



UPDATES ON METHODS AND MODEL IN IMPEDANCE SIMULATIONS FOR AN IVU AT PETRA





CONTENT

- Undulator Geometry
- Simulation Procedure
 - Solvers
 - New Domain Decomposition Solver
 - Shunt Impedance Calculation
- Results
 - Broadband
 - Modelling Impact
 - Domain Decomposition Solver
- Conclusion





GEOMETRY





Entry

Body

Exit

Grey: stainless steel: 1.4e6 S/m Yellow: aluminium: 3.56e7 S/m



GEOMETRY









GEOMETRY



SIMULATION PROCEDURE CHALLENGES

- Large model at high frequency
 - 1 GHz
 - 4.6 m length
 - 32 cm diameter
- Unknown material parameters
 - Multiple simulations required
- Large number of resonances (155)
- High quality factors (up to 10000)
 - Strong dependence on material parameters

- Challenging for all available solvers
 - CST wakefield solver
 - High quality factors
 - CST eigenmode solver
 - Large number of resonances
 - FELIS (in-house impedance solver in FD)
 - Large number of resonances









SIMULATION PROCEDURE APPLIED SOLVERS

- FELIS (in-house impedance solver in FD)
 - Longitudinal coupling impedances by integration
 - Transverse impedances by Panofsky-Wenzel theorem
 - Shunt impedances by rational decomposition
- CST wakefield solver
 - Established method
 - Inefficient for this problem
 - Reference for the coupling impedances
- CST eigenmode solver
 - Longitudinal shunt impedances by integration
 - Transverse shunt impedances by Panofsky-Wenzel theorem
 - Comparison of shunt impedances



SIMULATION PROCEDURE FELIS – PROBLEM FORMULATION

• Weak formulation per subdomain: find $E \in H(curl)$ such that $\forall v \in H(curl)$:

$$\int_{\Omega} \mu^{-1} \nabla \times E \cdot \nabla \times \nu \, \mathrm{d}V - \omega^2 \int_{\Omega} \varepsilon E \cdot \nu \, \mathrm{d}V =$$



• Waveguide BC: $\mu^{-1}n \times \nabla \times E = \mu^{-1}n \times \nabla \times E^{\text{Inc}} - j\omega \sum_{m=1}^{\infty} Y_m^{\text{TX}}(\omega) a_m(E - E^{\text{Inc}}) e_m^{\text{TX}}$

$$a_m(E) = \int_{\Sigma_{\rm WG}} e_m^{\rm TM} \cdot E \, \mathrm{d}S$$

• Resistive wall BC: $\mu^{-1}n \times \nabla \times E = j\omega Y_{SI}(\omega)n \times n \times E$



SIMULATION PROCEDURE FELIS – PROBLEM FORMULATION

 $\sum_{i=1}^{T \in M} \sum_{i=1}^{F}$

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• Resistive wall BC: $\mu^{-1}n \times \nabla \times E = j\omega Y_{SI}(\omega)n \times n \times E$



SIMULATION PROCEDURE FELIS – DOMAIN DECOMPOSITION

- Classical zeroth order OTC
 - $n \times H = -Y(\omega)n \times (n \times E))$
- MTC
 - $n \times H = -\sum_{m=1}^{\infty} Y_m^{\mathrm{TX}}(\omega) n \times (n \times E_m^{\mathrm{TX}}))$
 - $\mu^{-1}[n \times \nabla \times E] = \sum_{m=1}^{\infty} Y_m^{\mathrm{TX}}(\omega) \alpha_m^{\mathrm{TX}}(E) e_m^{\mathrm{TX}}$
- MTC with beam
 - $\mu^{-1}[n \times \nabla \times (E E^{\operatorname{Inc}})] = \sum_{m=1}^{\infty} Y_m^{\operatorname{TX}}(\omega) \alpha_m^{\operatorname{TX}}(E E^{\operatorname{Inc}}) e_m^{\operatorname{TX}})$
- Added GMRES iteration procedure
 - Required for resonant structures



SIMULATION PROCEDURE FELIS – IMPEDANCE CALCULATION



- Available data
 - Field solution *E*
 - Current of driving beam I_s on path L_s
 - Current of test beam I_t on path L_t
- Longitudinal impedance
 - $Z_{\parallel} = -\frac{1}{I_0^2} \int_L I(z) E_z(\mathbf{r}_{\perp}, z) \,\mathrm{d}z$
- Vertical impedance by Panofsky-Wenzel theorem

•
$$Z_y = \frac{c}{\omega} \frac{\partial}{\partial y} Z_{\parallel}(\mathbf{r}_{\perp})$$

• $Z_y \approx \frac{c}{\omega} \frac{(Z_{\parallel,t} - Z_{\parallel,s})}{\Delta y}$



SIMULATION PROCEDURE FELIS – SHUNT IMPEDANCE CALCULATION





SIMULATION PROCEDURE EIGENMODE SOLVER – SHUNT IMPEDANCES

- No modes propagating in the beam pipes
 - Direct integration
 - $V_i = \int_{z_0}^{z_1} E_{z,i} e^{jk_i z} \mathrm{d}z$
- Longitudinal shunt impedances

•
$$R_{\mathrm{s},i} = \frac{|V_i|^2 Q_i}{2W_i \omega_i}$$

Vertical shunt impedances by Panovsky-Wenzel theorem

•
$$R_{\text{sy},i} = \frac{c}{\omega} \frac{\partial}{\partial y} R_{\text{s},i}(\mathbf{r}_{\perp})$$

• $R_{\text{sy},i} \approx \frac{c}{\omega} \frac{(R_{\text{s},i}(\mathbf{r}_{\perp} + \Delta y e_y) - R_{\text{s},i}(\mathbf{r}_{\perp}))}{\Delta y}$



SIMULATION PROCEDURE CONFIGURATION

- Geometry
 - 5 mm gap
 - 1.25 mm vertical beam displacement
- Felis
 - 12 cells per wavelength
 - 240 000 hex and tet mesh cells
 - 4th order basis function







- CST Wakefield solver
 - 25 cells per wavelength
 - 14.5 million hex cells
 - 2 km of integrated wake length
 - 30 dB decay in field energy
- CST Eigenmode Solver
 - 30 cells per wavelength
 - 1.6 million tet cells
 - 2nd order basis functions



RESULTS LONGITUDINAL COUPLING IMPEDANCE

- Perfect match in the overall trend
- Perfect agreement in resonance frequencies
- Impedance peaks of wakefield solver are lower due to cos² window
- Small offset of 1.7 Ω





RESULTS LONGITUDINAL COUPLING IMPEDANCE



Perfect agreement

 Impedance peaks of wakefield solver are lower due to cos² window





RESULTS LONGITUDINAL SHUNT IMPEDANCES



- Normalized by beam displacement
- Some shunt impedances missing for the impedance solver due to vector fitting

- Perfect agreement at lower frequencies
- Increasing deviation due to mesh
- Highest shunt impedances around 200 MHz





RESULTS TRANSVERSE SHUNT IMPEDANCES

- Threshold for instability: 16.4 MΩ/m
- Highest shunt impedances around 200 MHz
- No critical shunt impedances above 300 MHz





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RESULTS TRANSVERSE SHUNT IMPEDANCES

- Threshold for instability: 16.4 MΩ/m
- Four shunt impedances above threshold

Index	F in Mhz	Q	$Z_{s,\perp}$ in M Ω/m
1	99.0	1265	8.25
2	133	1089	9.74
3	158	1069	25.7
4	185	1072	30.2
5	215	1095	26.0
6	247	1129	19.9
7	279	1168	13.9





RESULTS UNCERTAIN MATERIAL PROPERTIES



- AISI 317: σ = 1.4e6 S/m
- Steel 1010: σ = 7e6 S/m

 Steel conductivity causes some uncertainty in transverse shunt impedances





RESULTS IMPROVED MODEL

- Accurate modelling of copper parts
- Curved flexible taper







RESULTS IMPROVED MODEL

- Accurate modelling of copper parts
- Curved flexible taper
- Accurate pillar geometry
- Still unknown steel material properties
 - $\sigma = 1.4e6 \text{ S/m}$





RESULTS IMPROVED MODEL

- Moderate change of shunt impedances
- Strong change of resonance frequencies
 - Mainly due to new pillar model







RESULTS DOMAIN DECOMPOSITION SOLVER





- Old Geometry
- ~700.000 vol. elements (4th order)
 - 38e6 DOFs, 1.5 TB Ram
- 8 MPI nodes at Lichtenberg II
 - 96 cores and 384 GB / node

- MTC with 22 modes
- ~30 DMM iterations / frequency point



RESULTS DOMAIN DECOMPOSITION SOLVER

- Reference: CST Wakefield solver
 - Perfect agreement



Less than 5h of computations





CONCLUSION

- Shunt impedances up to 1 Ghz with high solver accuracy
- Transverse shunt impedances around 200 MHz are critical
- Unknown steel conductivity creates small uncertainty
- Improved model
 - Moderate change of shunt impedances
 - Strong change of resonance frequencies
- Domain decomposition solver
 - Enables application of HPC ressources
 - Fast results at high accuracy