Coupled Space Charge and Wakefield Simulation of a Retracted Gun

J. Christ and E. Gjonaj

jonas.christ@tu-darmstadt.de

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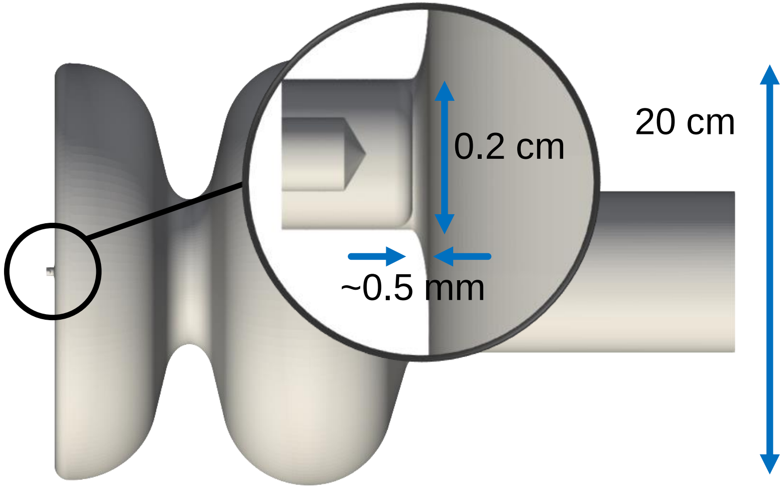
Outline

- Retracted Cathode
- Scattered Field Formulation
- Coupling to Beam Dynamics
- Results

Retracted Cathode

- **Idea of retracted cathode: built-in RF focusing for emittance compensation**
  - Strong coupling between wakefields and space-charge interaction
  - Kick-wise application of wakefields is inaccurate
  - Full-scale EM PIC not feasible



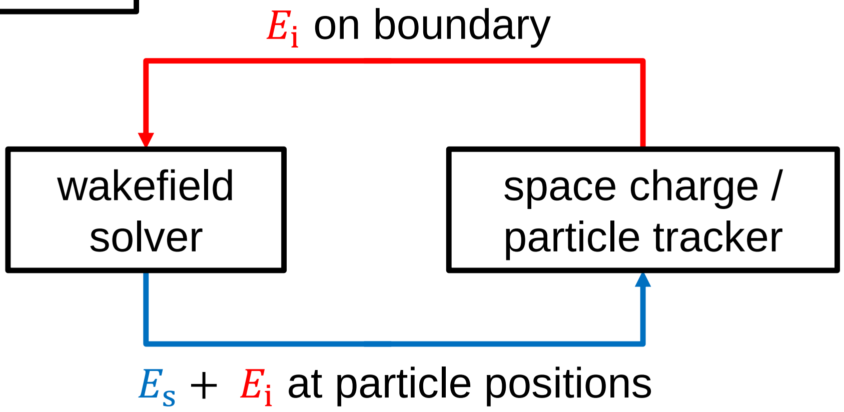
Credit: Bazyl, Gjonaj; Vennekate

Idea

- **Strong coupling of space-charge and wakefield calculations**

  → Scattered field formulation: $E = E_\mathrm{i} + E_\mathrm{s}$

- Employ available specialized solvers
  - Space-charge: Green function in rest frame
  - Wakefields: FIT in moving window



- For arbitrary particle dynamics
- Avoids current interpolation step (in PIC)
- Allows better resolution of space-charge fields (than PIC)

Scattered Field Formulation

- Incident field is a matter of choice
- Fulfills Maxwell's eqs. for given current

$$\frac{d}{dt} E = \varepsilon^{-1} \operatorname{curl} H - \varepsilon^{-1} J$$
$$\frac{d}{dt} H = -\mu^{-1} \operatorname{curl} E$$

BC: $E_t = 0$ on Γ_{PEC}

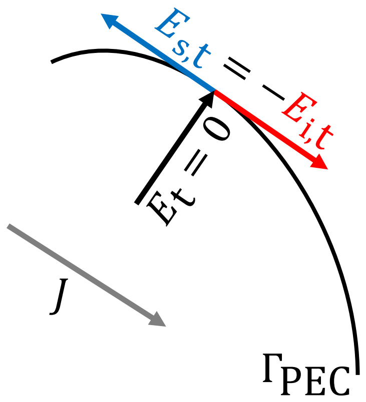
$E = E_s + E_i$

$$\frac{d}{dt} E_i = \varepsilon^{-1} \operatorname{curl} H_i - \varepsilon^{-1} J$$
$$\frac{d}{dt} H_i = -\mu^{-1} \operatorname{curl} E_i$$

arbitrary BC

$$\frac{d}{dt} E_s = \varepsilon^{-1} \operatorname{curl} H_s$$
$$\frac{d}{dt} H_s = -\mu^{-1} \operatorname{curl} E_s$$

BC: $E_{s,t} = -E_{i,t}$ on Γ_{PEC}



Initial conditions depend on choice of incident field:

$$E_s(0) = E(0) - E_i(0)$$

Scattered Field Formulation in PBCI

- **Staircase approximation**
  - Discretization of Faraday's eq. at a PEC boundary

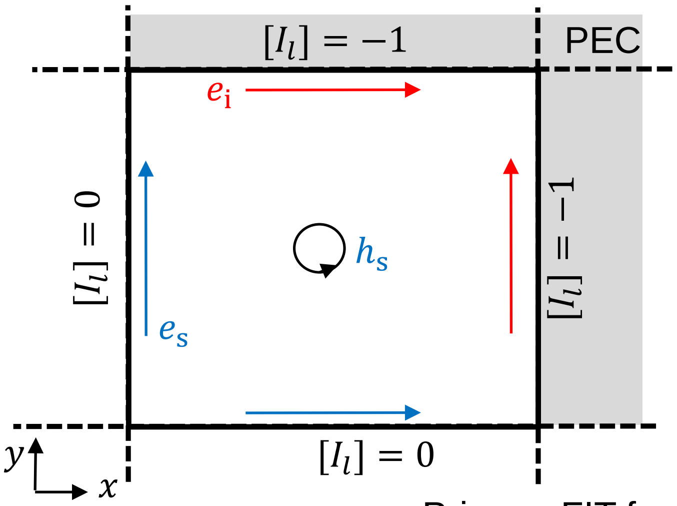
$$\frac{d}{dt}\begin{pmatrix} h_s \\ e_s \end{pmatrix} = \begin{pmatrix} 0 & -M_\mu^{-1}C \\ M_\varepsilon^{-1}C^T & 0 \end{pmatrix}\begin{pmatrix} h_s \\ e_s \end{pmatrix} - \begin{pmatrix} M_\mu^{-1}\, j_{mag} \\ 0 \end{pmatrix}$$

  - Equivalent magnetic current at the boundary

$$j_{mag} = C\, I_l\, e_i$$

  - With local interpolation matrix $I_l = \begin{cases} -1 & \text{if edge in PEC} \\ 0 & \text{else} \end{cases}$
  - Material matrices remain the same as for PEC-boundaries

$$e = e_s + e_i$$



Primary FIT face

Scattered Field Formulation in PBCI II

- **Conformal approximation**
  - Change computation of magnetic current only
  - Two variants:

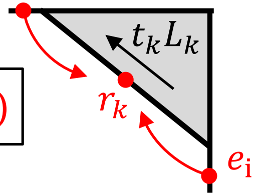
1. Reduction of incident field to conformal lengths / areas

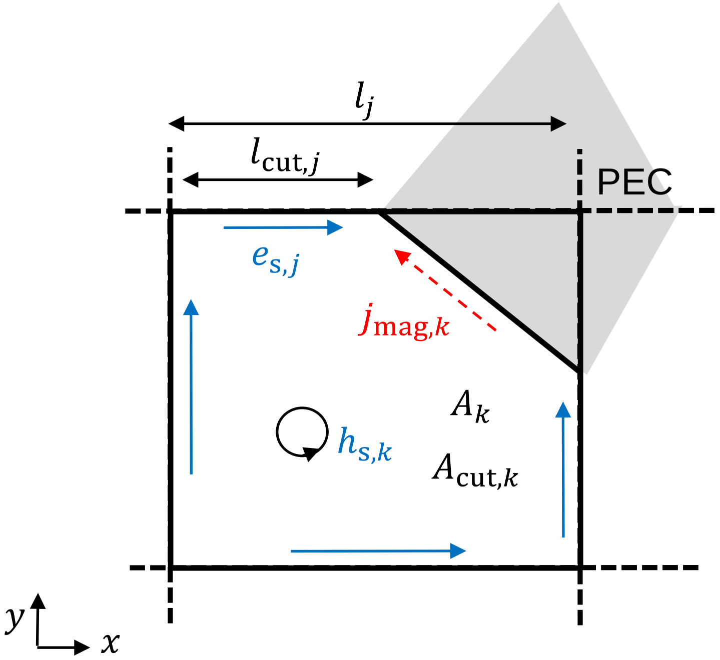
$$e_j = e_{s,j} + \frac{l_{cut,j}}{l_j} e_{i,j} \qquad b_k = b_{s,k} + \frac{A_{cut,k}}{A_k} b_{i,k}$$

$$j_{mag,k} = \sum C_{kj} \frac{l_{cut,j}}{l_j} e_{i,j} - \frac{A_{cut,k}}{A_k} C_{kj} e_{i,j}$$

2. Interpolation to cut edge center

$$j_{mag,k} = -L_k t_k \cdot E_i(r_k)$$

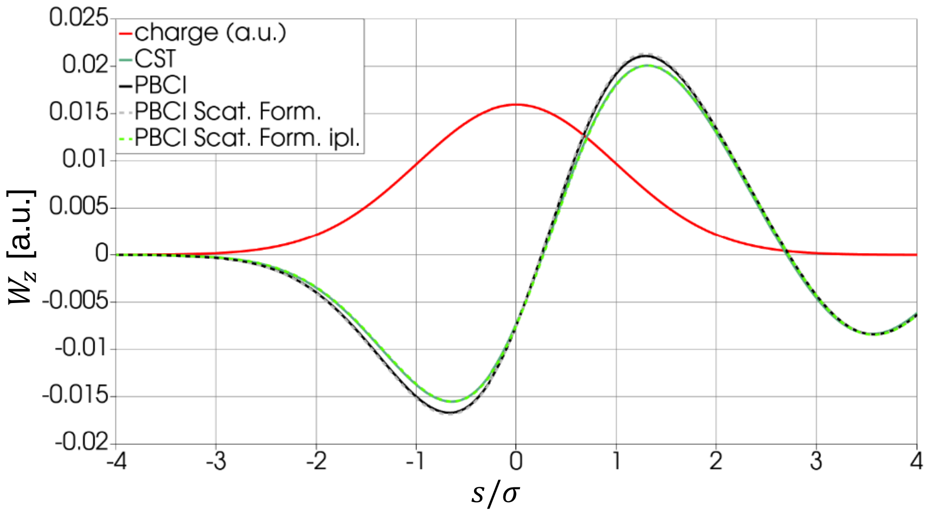


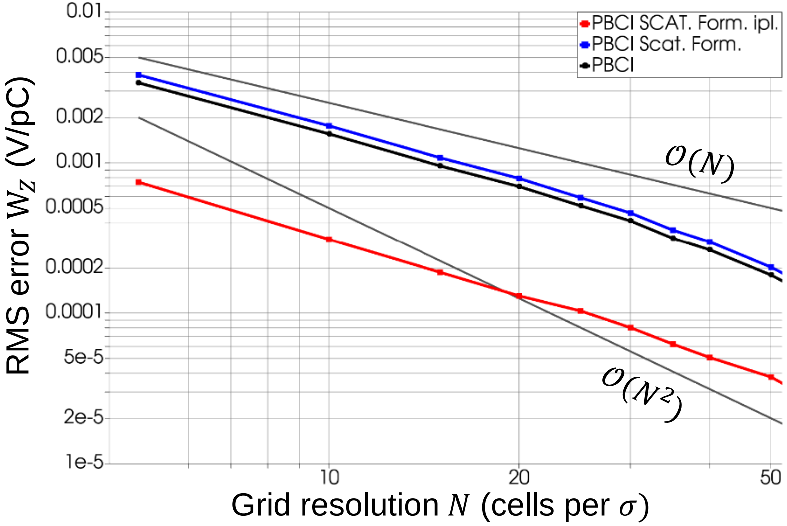


Primary FIT face

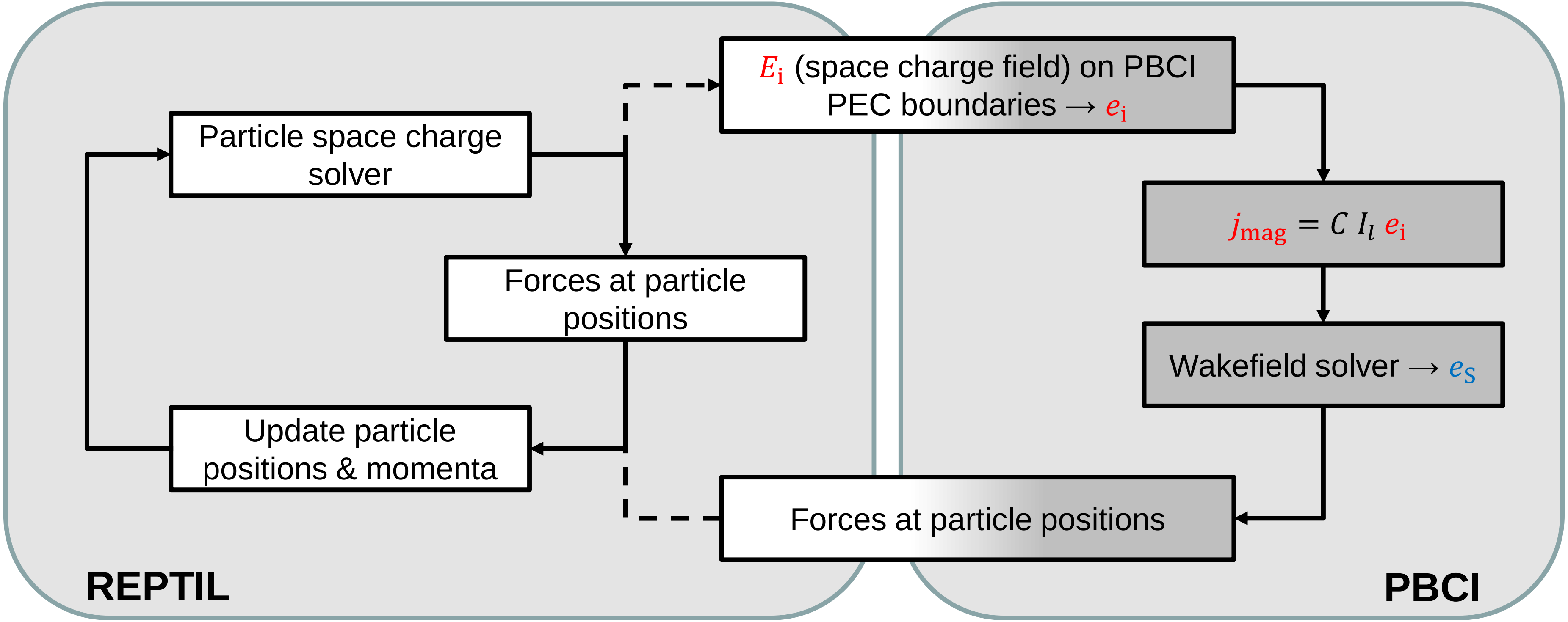
Scattered Field Formulation in PBCI III

- **Validation: tapered collimator (conformal)**
  - Good agreement with PBCI and CST
  - Same convergence behavior as PBCI

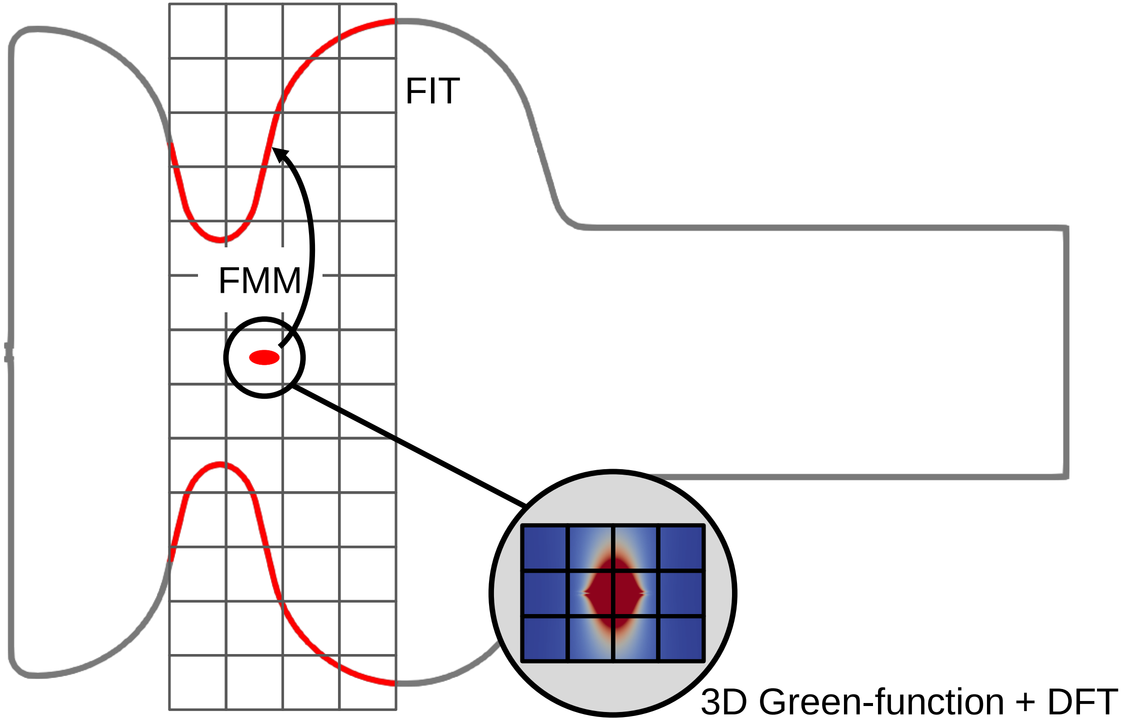




Coupling: PBCI + REPTIL



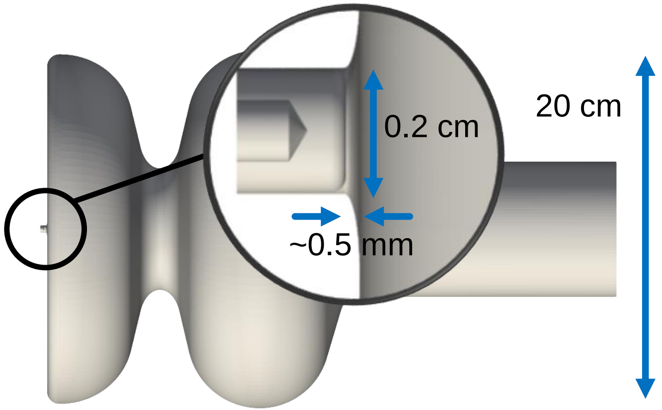
Coupling: PBCI + REPTIL II

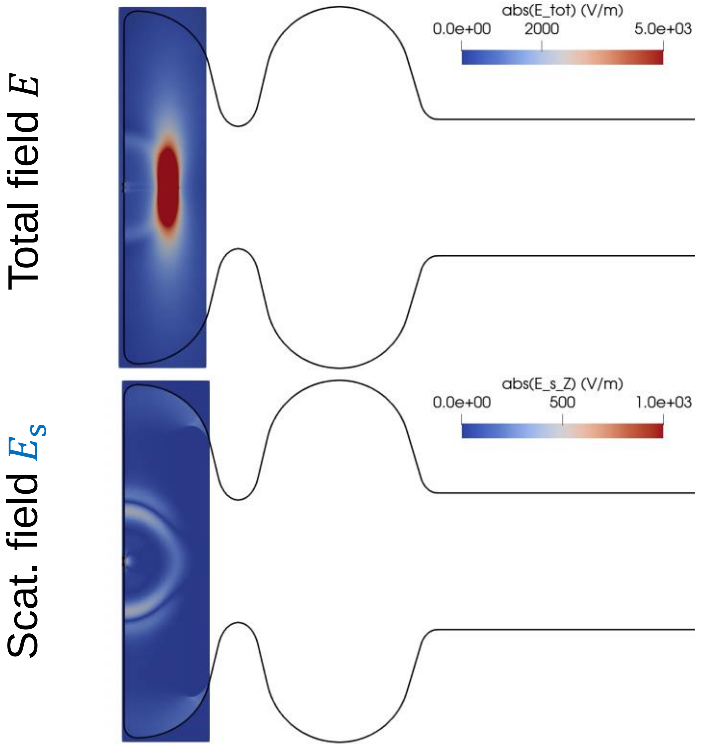


Results: Retracted Cathode

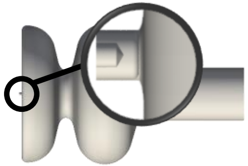
- Idea of retracted cathode: built-in RF focusing to compensate space-charge forces

- Coupled simulation yields the wakefield



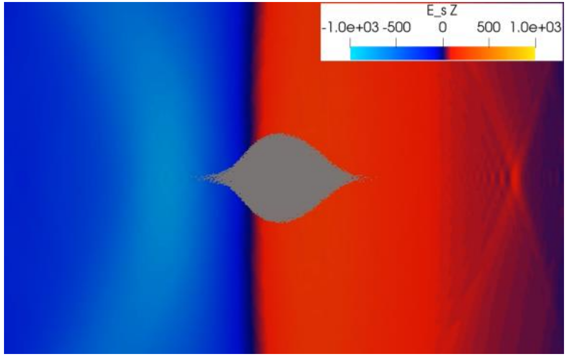


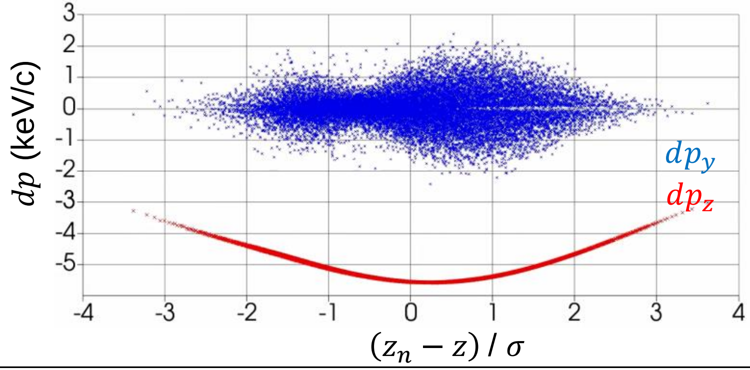
Results: Retracted Cathode II

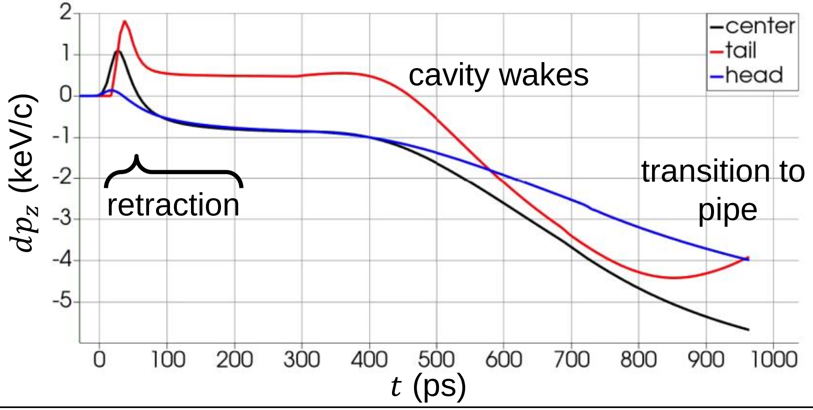


- **Cathode with retraction by 0.45 mm**
  - No back-coupling on trajectory
  - Particle-wise momentum kick computation

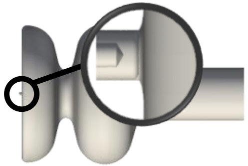
$$dp_i = q_i \int E_s + v_i \times B_s \,\mathrm{d}t$$







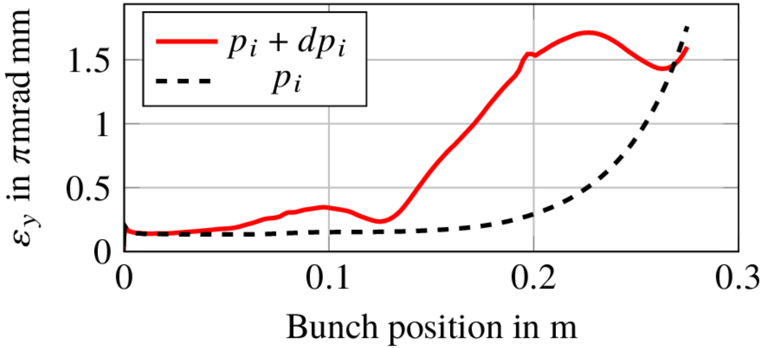
Results: Retracted Cathode III

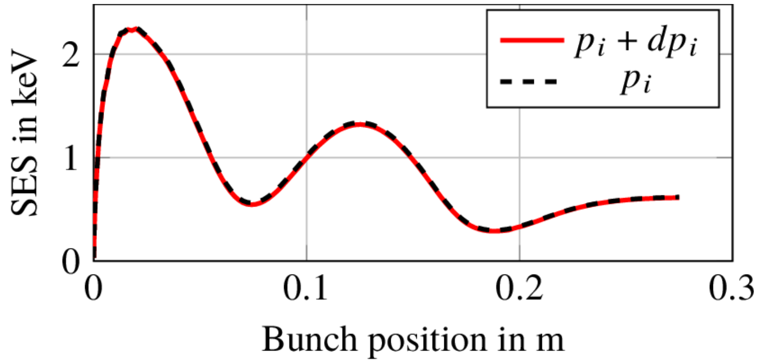


- **Cathode with retraction by 0.45 mm**
  - No back-coupling on trajectory
  - Particle-wise momentum kick computation

$$dp_i = q_i \int E_s + v_i \times B_s \, \mathrm{d}t$$

  - Wakefields influence transv. emittance
  - SES remains unaffected
  - Dominant wakefields from iris and pipe transition





Discussion & Outlook

- Scattered field formulation for FIT
- Successful implementation in wakefield code PBCI, coupled with space charge solver REPTIL
- First results for retracted cathode
- Outlook
  - Back-coupling of wakefields directly onto the particles
  - Surface impedance BC, adaptive time steps, 2D field maps
  - CSR wakefields in the bunch compressor