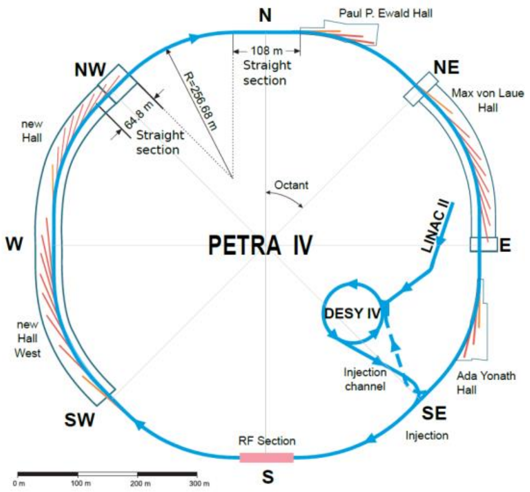
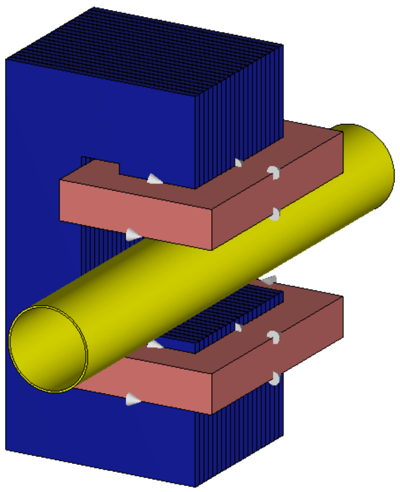
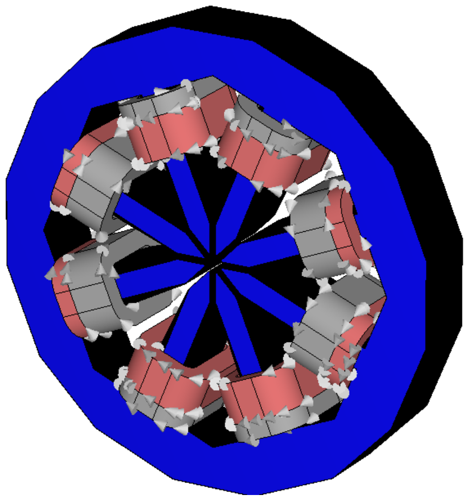
TOWARDS EFFICIENT NONLINEAR SIMULATIONS OF THE FAST CORRECTOR MAGNETS FOR PETRA IV

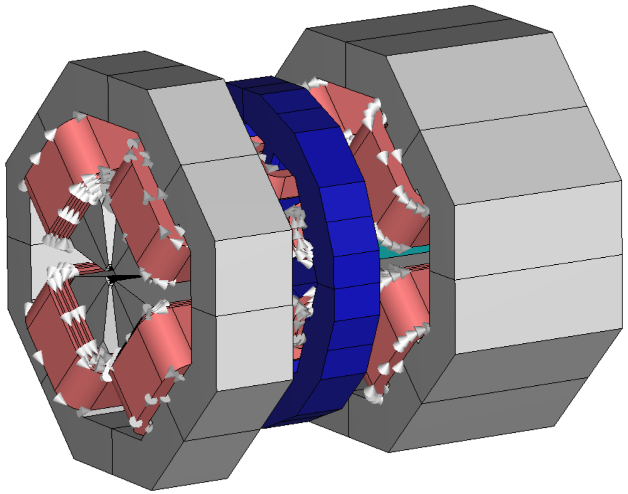
J. Christmann[1], L. A. M. D'Angelo[1], H. De Gersem[1], A. Aloev[2], S. H. Mirza[2], S. Pfeiffer[2], H. Schlarb[2], and M. Thede[2]



PETRA IV Conceptual Design Report







[1]TEMF, TU Darmstadt, Germany
[2]DESY, Hamburg, Germany

CONTENTS

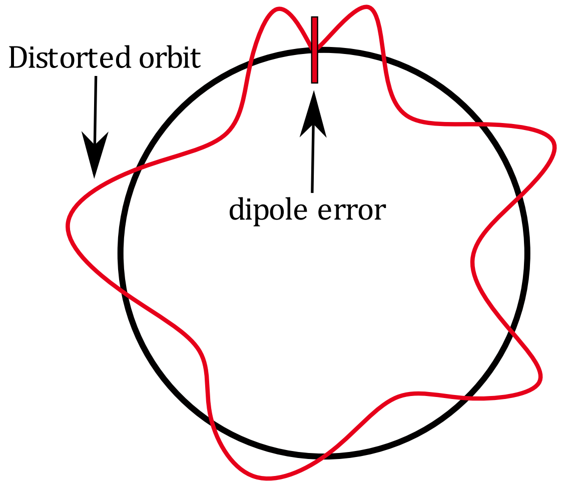
INTRODUCTION

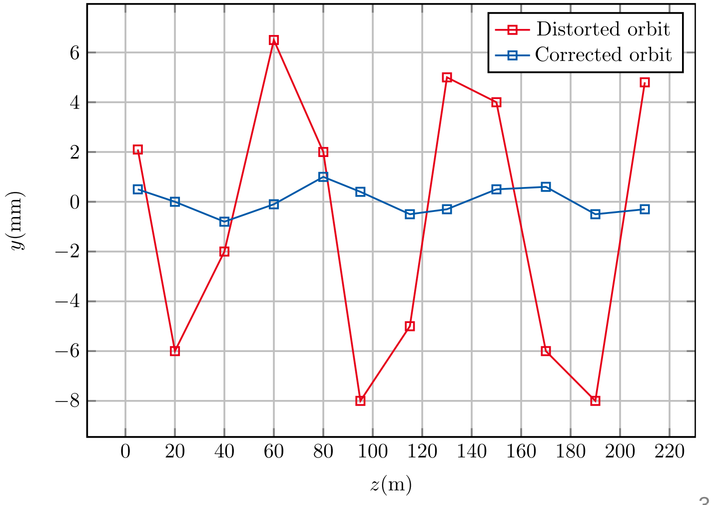
- Circular accelerators need dipole magnets to correct orbit distortions
- **PETRA IV**: ultra-low emittance synchrotron radiation source

➔ Fast orbit feedback system, **corrector magnets with frequencies in kHz range**

- **Strong eddy currents ➔** power losses, time delay, and field distortion
- **Simulation challenging** due to small skin depths and laminated yoke

➔ Need for technique to simplify simulations





TECHNISCHE UNIVERSITÄT DARMSTADT

CONTENTS

TECHNISCHE UNIVERSITÄT DARMSTADT

THEORY

- Magnetoquasistatic PDE: $\nabla \times (\nu(\vec{r})\nabla \times \underline{\vec{A}}(\vec{r})) + j\omega\sigma(\vec{r})\underline{\vec{A}}(\vec{r}) = \underline{\vec{J}}_s(\vec{r})$
- Replace reluctivity $\nu(\vec{r})$ and conductivity $\sigma(\vec{r})$ in the laminated yoke with spatially constant tensors

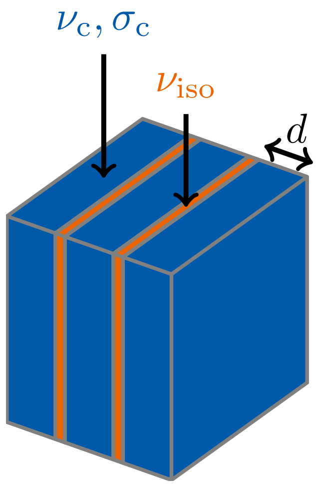
$$\nu(\vec{r}) \rightarrow \underline{\bar{\bar{\nu}}} = \frac{1}{8}\sigma_c d\delta\omega(1+j)\frac{\sinh((1+j)\delta^{-1}d)}{\sinh^2\left((1+j)\delta^{-1}\,d/2\right)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix} + \nu_c\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

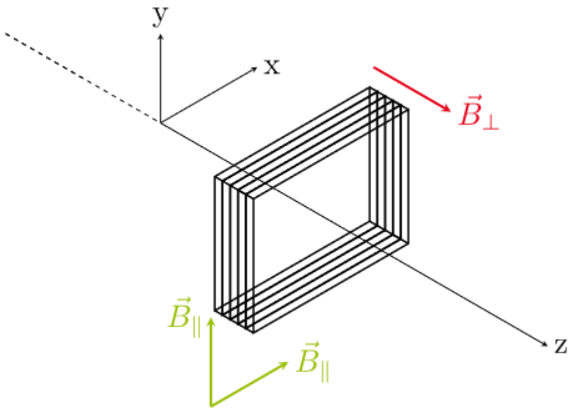
$$\sigma(\vec{r}) \rightarrow \bar{\bar{\sigma}} = \gamma\sigma_c\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$$

Skin depth $\delta = \sqrt{2/\omega\sigma_c\mu_c}$

Stacking factor $\gamma = \frac{V_c}{V_{\text{Yoke}}}$

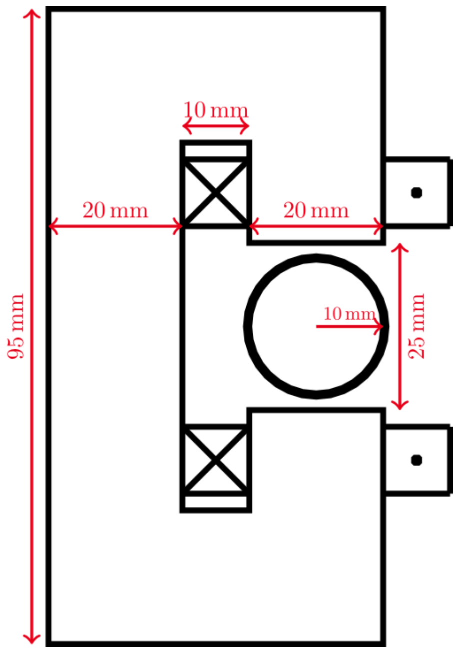
P. Dular et al., 2003
L. Krähenbühl et al., 2004
H. De Gersem et al., 2012

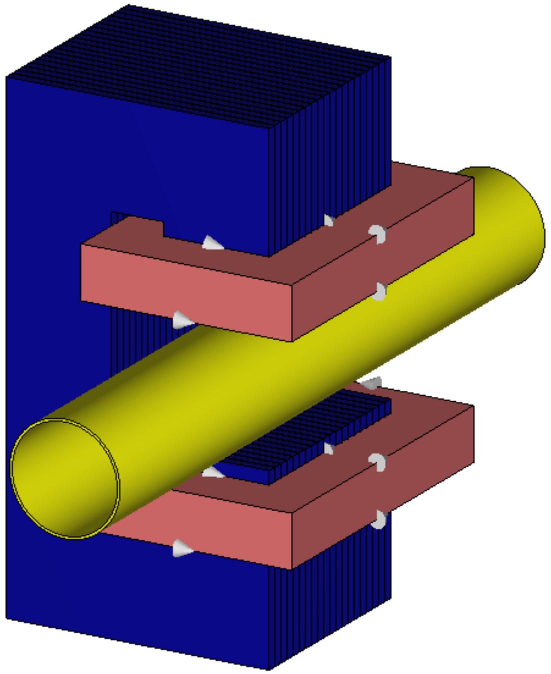




VERIFICATION

- **Model specifics:**
  - Iron yoke**:** length = 40 mm, lamination thickness = 1.83 mm
  - Copper beam pipe**:** thickness = 0.5 mm, length = 140 mm
  - Coils: current **=** 10 A (peak), # turns = 250
  - Frequency domain simulation via CST Studio Suite®

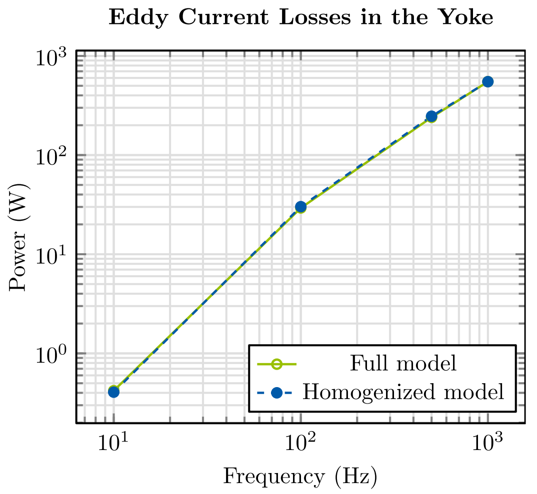


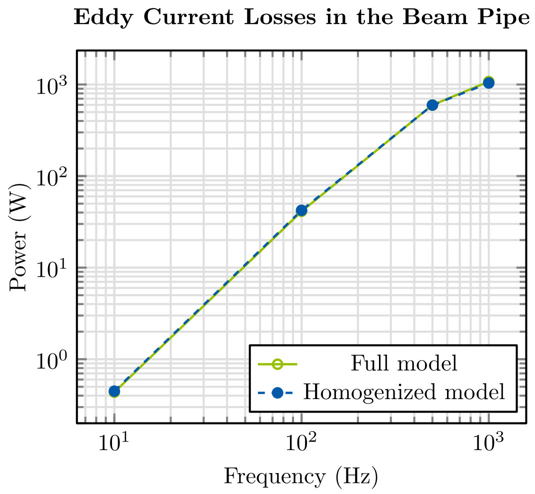


TECHNISCHE UNIVERSITÄT DARMSTADT

VERIFICATION

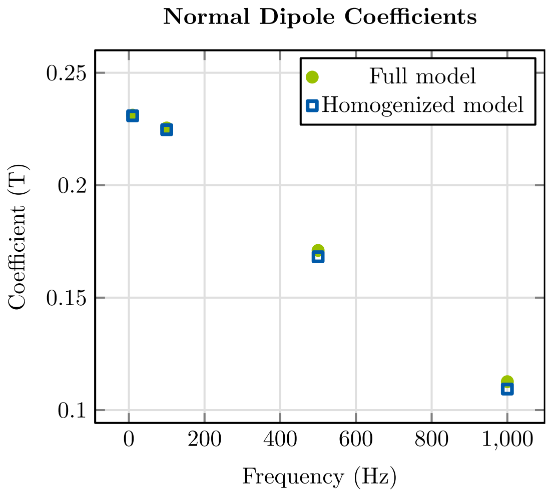
EDDY CURRENT LOSSES





- Eddy current losses well approximated

MULTIPOLE COEFFICIENTS



Multipole coeff.	Average rel. error
Dipole	1 %
Quadrupole	5 %
Sextupole	2 %

- Aperture field well approximated

- Simulation time is reduced from several hours to just a few minutes!

➔ After comparing to other techniques, we decided to **use this technique to simulate the corrector magnets**

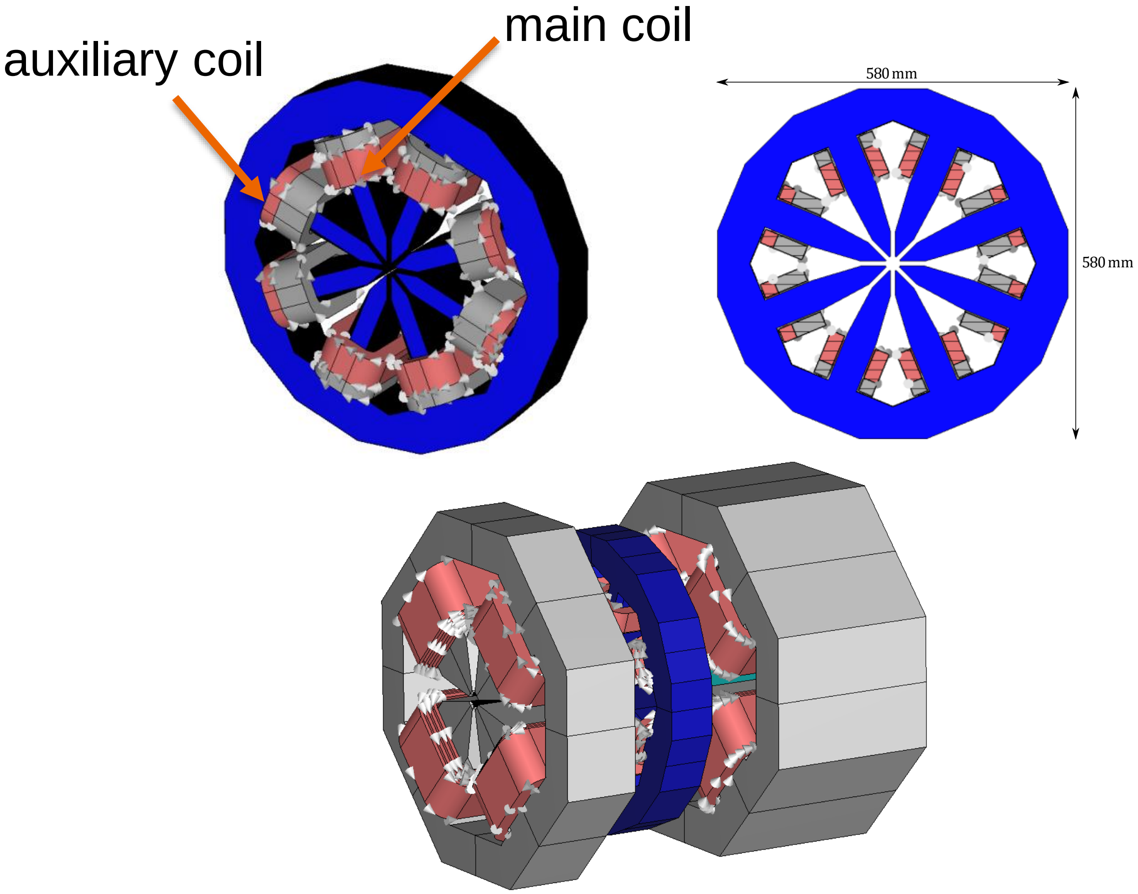
CONTENTS

TECHNISCHE UNIVERSITÄT DARMSTADT

MODEL DESCRIPTION

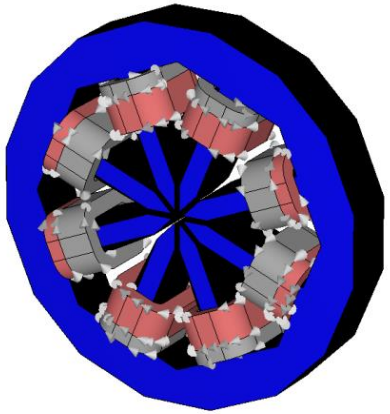
- **Dipole corrector**
  - Octupole-like design
  - Main coils with 975 At, auxiliary coils with 405 At (AC)
  - Laminated yoke with 580 mm diameter, 90 mm length

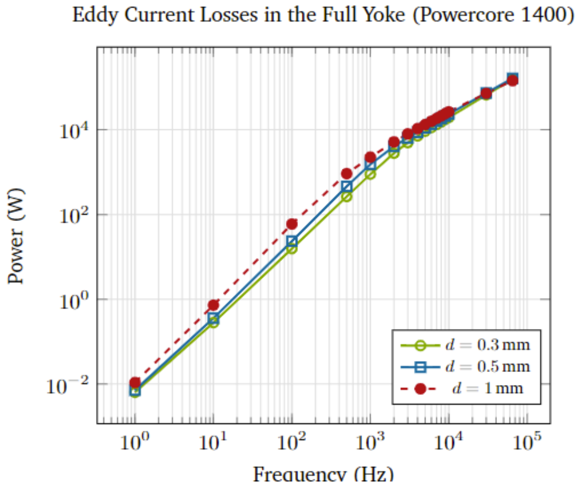
- **Neighboring quadrupoles**
  - DC currents
  - Not laminated
  - Distance to corrector yoke $\sim 11.5\ \mathrm{cm}$ ➔ cross-talk?



TECHNISCHE UNIVERSITÄT DARMSTADT

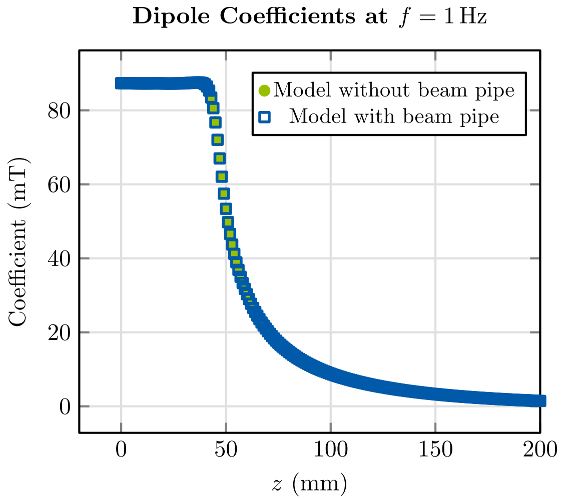
EDDY CURRENT LOSSES & MULTIPOLE COEFFICIENTS

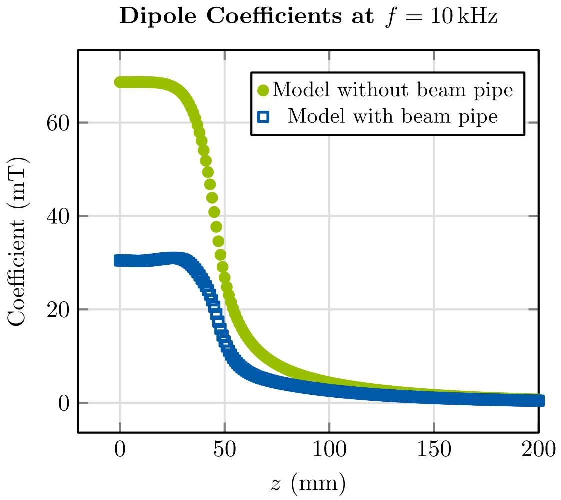




- At lower frequencies: $P_{\text{eddy}}\ \alpha\ f^2$, as expected from theory*
- Lamination thickness only important at frequencies ≤ 1 kHz

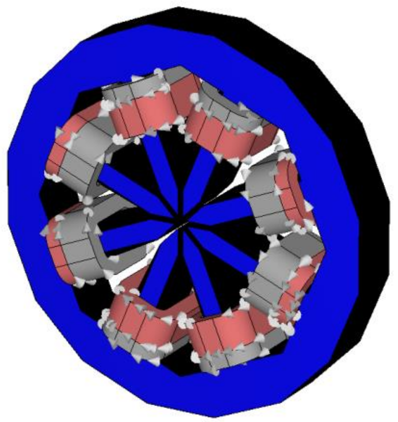
* R. L. Stoll, *The Analysis of Eddy Currents.* 1974.
J. Lammeraner and M. Štafl, *Eddy Currents.* 1966.

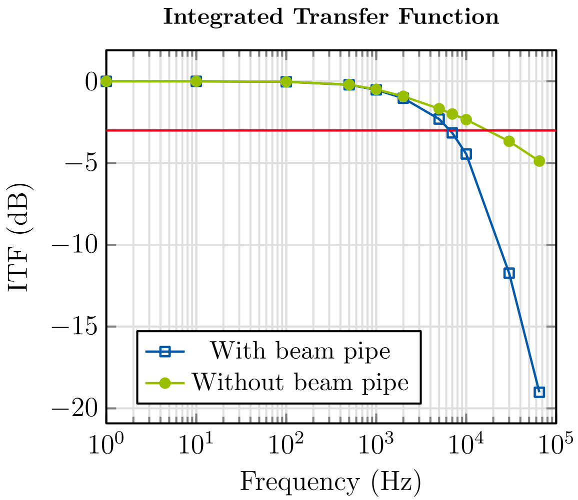


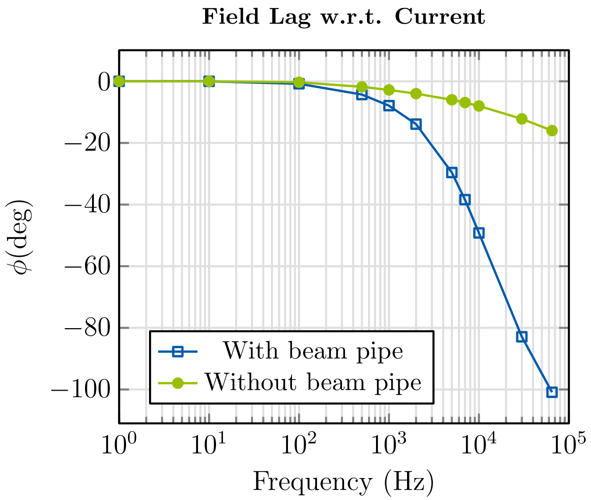


- Dipole field is attenuated due to eddy currents
- Beam pipe ➔ stronger attenuation at higher frequencies

INTEGRATED TRANSFER FUNCTION & FIELD LAG



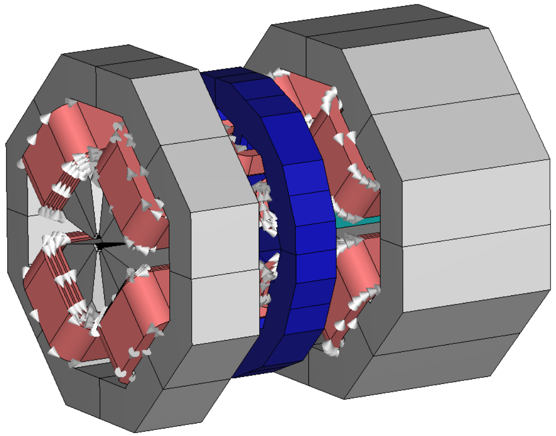


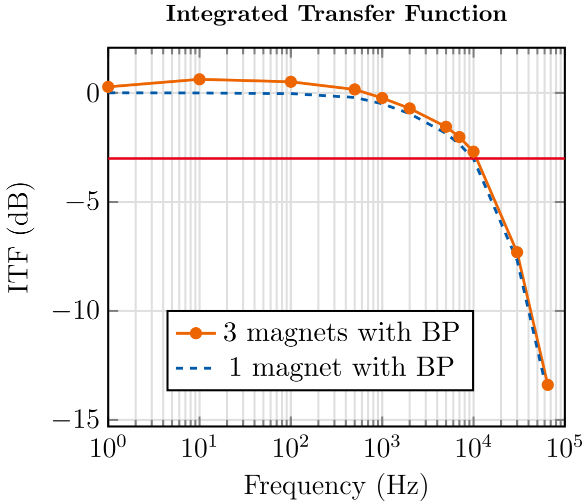


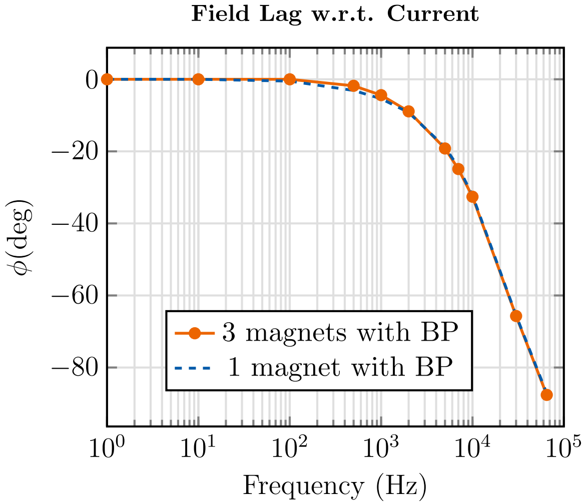
$$\mathrm{ITF}(f) = \frac{\int_l B_1(z,f)\,\mathrm{d}z}{\int_l B_{1,\mathrm{DC}}(z)\,\mathrm{d}z}$$

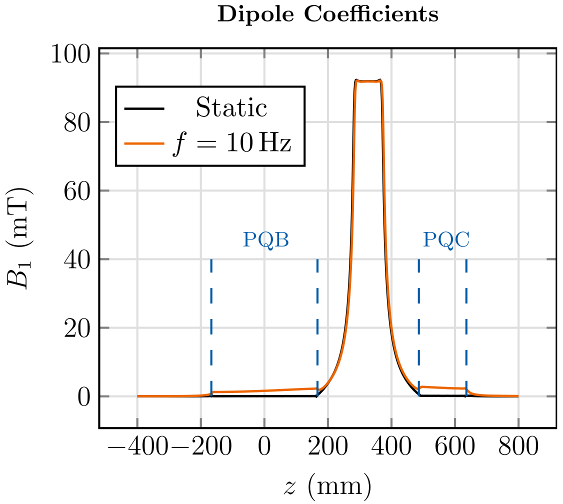
- Integrated transfer function and field lag (phase difference between current and aperture field) are of high interest for design of feedback control
- We compute both from our simulations for different yoke materials, different lamination thicknesses, etc.

CROSS-TALK







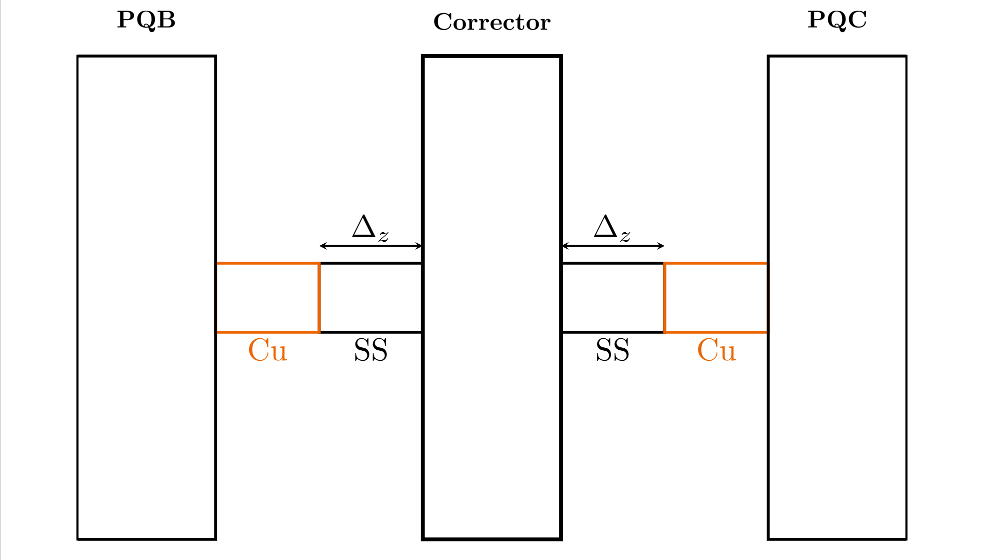


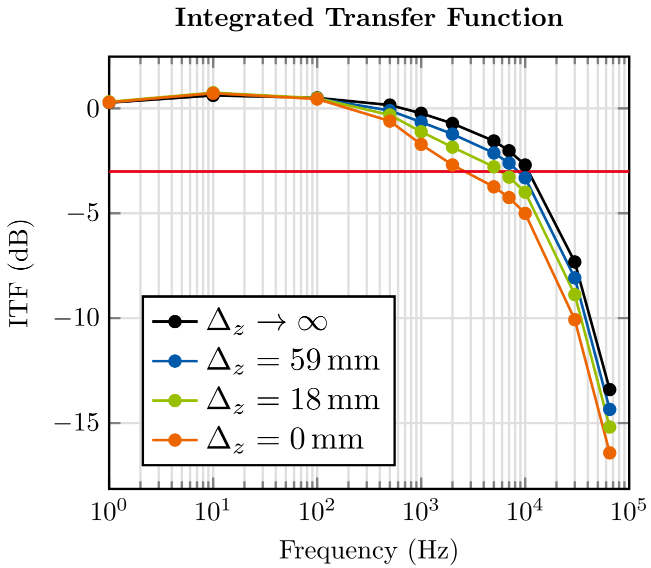
- Analyze ITF and field lag for **model with neighboring quadrupoles, compare to stand-alone corrector**
- Main difference: at low frequencies, a ~0.7 dB peak is occurring in the ITF of the model with the neighboring quadrupoles
- Reason for the peak in the ITF is parasitic dipole component inside the quadrupole magnets

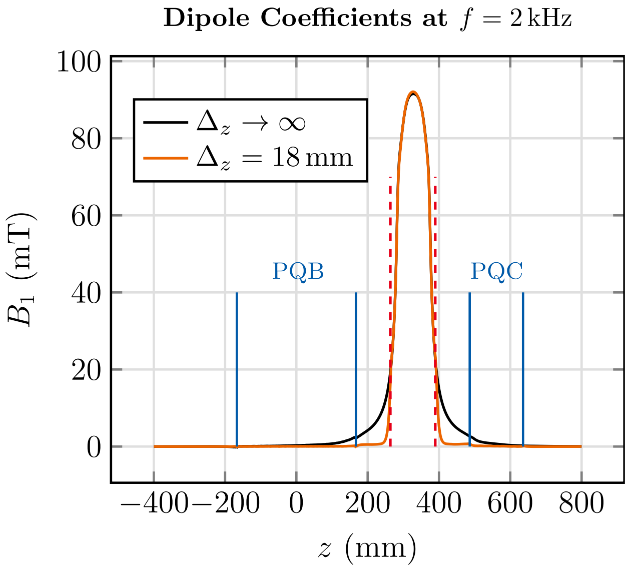
TECHNISCHE UNIVERSITÄT DARMSTADT

BEAM PIPE MATERIAL TRANSITION

Investigate different scenarios characterized by **distance Δ_z** from corrector yoke to copper parts of beam pipe







- The closer the copper parts are to the corrector yoke, the smaller the bandwidth
- Copper parts have a higher conductivity than SS part ➔ stronger eddy currents ➔ stronger field attenuation

CONTENTS

TECHNISCHE UNIVERSITÄT DARMSTADT

WITHOUT DC BIAS - THEORY

- To incorporate **non-linear BH-curves** into simulations: **combine homogenization technique and harmonic balance FEM (HBFEM)**
- HBFEM is a technique to approximate periodic solutions of nonlinear transient PDEs in frequency domain
- Example: excitation current with 1st and 3rd harmonic, include field quantities up to 3rd harmonic

$$\nabla \times (\nu(t) \nabla \times \vec{A}(t)) + \sigma \frac{\partial \vec{A}(t)}{\partial t} = \vec{J}_s(t)$$

$$\nabla \times (\underline{\nu}(\omega) \circledast \nabla \times \underline{\vec{A}}(\omega)) + j\omega\sigma \underline{\vec{A}}(\omega) = \underline{\vec{J}}_s(\omega)$$

+ Homogenization

$$\begin{bmatrix} K_{\underline{\overline{\overline{\nu}}}_0(3\omega_\mathrm{f})} + 3j\omega_\mathrm{f} M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 & 0 \\ K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_\mathrm{f})} + j\omega_\mathrm{f} M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 \\ 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_\mathrm{f})} - j\omega_\mathrm{f} M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} \\ 0 & 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(3\omega_\mathrm{f})} - 3j\omega_\mathrm{f} M_{\overline{\overline{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 \\ \underline{j}_1 \\ \underline{j}_{-1} \\ \underline{j}_{-3} \end{bmatrix}$$

S. Yamada and K. Bessho (1988)
H. De Gersem, H. Vande Sande, K. Hameyer (2001)

TECHNISCHE UNIVERSITÄT DARMSTADT

WITHOUT DC BIAS - THEORY

- To resolve the nonlinearity: bring off-diagonal terms to the right-hand side
- Iterate until energy does not change anymore

$$\begin{bmatrix} K_{\underline{\bar{\bar{\nu}}}_0(3\omega_f)} + 3j\omega_f M_{\bar{\bar{\sigma}}} & K_{\underline{\nu}_2} & 0 & 0 \\ K_{\underline{\nu}_{-2}} & K_{\underline{\bar{\bar{\nu}}}_0(\omega_f)} + j\omega_f M_{\bar{\bar{\sigma}}} & K_{\underline{\nu}_2} & 0 \\ 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\bar{\bar{\nu}}}_0(\omega_f)} - j\omega_f M_{\bar{\bar{\sigma}}} & K_{\underline{\nu}_2} \\ 0 & 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\bar{\bar{\nu}}}_0(3\omega_f)} - 3j\omega_f M_{\bar{\bar{\sigma}}} \end{bmatrix} \underbrace{\begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix}}_{\vec{\underline{a}}} = \begin{bmatrix} \underline{j}_3 \\ \underline{j}_1 \\ \underline{j}_{-1} \\ \underline{j}_{-3} \end{bmatrix}$$

Initialize $K_{\underline{\bar{\bar{\nu}}}_0(3\omega_f)}, K_{\underline{\bar{\bar{\nu}}}_0(\omega_f)}, K_{\underline{\nu}_2}, K_{\underline{\nu}_{-2}}$ and solve for $\vec{\underline{a}}^{(1)}$

For $i = 1, 2, 3 \ldots$ **until convergence**

Compute $\vec{\underline{b}}^{(i)} = \nabla \times \vec{\underline{a}}^{(i)}$

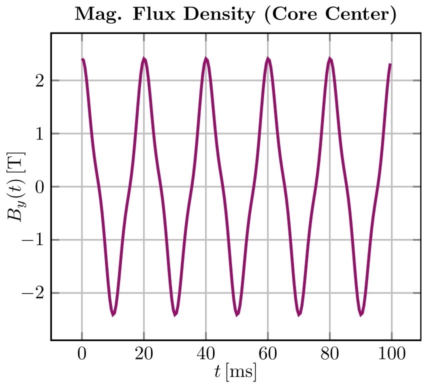
Transform in time domain $\vec{\underline{b}}^{(i)} \to \vec{B}(t)$

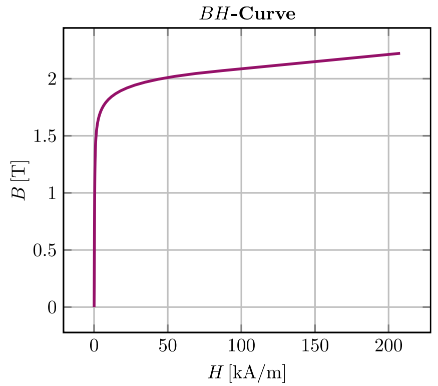
Insert $\|\vec{B}(t)\|$ in BH-curve $\to \|\vec{H}(t)\| \to \nu(t)$

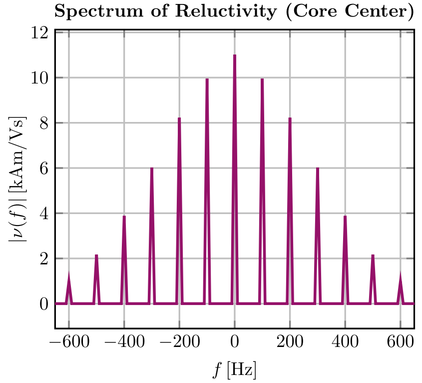
Fourier transform $\nu(t) \to \underline{\nu}_0^{(i)}, \underline{\nu}_2^{(i)}, \underline{\nu}_{-2}^{(i)}$

Solve

$$\begin{bmatrix} K_{\underline{\bar{\bar{\nu}}}_0^{(i)}(3\omega_f)} + 3j\omega_f M_{\bar{\bar{\sigma}}} & 0 & 0 & 0 \\ 0 & K_{\underline{\bar{\bar{\nu}}}_0^{(i)}(\omega_f)} + j\omega_f M_{\bar{\bar{\sigma}}} & 0 & 0 \\ 0 & 0 & K_{\underline{\bar{\bar{\nu}}}_0^{(i)}(\omega_f)} - j\omega_f M_{\bar{\bar{\sigma}}} & 0 \\ 0 & 0 & 0 & K_{\underline{\bar{\bar{\nu}}}_0^{(i)}(3\omega_f)} - 3j\omega_f M_{\bar{\bar{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3^{(i+1)} \\ \underline{a}_1^{(i+1)} \\ \underline{a}_{-1}^{(i+1)} \\ \underline{a}_{-3}^{(i+1)} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 - K_{\underline{\nu}_2^{(i)}} \underline{a}_1^{(i)} \\ \underline{j}_1 - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_3^{(i)} - K_{\underline{\nu}_2^{(i)}} \underline{a}_{-1}^{(i)} \\ \underline{j}_{-1} - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_1^{(i)} - K_{\underline{\nu}_2^{(i)}} \underline{a}_{-3}^{(i)} \\ \underline{j}_{-3} - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_{-1}^{(i)} \end{bmatrix}$$

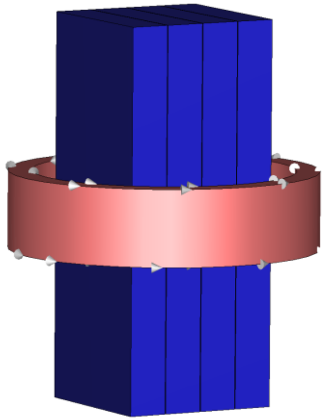


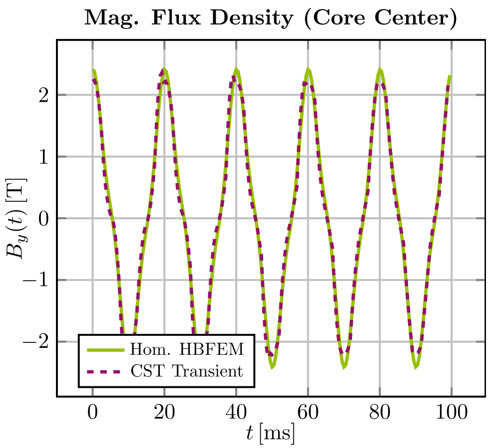


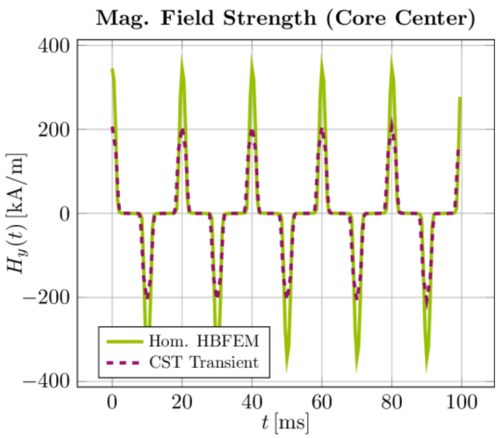


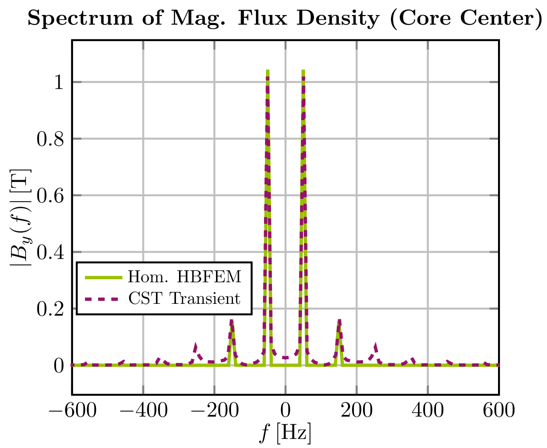
TECHNISCHE UNIVERSITÄT DARMSTADT

WITHOUT DC BIAS - VERIFICATION







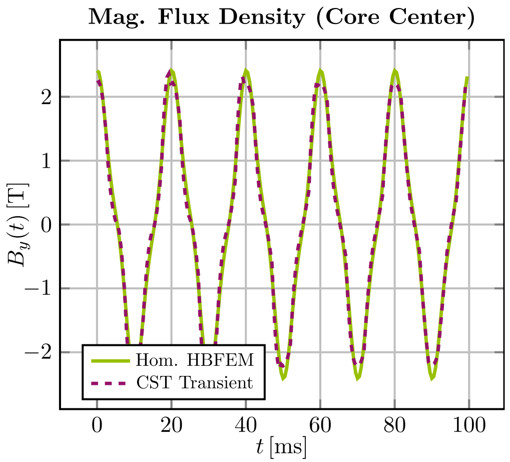


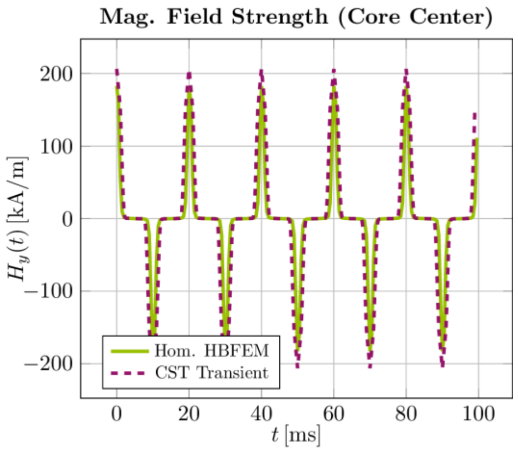
- Simple inductor with laminated core, excitation current: $I_s(t) = 1.5\ \text{kA} \cos(2\pi 50\text{Hz}\ t) + 0.24\ \text{kA} \cos(2\pi 150\text{Hz}\ t)$
- Compare results of HBFEM + homogenization (GetDP + Python) to transient CST simulation with individually resolved laminations

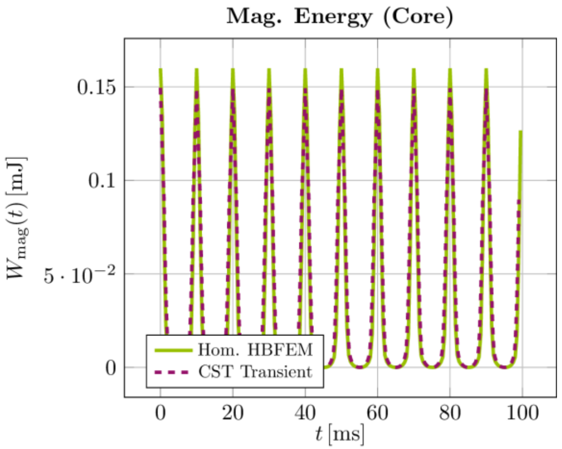
➔ Good agreement in magnetic flux density
➔ Larger differences in magnetic field strength
➔ **Suspicion: differences in magnetic field strength are due to not having included enough harmonics**

TECHNISCHE UNIVERSITÄT DARMSTADT

WITHOUT DC BIAS - VERIFICATION







- Include 5th harmonic in the analysis

➔ Still good agreement in magnetic flux density, large differences in magnetic field strength vanish

➔ Decent agreement in magnetic energy

WITH DC BIAS - THEORY

- Current signal of corrector magnet: DC current + oscillations ➔ modify HBFEM method to include DC bias
- Again, we combine HBFEM with a homogenization technique

$$\nabla \times \left(\underline{\nu}(\omega) \circledast \nabla \times \underline{\vec{A}}(\omega) \right) + j\omega\sigma \underline{\vec{A}}(\omega) = \underline{\vec{J}_s}(\omega) \;\Rightarrow\; \nabla \times \left(\underline{\nu_d}(\omega) \circledast \nabla \times \underline{\vec{A}}(\omega) \right) + j\omega\sigma \underline{\vec{A}}(\omega) = \underline{\vec{J}_s}(\omega) - \nabla \times \underline{\vec{H}_c}(\omega)$$

chord reluctivity — differential reluctivity — magnetizing field strength

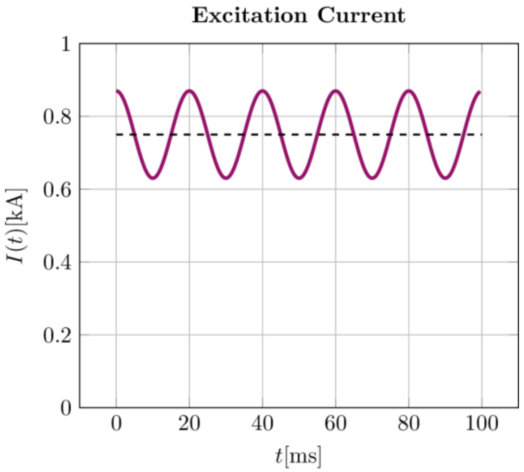
HOMOGENIZATION

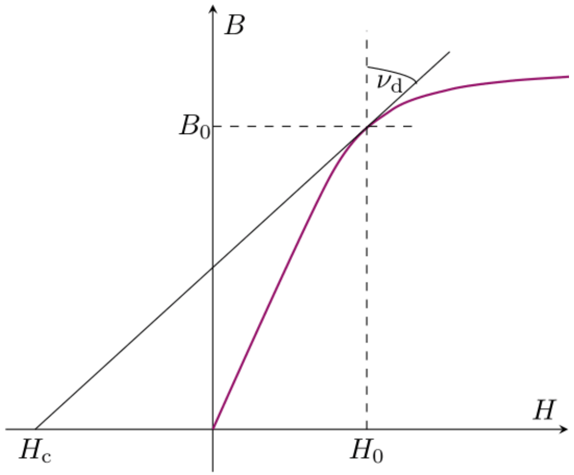
$$\nabla \times (\bar{\bar{\nu}} \nabla \times \vec{A}) + \nabla \times \left(\bar{\bar{\xi}} \nabla \times \frac{\partial \vec{A}}{\partial t} \right) + \bar{\bar{\sigma}} \frac{\partial \vec{A}}{\partial t} = \vec{J}_s$$

$$\bar{\bar{\nu}} = \frac{1}{\frac{\gamma}{\nu_{\mathrm{Fe}}} + \frac{1-\gamma}{\nu_{\mathrm{Iso}}}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (\nu_{\mathrm{Fe}}\gamma + \nu_{\mathrm{Iso}}(1-\gamma)) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{\xi}} = \frac{1}{12} \sigma_{\mathrm{Fe}} d^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

J. Gyselinck et al., 1999

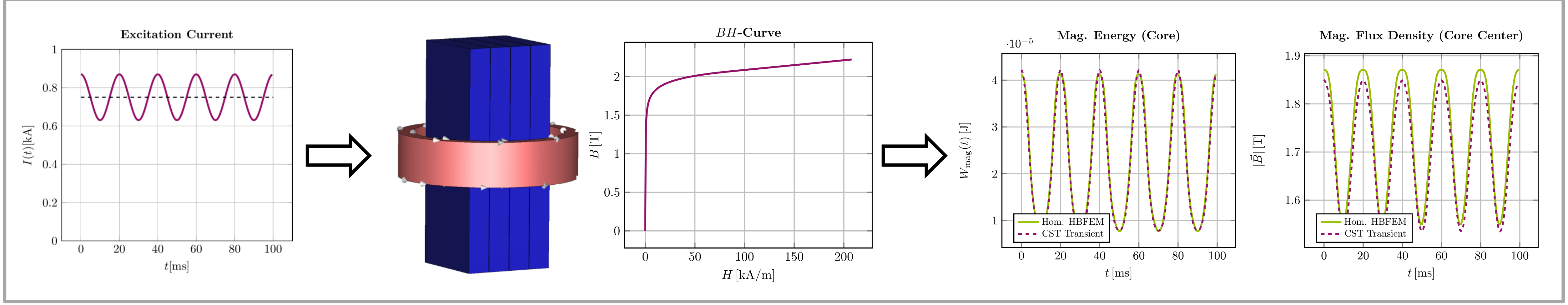




TECHNISCHE UNIVERSITÄT DARMSTADT

WITH DC BIAS - VERIFICATION

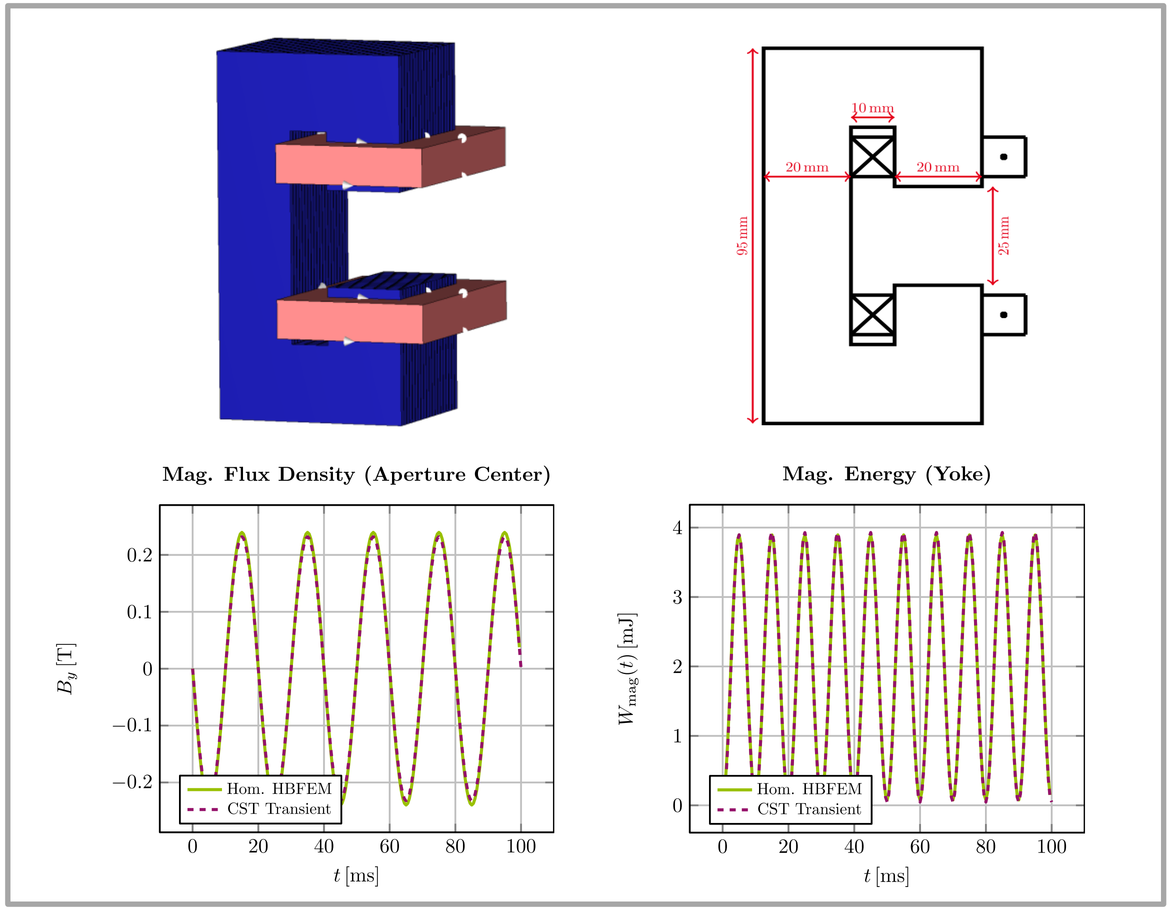
- Excitation current: $I_s(t) = 750\text{A} + 120\text{A}\cos(2\pi 50\text{Hz}\ t)$
- Comparison to transient CST simulation of toy model:
  - Very good **agreement in magnetic energy** in the core
  - Decent **agreement in magnetic flux densities** at individual points inside the core (average rel. error 3.7 %)



TECHNISCHE UNIVERSITÄT DARMSTADT

APPLICATION TO C-DIPOLE

- Same magnet as before, lam. thickness $d = 0.5\ \mathrm{mm}$
- Excitation current for both coils:
  $I_\mathrm{s}(t) = 2.5\ \mathrm{kA}\cos(2\pi 50\mathrm{Hz}\ t)$
- Agreement in **aperture field** and **magnetic energy** in the yoke
- **Eddy current losses** well approximated:
  1.36 W with Hom. HBFEM vs. 1.32 W with CST
  ➔ 3 % relative error
- **Higher order finite elements*** to achieve good approximation of losses and energy



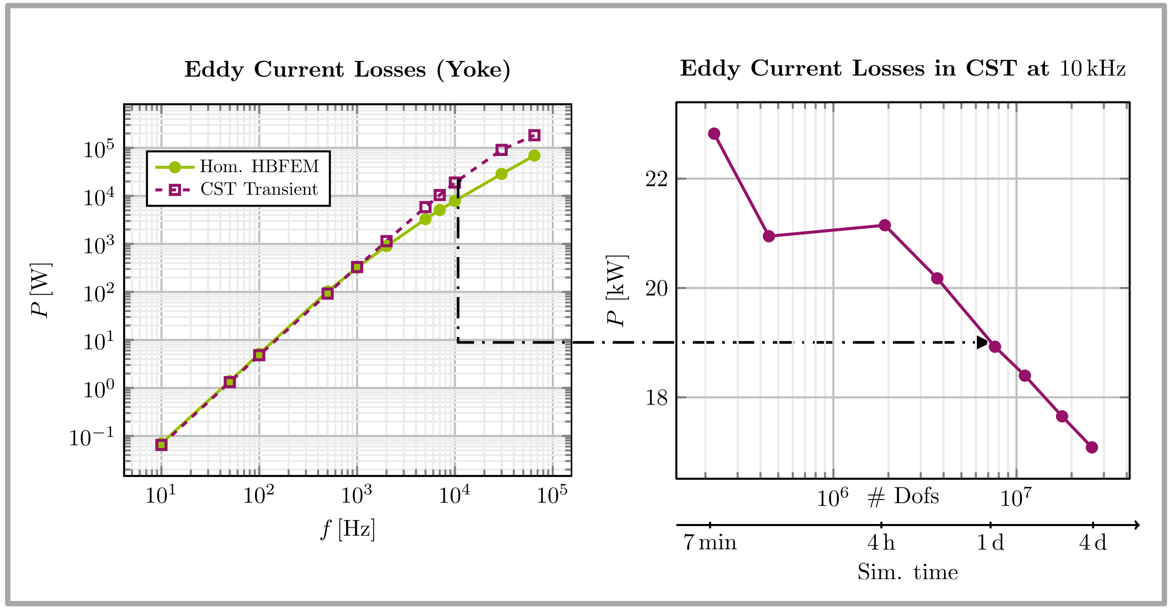
*J.P. Webb and B. Forghani, “Hierarchal Scalar and Vector Tetrahedra“, 1993

APPLICATION TO C-DIPOLE

- Compute eddy current losses up to $f = 65\,\mathrm{kHz}$
  ➔ Scaling behavior as expected from theory*
  ➔ Good agreement with CST results up to $f \approx 1\,\mathrm{kHz}$

- Differences between Hom. HBFEM and CST at higher frequencies are due to mesh dependence of CST results

- Hom. HBFEM reduces simulation time for nonlinear simulations in kilohertz range from days to hours



* R. L. Stoll, *The Analysis of Eddy Currents.* 1974.
J. Lammeraner and M. Štafl, *Eddy Currents.* 1966.

TECHNISCHE UNIVERSITÄT DARMSTADT

CONTENTS

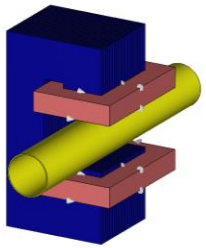
TECHNISCHE UNIVERSITÄT DARMSTADT

CONCLUSION/OUTLOOK

1. Verification of Homogenization

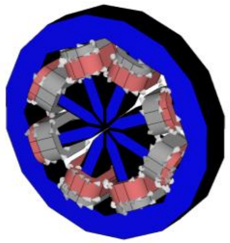
- Good approximation of multipoles & power losses

➔ Simulation time for linear simulations reduced from hours to minutes



2. Application of Homogenization to Corrector

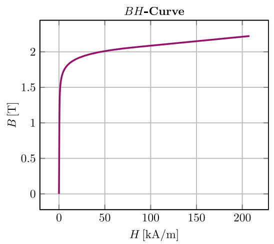
- Power losses, multipoles along axis
- Integrated transfer function & field lag
- Cross-talk with neighboring magnets
- Beam pipe material transition



3. Implementation & Verification of Hom. HBFEM

- Toy model with and without DC bias
- C-Dipole without DC bias up to 65 kHz

➔ Simulation time for nonlinear simulations reduced from days to hours



Application of Hom. HBFEM to Corrector

REFERENCES

[1] PETRA IV Conceptual Design Report.

[2] K. Wille, *Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen*. Stuttgart, Germany: Teubner, 1992.

[3] P. Dular et al., “A 3-D Magnetic Vector Potential Formulation Taking Eddy Currents in Lamination Stacks Into Account*,” IEEE Trans. Magn*., vol. 39, no. 3, pp. 1424-1427, May 2003.

[4] L. Krähenbühl et al., “Homogenization of Lamination Stacks in Linear Magnetodynamics,” *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 912 - 915 Mar. 2004.

[5] H. De Gersem, S. Vanaverbeke, and G. Samaey, “Three-Dimensional-Two-Dimensional Coupled Model for Eddy Currents in Laminated Iron Cores,” *IEEE. Trans. Magn*., vol. 48, no. 2, pp.815 – 818, Feb. 2012.

[6] B. Podobedov et al., “Eddy Current Shielding by Electrically Thick Vacuum Chambers,” *Proceedings of PAC09*, *Vancouver, BC, Canada,* 2009.

[7] S. Yamada and K. Bessho, “Harmonic Field Calculation by the Combination of Finite Element Analysis and Harmonic Balance Method,” in *IEEE Trans. Mag.,* vol.24, no. 6, pp. 2588-2590, Nov. 1988.

[8] H. De Gersem, H. Vande Sande, and K. Hameyer, “Strong Coupled Multi-Harmonic Finite Element Simulation Package”, COMPEL, vol. 20, no.2, pp. 335-546, June 2001.

[9] J. Gyselinck, L. Vandevelde, and J. Melebeek, “Calculation of Eddy Currents and Associated Losses in Electrical Steel Laminations,” in *IEEE Trans. Mag.,* vol. 35, n. 3, May 1999

[10] J. P. Webb and B. Forghani, "Hierarchal scalar and vector tetrahedra," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1495-1498, Mar. 1993, doi: 10.1109/20.250686.

[11] R. L. Stoll, *The Analysis of Eddy Currents*. Oxford, U. K., Clarendon Press, 1974.

[12] J. Lammeraner and M. Štafl, *Eddy Currents.* Iliffe books, 1966.