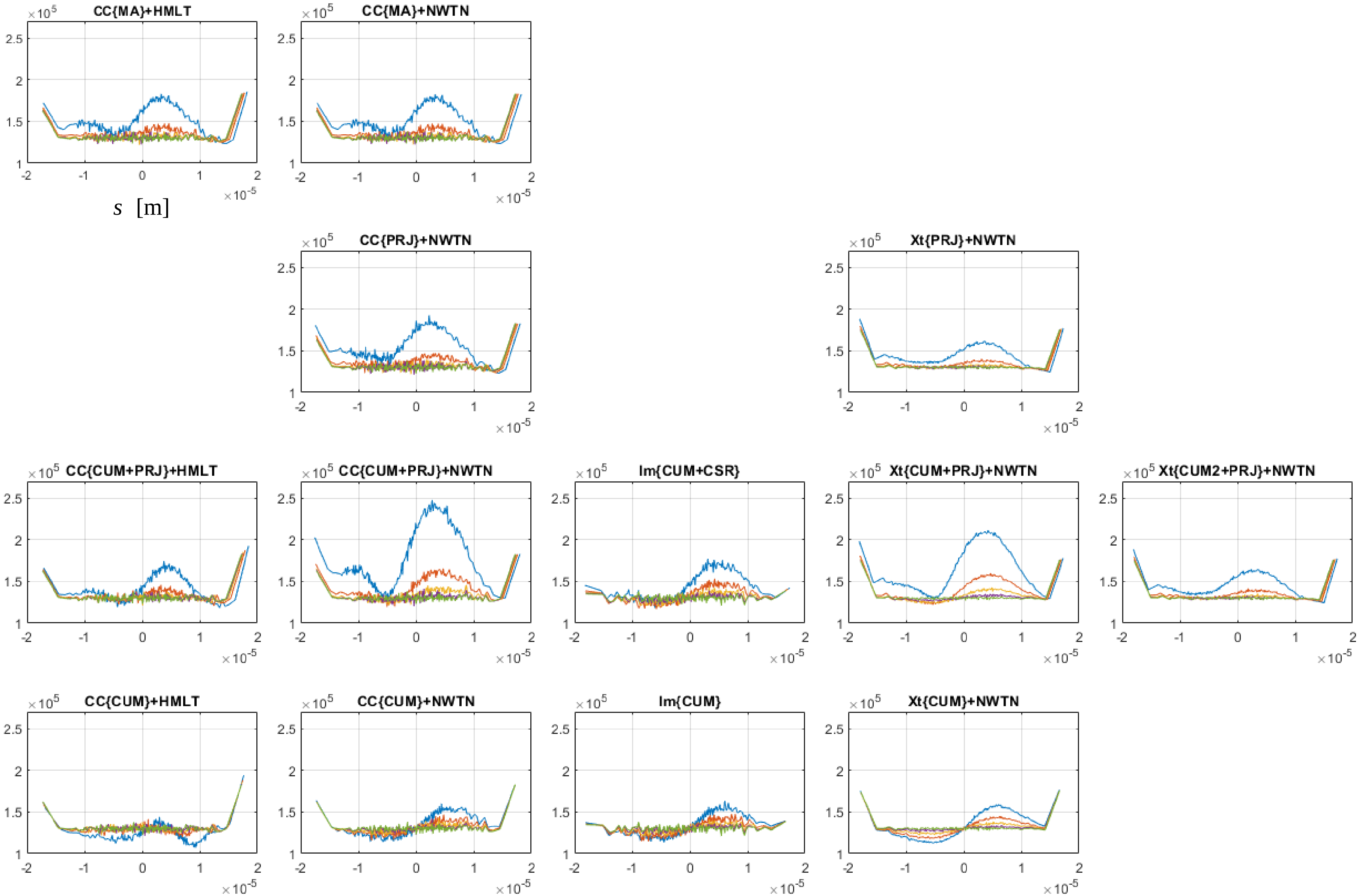
“reasonable” approximation to reference method

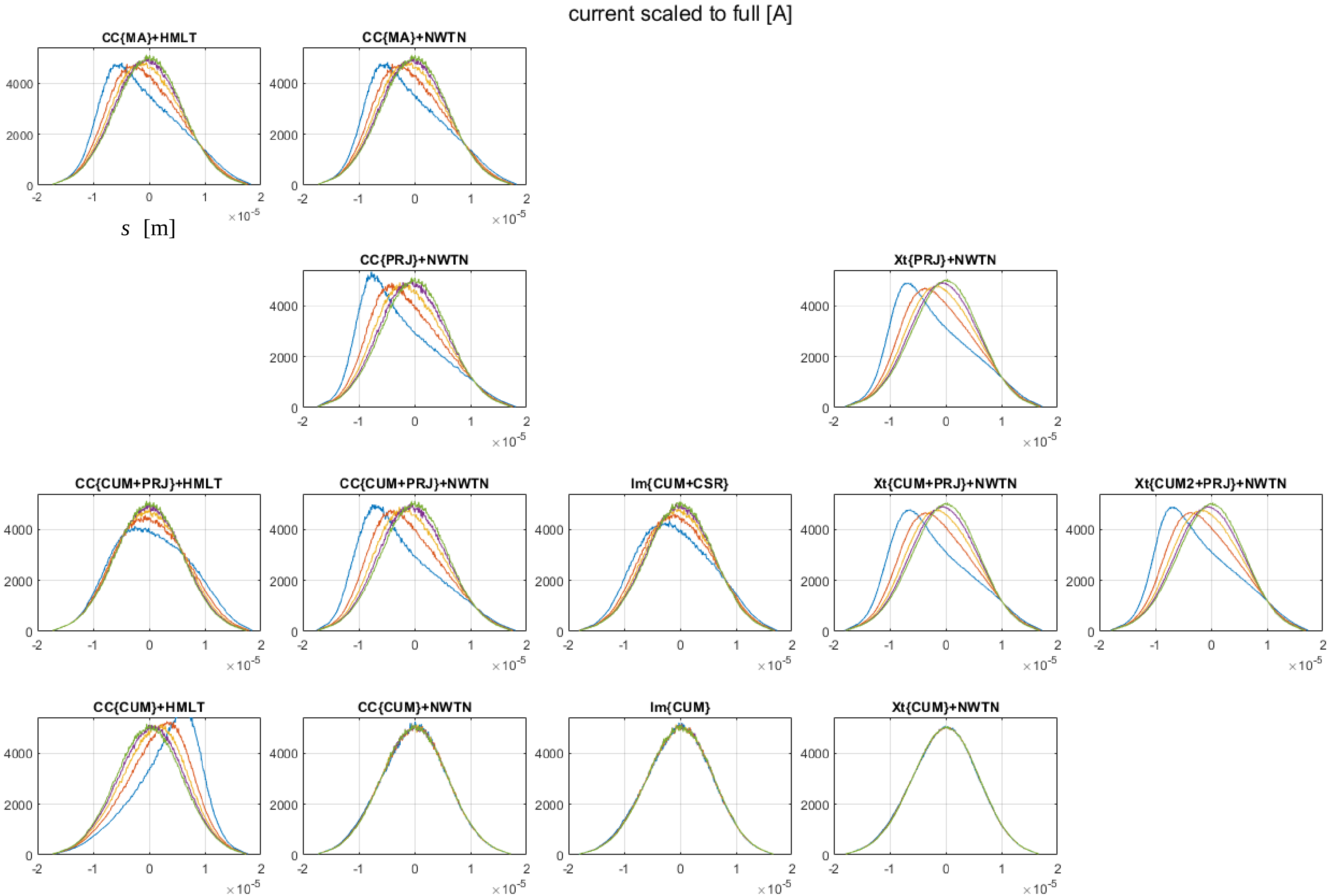

norm. ϵ [m]
s [m]
CC{MA}+HMLT
CC{MA}+NWTN
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10^-7
×10^-5


slice σ_E [eV]
CC{MA}+HMLT
CC{MA}+NWTN
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
$\times 10^6$
$\times 10^{-5}$
s [m]

slice σ_E/C [eV]

$C(s)$ see local compression

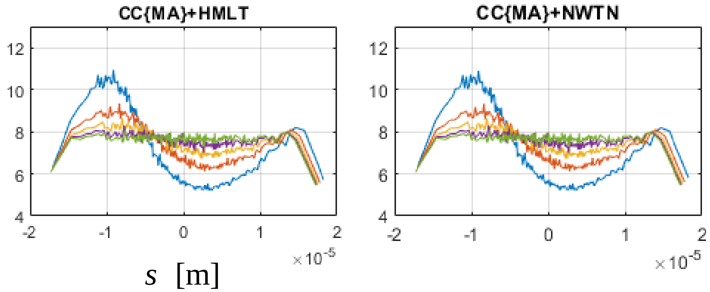


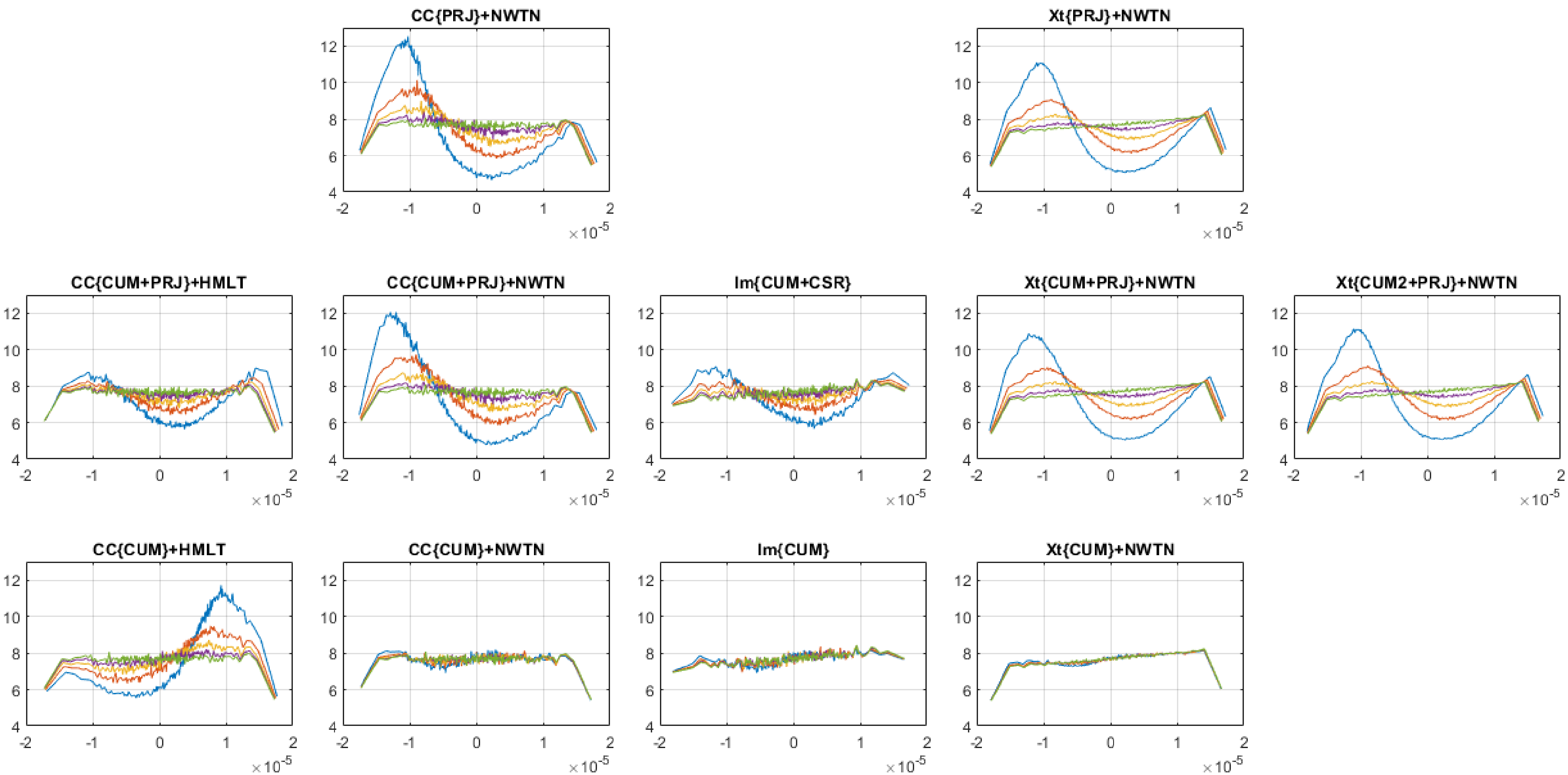

current scaled to full [A]
CC{MA}+HMLT
CC{MA}+NWTN
s [m]
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10^-5

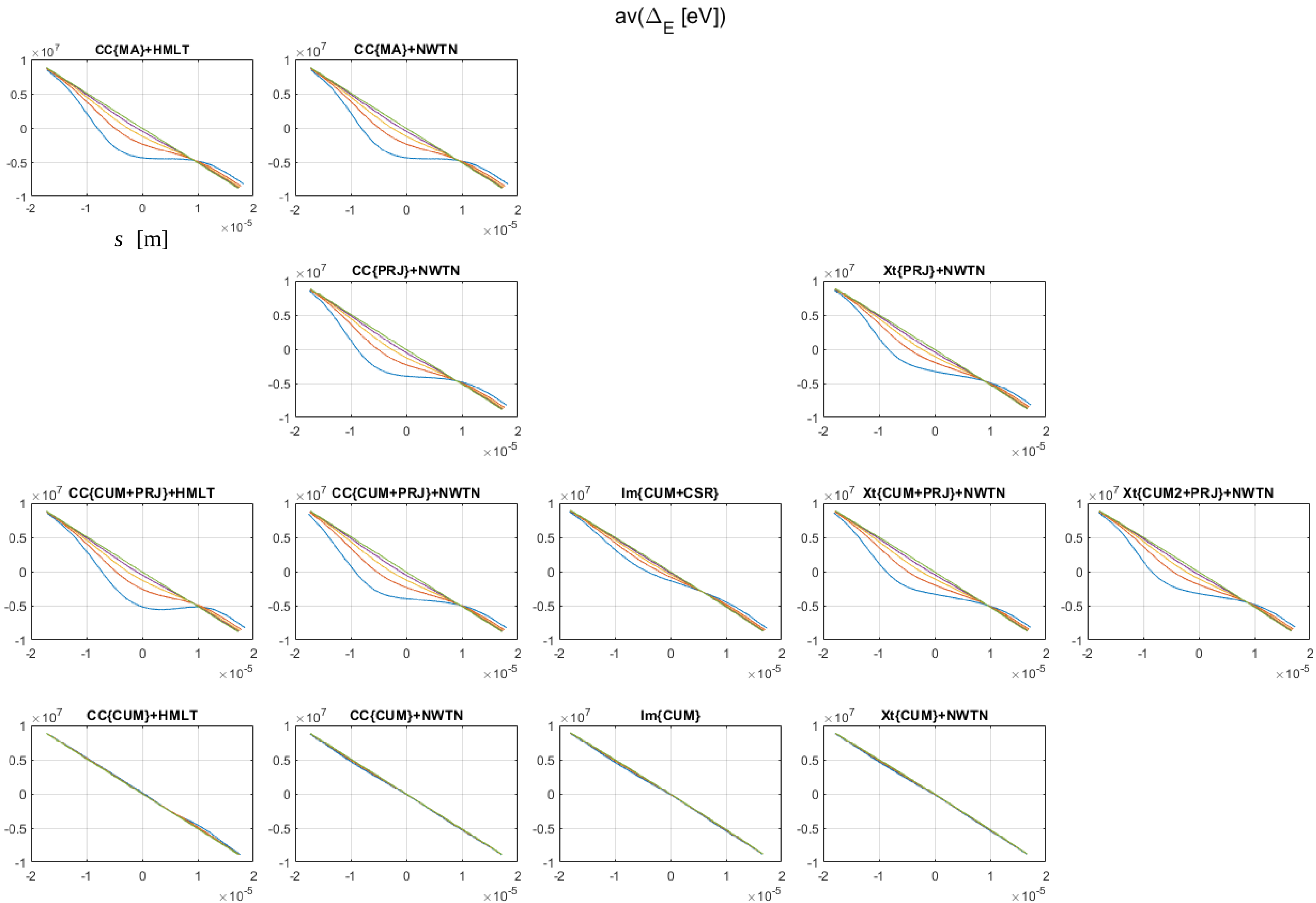
C=local compression

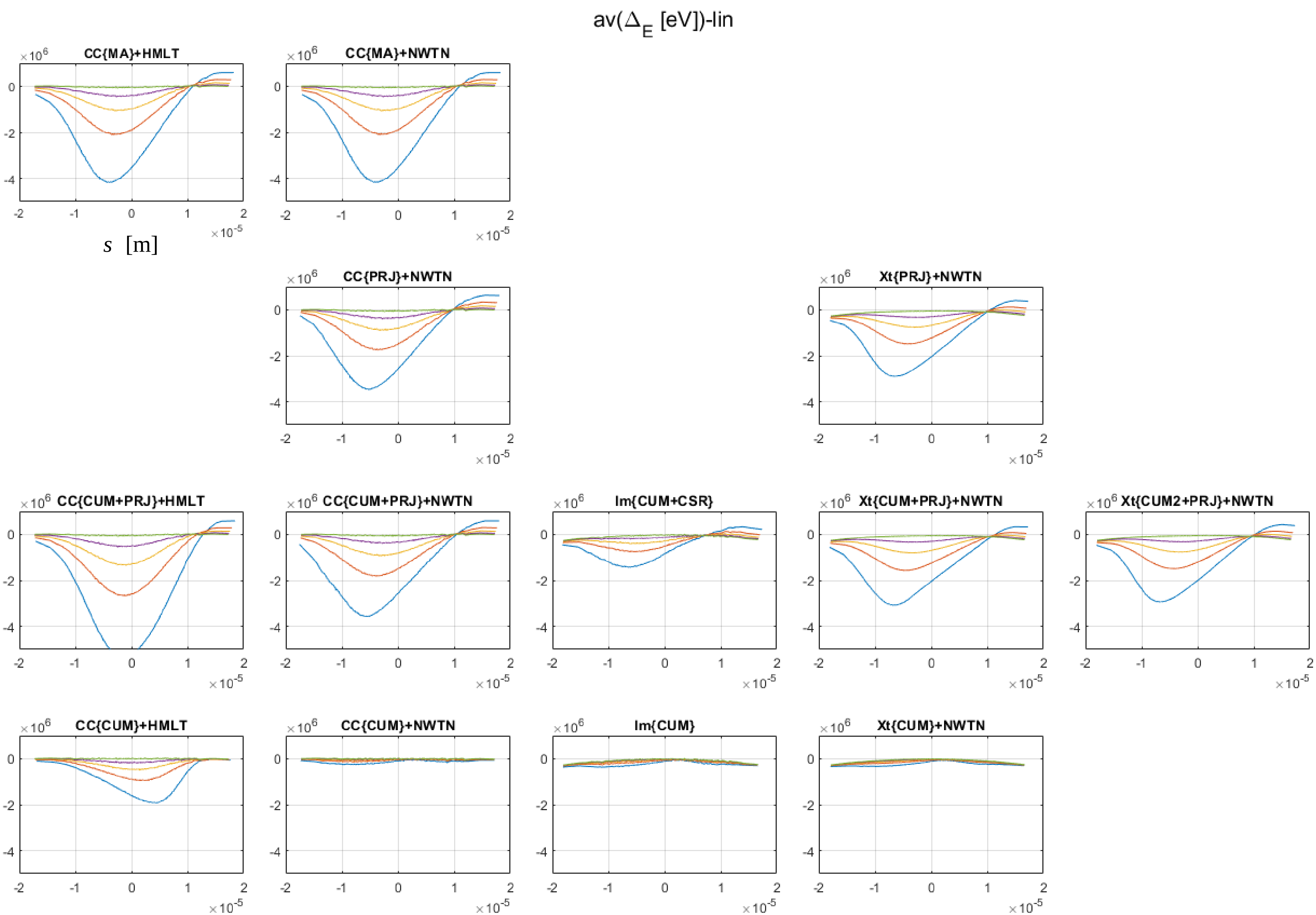
about $C(s)$: $C(s)=\dfrac{\lambda_2(s)}{\lambda_1(\textcolor{red}{s_1})}$ with

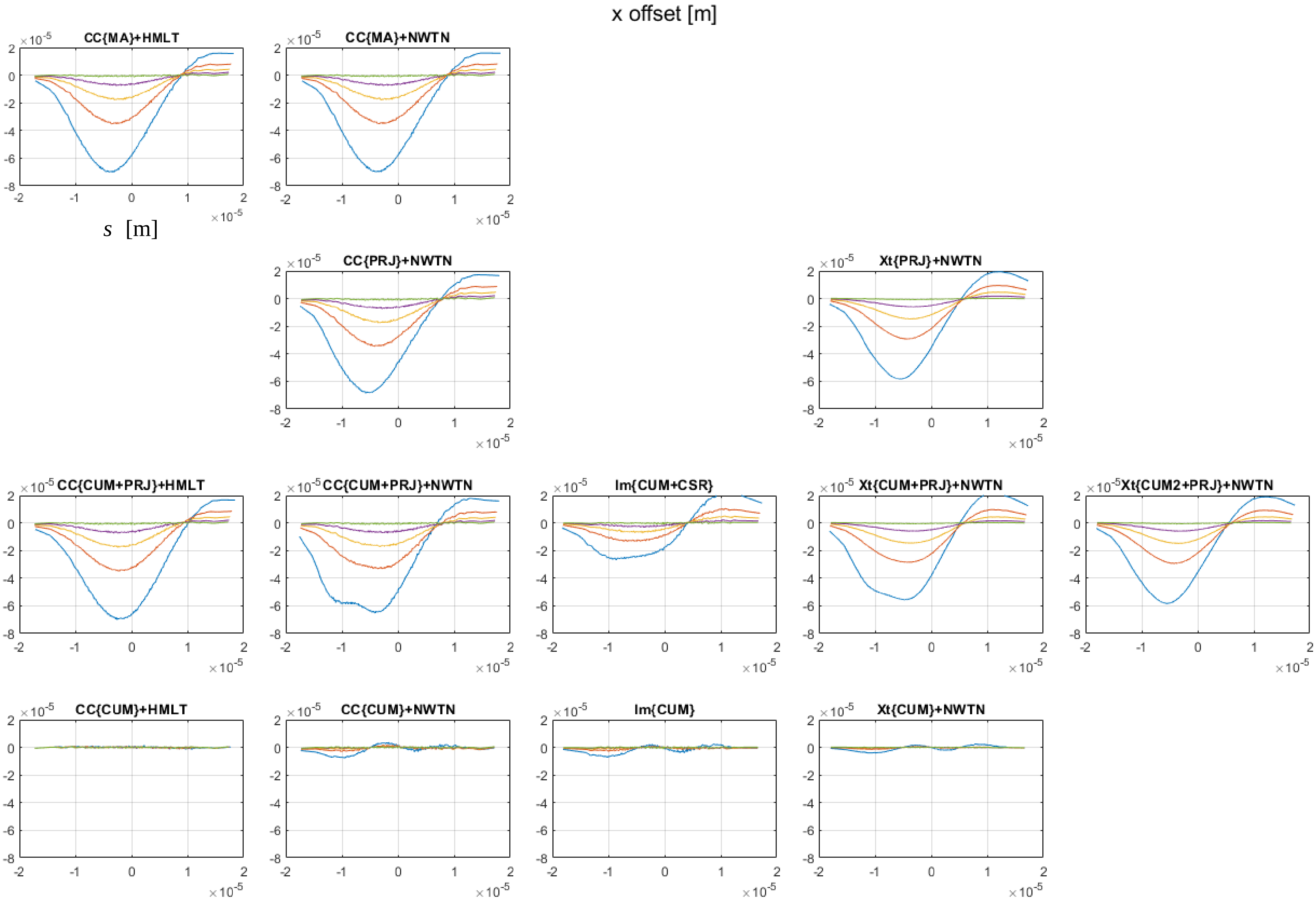
$$\int_{-\infty}^{\textcolor{red}{s_1}} \lambda_1(\tilde{s})\,d\tilde{s} = \int_{-\infty}^{s} \lambda_2(\tilde{s})\,d\tilde{s}$$

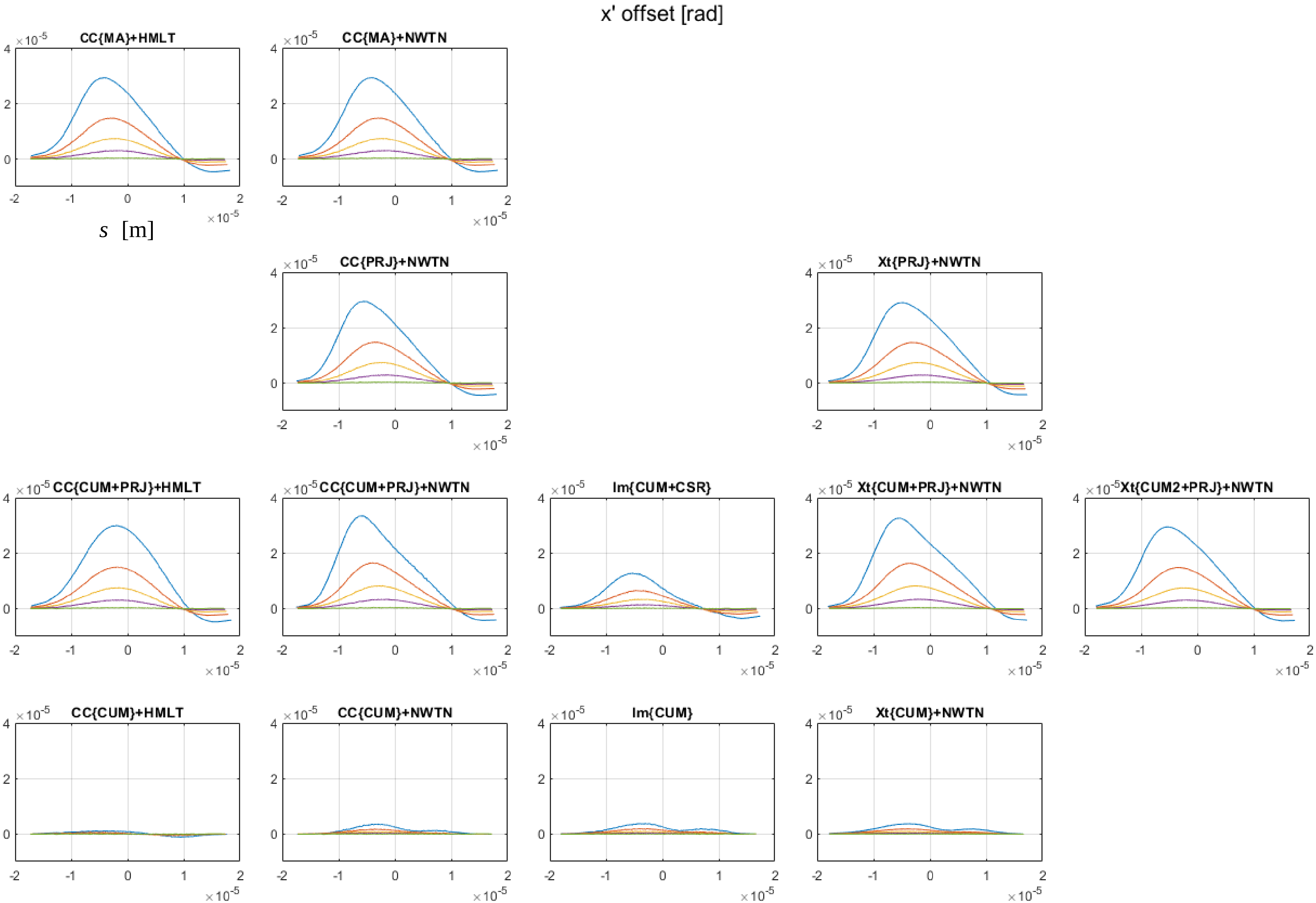


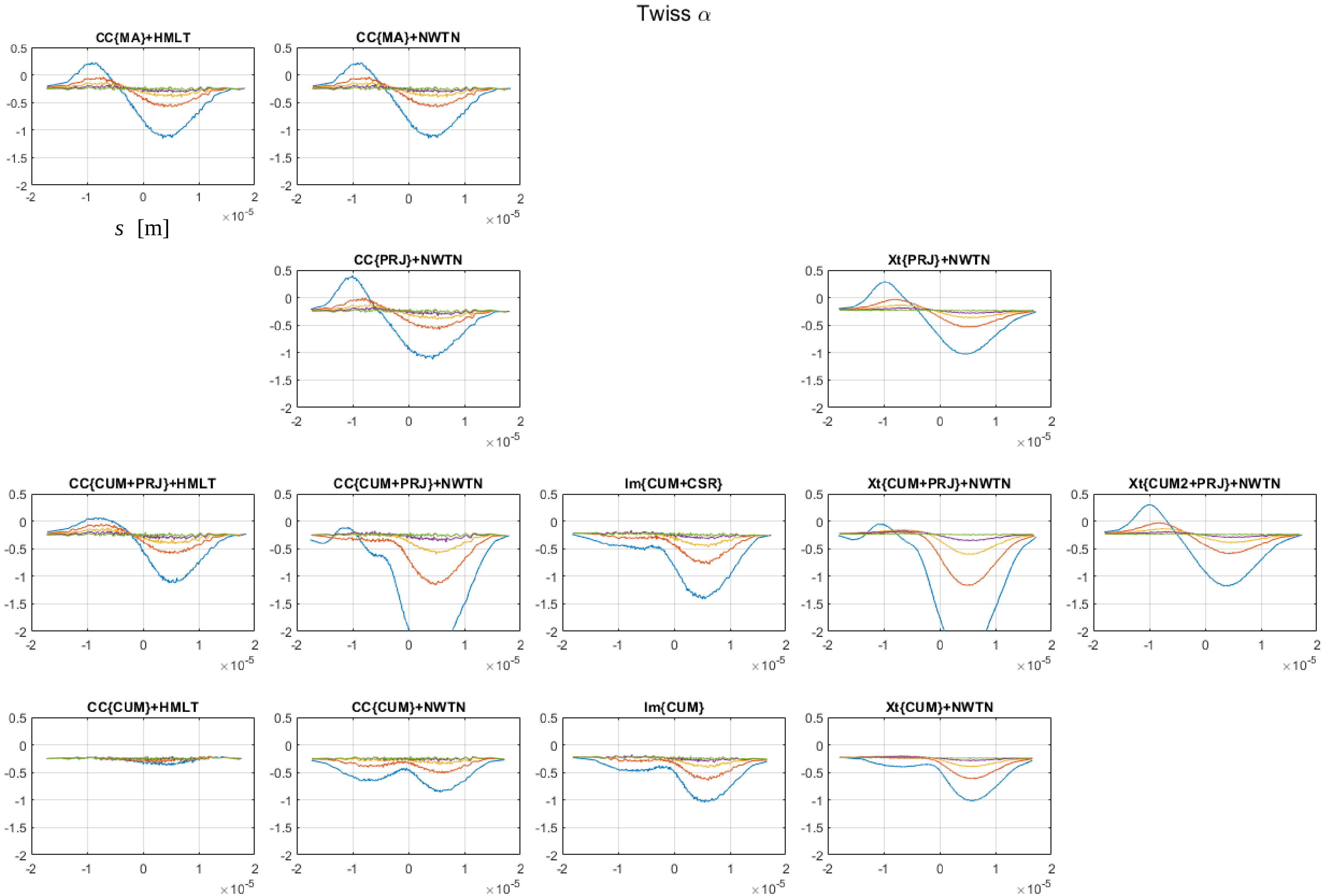


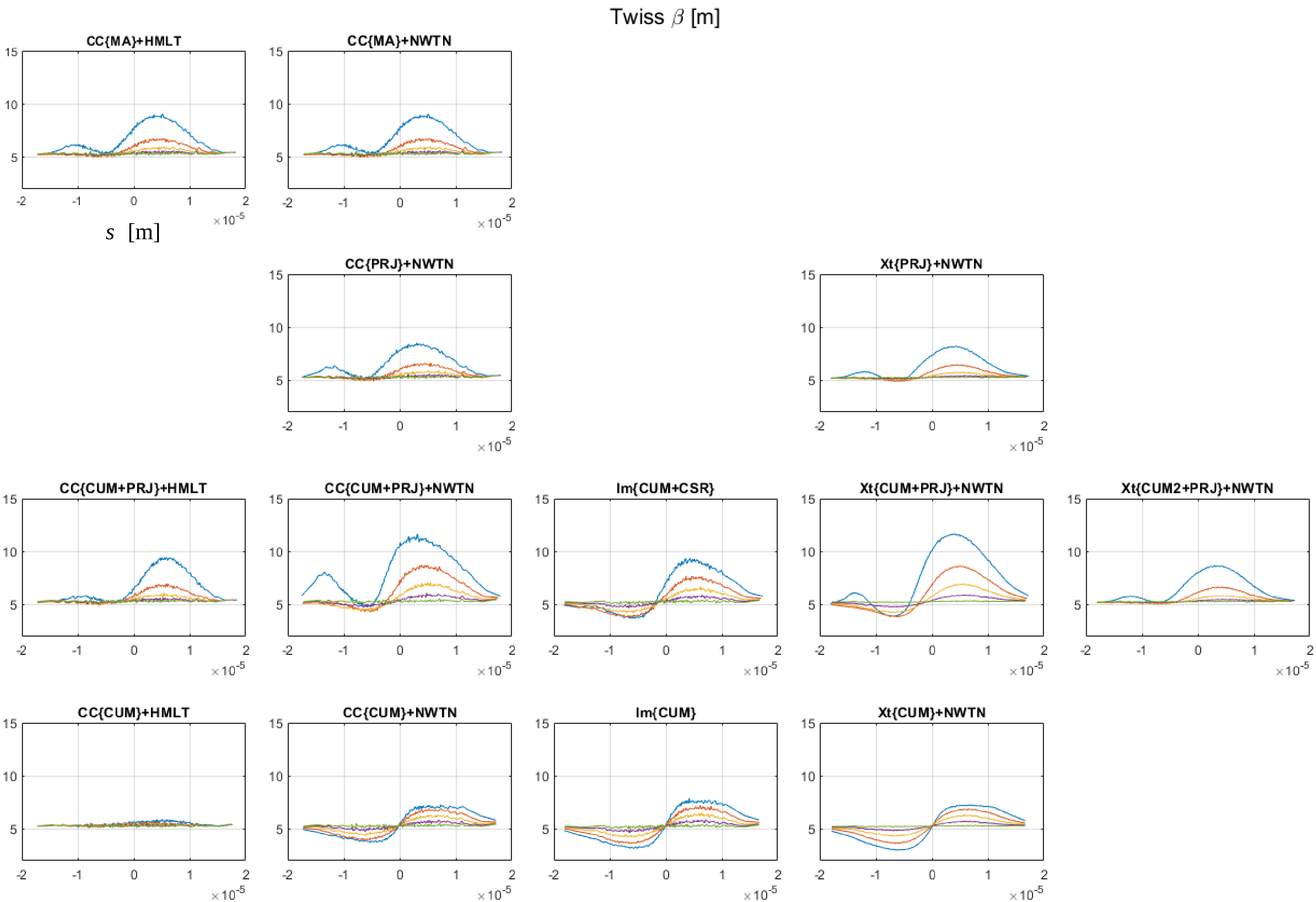

av(Δ_E [eV])
CC{MA}+HMLT
CC{MA}+NWTN
s [m]
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10^7
×10^-5


av(Δ_E [eV])-lin
CC{MA}+HMLT
CC{MA}+NWTN
s [m]
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10^6
×10^-5


x offset [m]
s [m]
CC{MA}+HMLT
CC{MA}+NWTN
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10^-5


x' offset [rad]
CC{MA}+HMLT
CC{MA}+NWTN
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
s [m]
×10^-5


Twiss α
CC{MA}+HMLT
CC{MA}+NWTN
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
s [m]
$\times 10^{-5}$



Twiss β [m]
CC{MA}+HMLT
CC{MA}+NWTN
s [m]
CC{PRJ}+NWTN
Xt{PRJ}+NWTN
CC{CUM+PRJ}+HMLT
CC{CUM+PRJ}+NWTN
Im{CUM+CSR}
Xt{CUM+PRJ}+NWTN
Xt{CUM2+PRJ}+NWTN
CC{CUM}+HMLT
CC{CUM}+NWTN
Im{CUM}
Xt{CUM}+NWTN
×10-5