Calculating Losses in AC Accelerator Magnets

TECHNISCHE UNIVERSITÄT DARMSTADT

DESY-TEMF-CANDLE meeting, 21th September 2021

Herbert De Gersem
herbert.degersem@tu-darmstadt.de

Overview

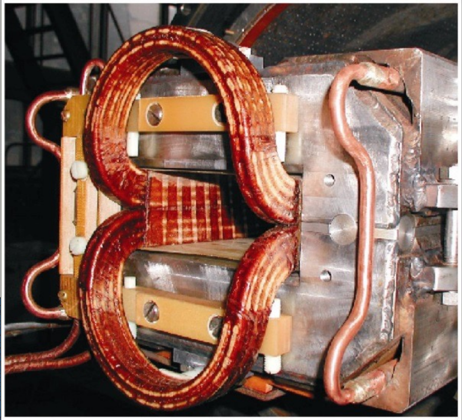
- Finite-element simulation
- Losses in accelerator magnets
- Modelling and simulation challenges
- Homogenisation of coils
- Homogenisation of lamination stacks

Finite-element simulation

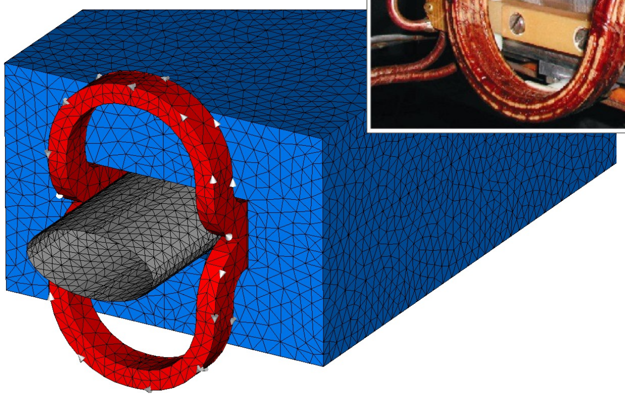
$$\nabla\times\left(\upsilon\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}=\vec{J}_s$$

$\vec{J}_s$	applied current density (coils)
υ	reluctivity
σ	conductivity
$\vec{A}$	magnetic vector potential

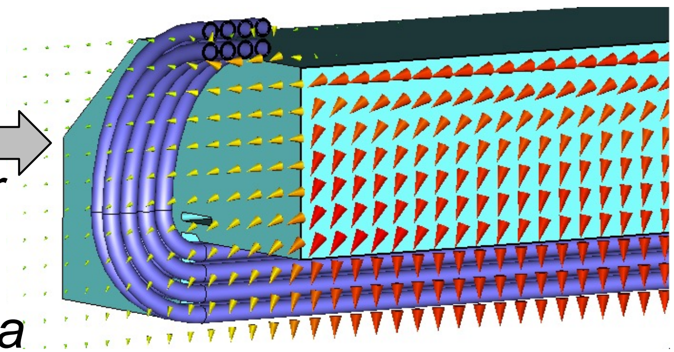
$\vec{B}=\nabla\times\vec{A}$	magnetic flux density
$\vec{J}_e=-\sigma\frac{\partial\vec{A}}{\partial t}$	eddy-current density
$p=\frac{1}{\sigma}\left\lvert\vec{J}_s+\vec{J}_e\right\rvert^2$	loss density
$\vec{J}_e\rightarrow\vec{A}_e\rightarrow\vec{B}_e$	parasitic field



mesher



FE solver



2nd order FEs, multigrid+CG, implicit Runge Kutta

Losses in Accelerator Magnets

TECHNISCHE UNIVERSITÄT DARMSTADT

ramped/AC magnets

ramped magnets	: increase during acceleration
AC magnets	: orbit corrections

losses in ramped/AC magnets

coil	Joule losses
coil	eddy-current losses
yoke	eddy-current losses
yoke	hysteresis losses

prevention

coil	many thin wires, twisting, transposition
yoke	many thin laminations

appropriate cooling

Modelling and Simulation Challenges

time dependence

nonlinear material

steep ramping

→ adaptive time stepping needed

many geometric details (prevention of losses)

homogenisation inavoidable

coil	conductor model
lamination	bulk model for lamination stack

numerical accuracy

looking for a secondary effect → higher accuracy needed

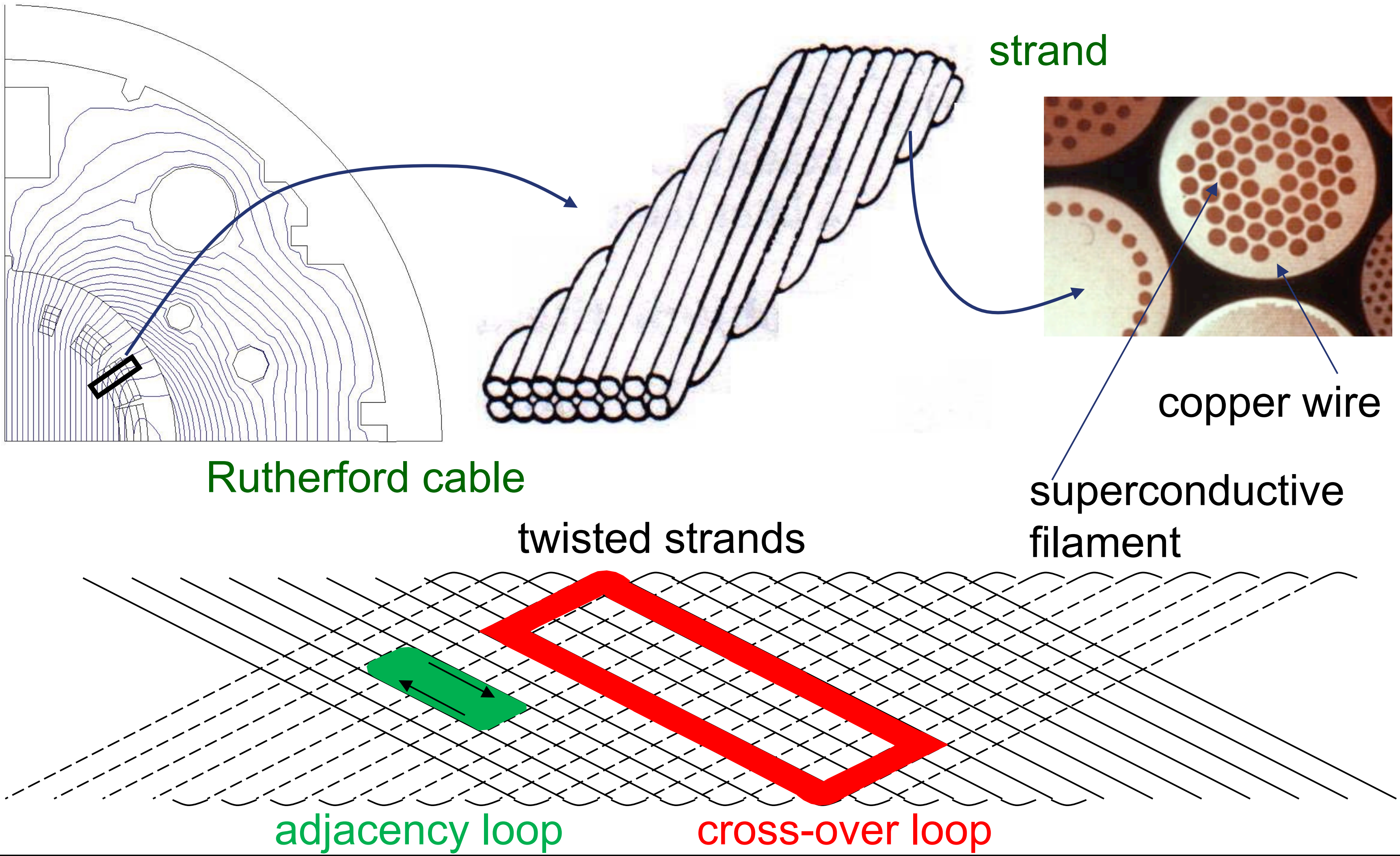
spatial scale, related to skin depth → large FE mesh

temporal scale, related to ramping → many time steps

Overview

- Finite-element simulation
- Losses in accelerator magnets
- Modelling and simulation challenges
- Homogenisation of coils
- Homogenisation of lamination stacks

Rutherford Cable (1)



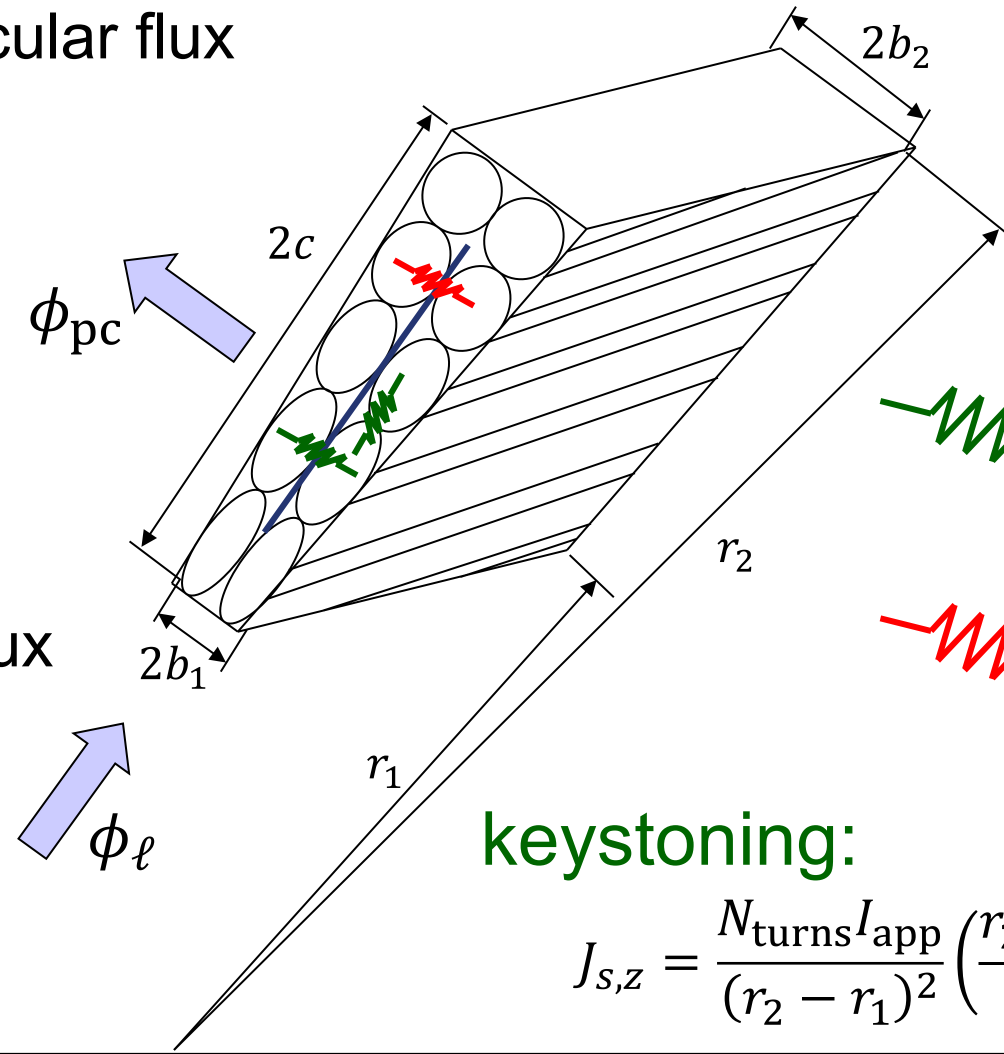
Rutherford Cable (2)

perpendicular flux

$B_p(r,\theta)$

parallel flux

$B_\ell(r,\theta)$



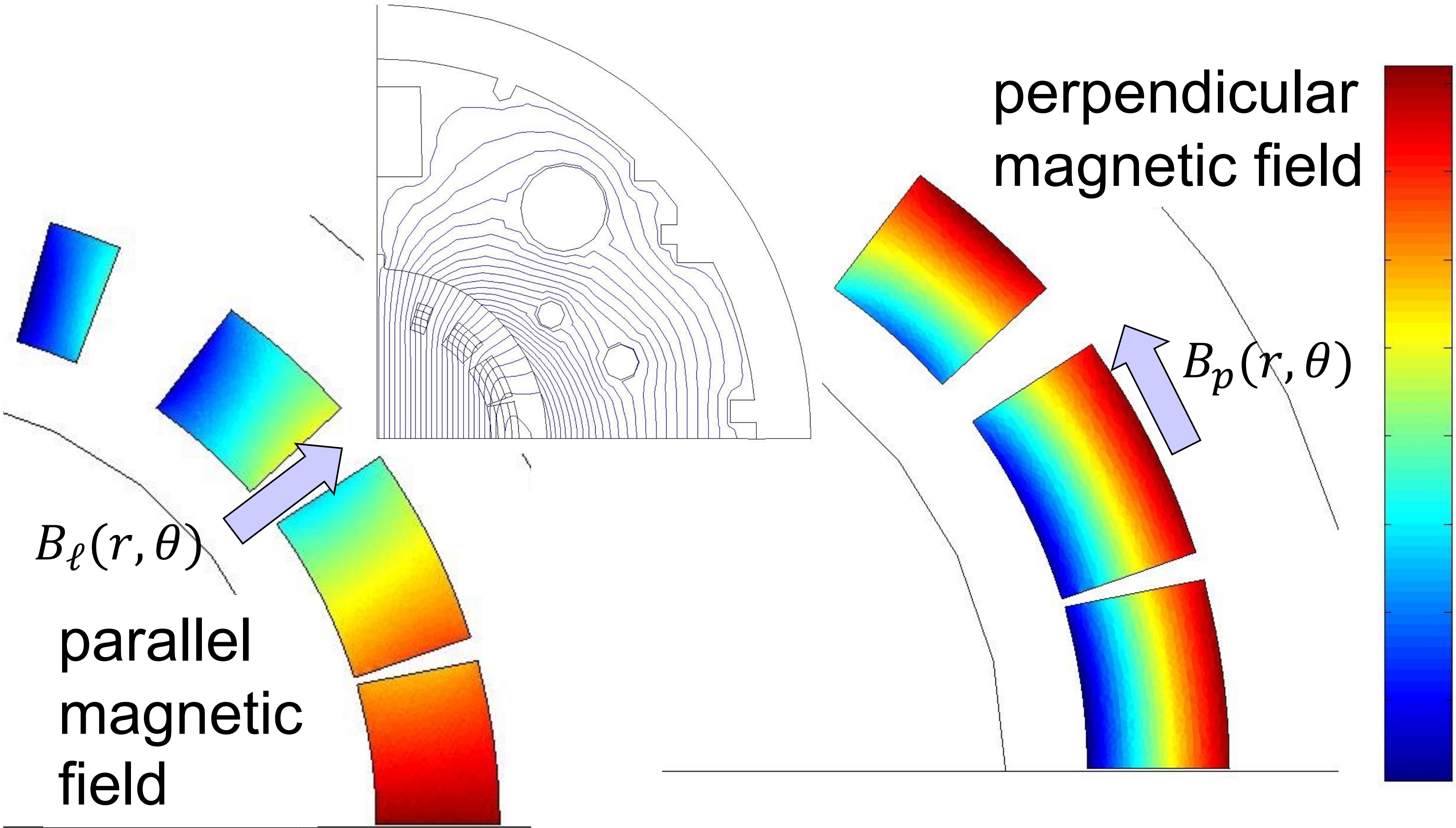
adjacency resistance

cross-over resistance

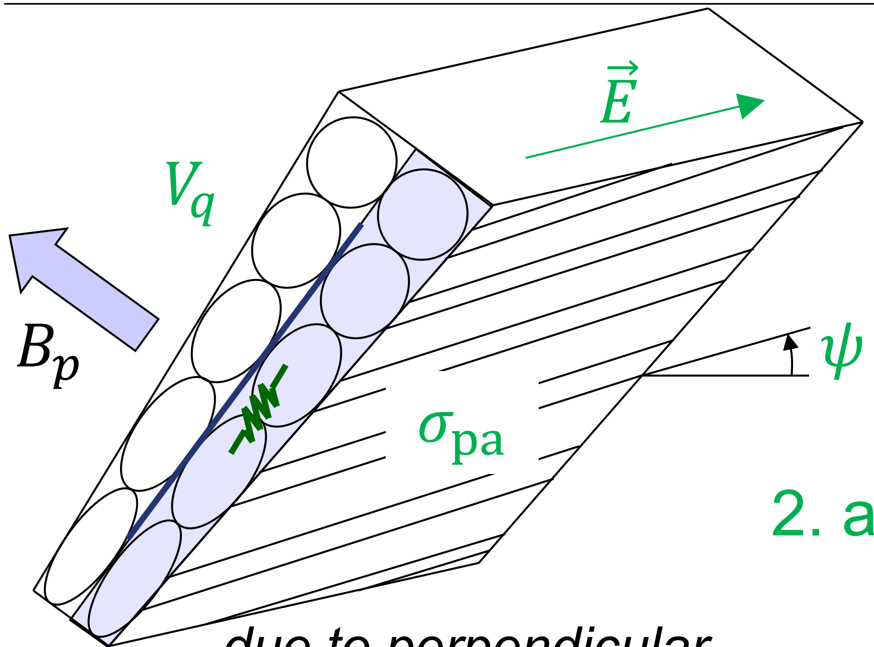
keystoning:

$$J_{s,z} = \frac{N_{\text{turns}} I_{\text{app}}}{(r_2 - r_1)^2}\left(\frac{r_2 - r}{2b_1} + \frac{r - r_1}{2b_2}\right)$$

Magnetic Flux Density



Adjacency Eddy Currents (1)



due to perpendicular magnetic field

1. additional discretisation for unknown electric field

$$\vec{E}_{pa}(\vec{r},t) = \sum_q u_q(t)\vec{\xi}_q(\vec{r})$$

2. adjacency eddy current density :

$$\vec{J}_{pa} = \sigma_{pa}\left(\vec{E}_{pa} - \frac{\partial \vec{A}}{\partial t}\right)$$

3. netto current through $V_q = 0$

$$I_{pa,q} = \int_{V_q} \vec{J}_{pa} \cdot \vec{\xi}_q(\vec{r})\mathrm{d}V' = 0$$

} *additional constraint !*

Adjacency Eddy Currents (2)

additional load term for magnetic FE model

$$g_{\mathrm{pa}} = M_{\mathrm{pa}} \frac{\mathrm{d}\hat{a}}{\mathrm{d}t} - Z_{\mathrm{pa}} u_{\mathrm{pa}}$$

additional constraint

$$-Z_{\mathrm{pa}}^T \frac{\mathrm{d}\hat{a}}{\mathrm{d}t} + G_{\mathrm{pa}} u_{\mathrm{pa}} = 0$$

degrees of freedom for u_{pa}

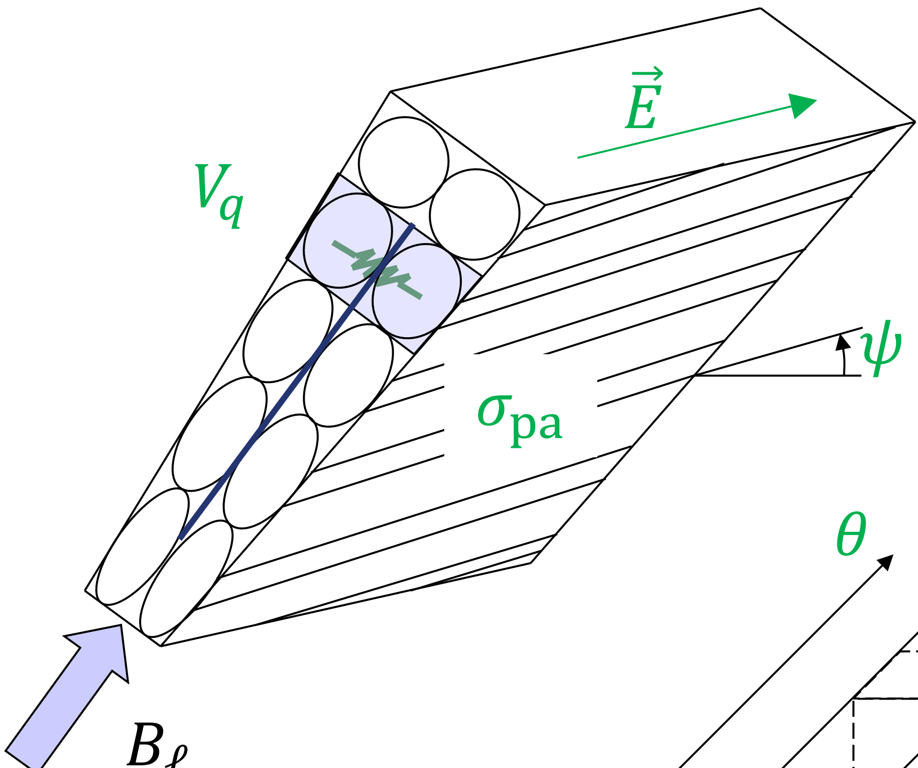
$$\begin{bmatrix} M_{\mathrm{pa}} & 0 \\ Z_{\mathrm{pa}}^T & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \hat{a} \\ u_{\mathrm{pa}} \end{bmatrix} + \begin{bmatrix} K & Z_{\mathrm{pa}} \\ 0 & G_{\mathrm{pa}} \end{bmatrix} \begin{bmatrix} \hat{a} \\ u_{\mathrm{pa}} \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$$M_{\mathrm{pa},ij} = \int_V \sigma_{\mathrm{pa}} \vec{w}_i \cdot \vec{w}_j \mathrm{d}V'$$

$$Z_{\mathrm{pa},iq} = \int_V \sigma_{\mathrm{pa}} \vec{w}_i \cdot \vec{\xi}_q \mathrm{d}V'$$

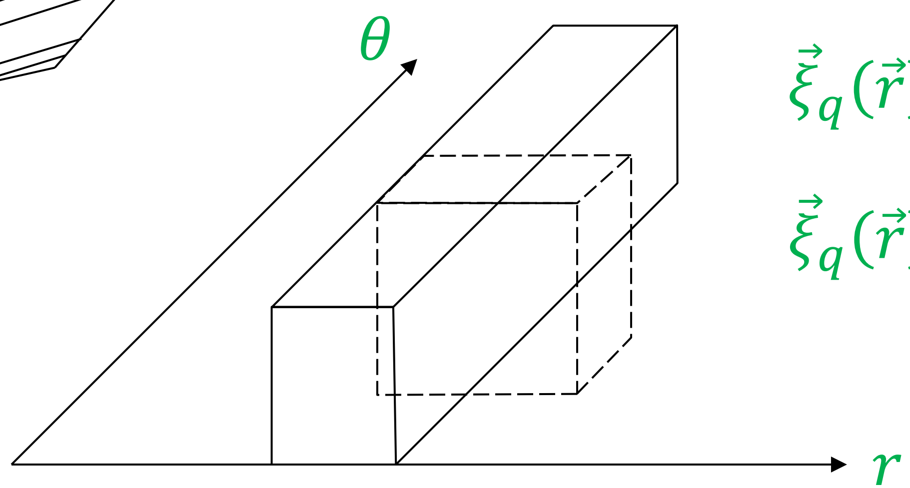
$$G_{\mathrm{pa},pq} = \int_V \sigma_{\mathrm{pa}} \vec{\xi}_p \cdot \vec{\xi}_q \, \mathrm{d}V'$$

Adjacency Eddy Currents (3)



due to parallel magnetic field

shape functions $\vec{\xi}_q(\vec{r})$ are related (but not necessarily equal) to the zones of current redistribution

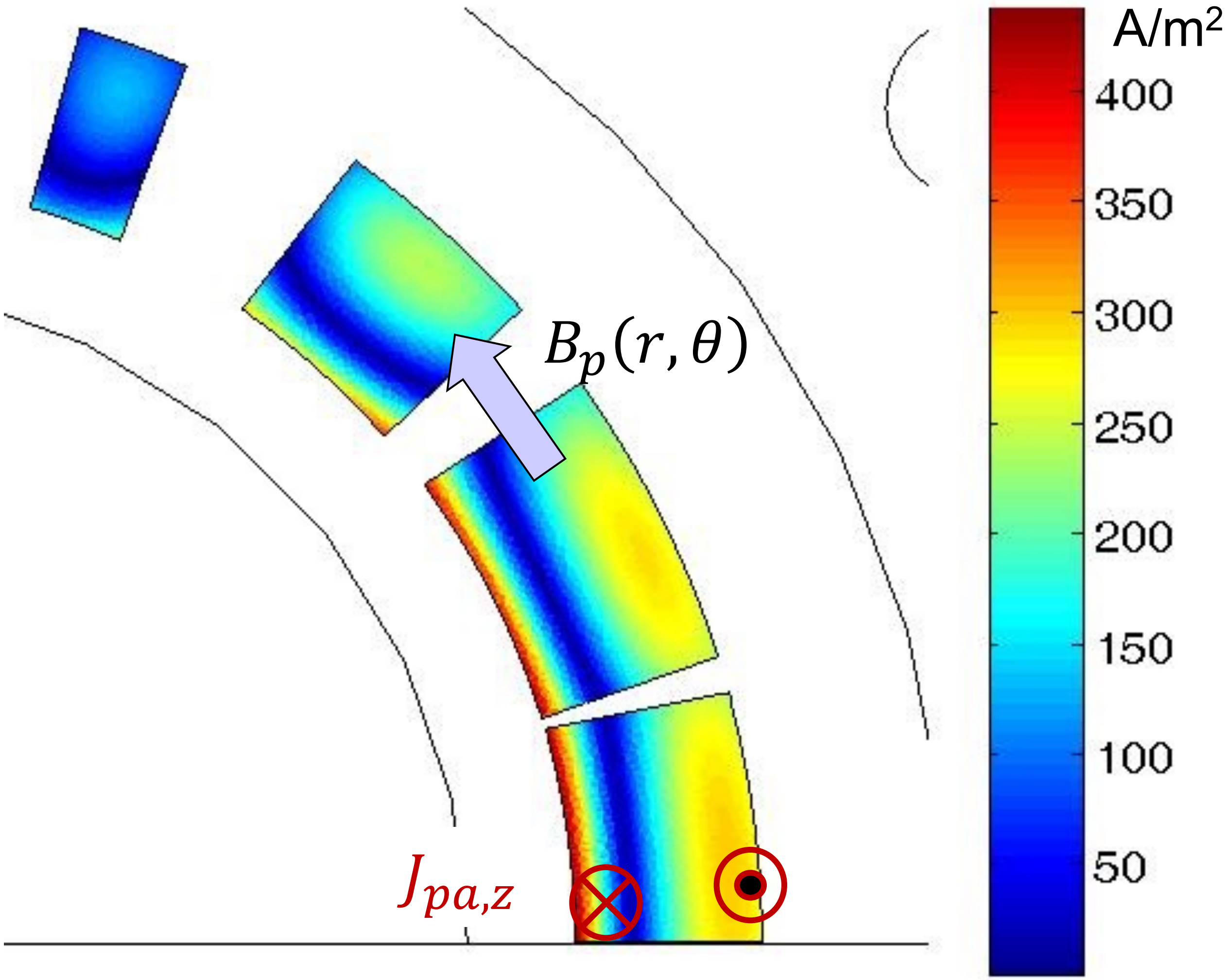


$$\vec{\xi}_q(\vec{r}) = \frac{\vec{e}_z}{\ell_z} \quad \text{in } V_q$$

$$\vec{\xi}_q(\vec{r}) = 0 \text{ outside } V_q$$

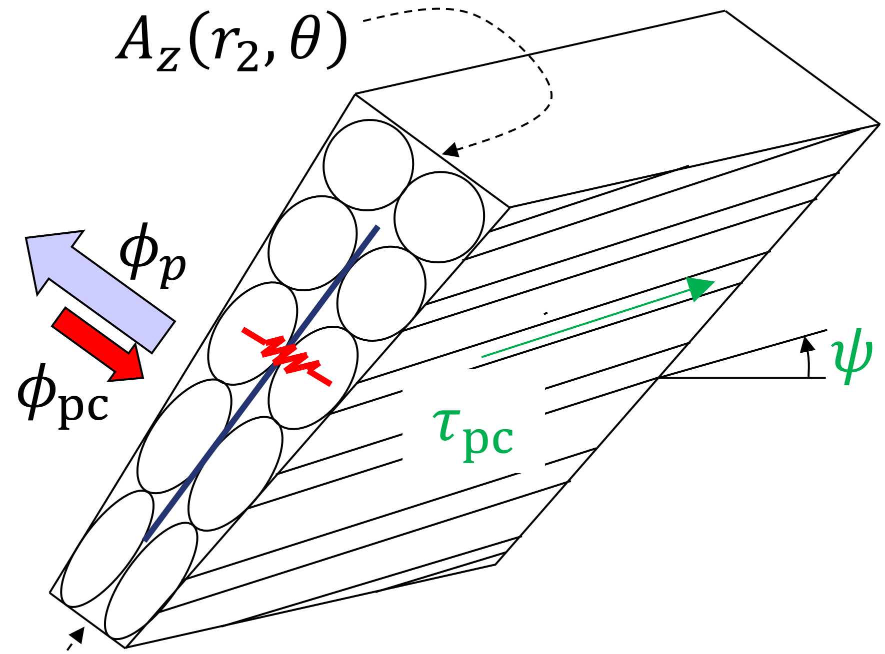
Adjacency Eddy Currents (4)

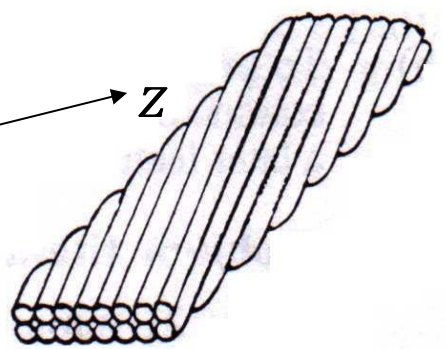
adjacency eddy currents due to perpendicular magnetic field

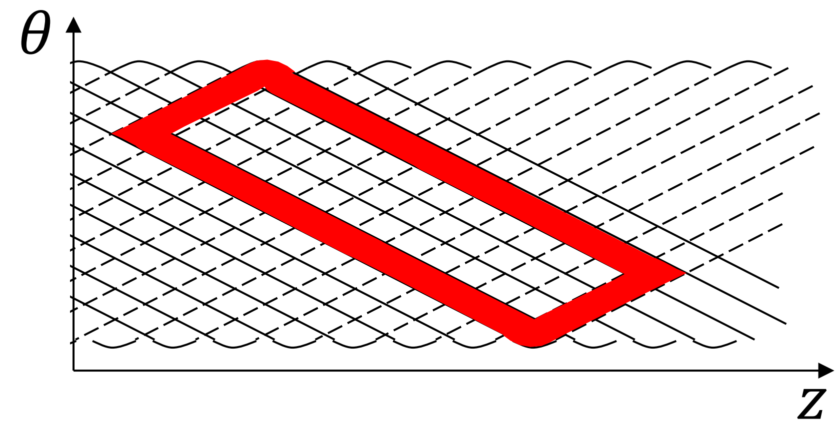


Cross-over Eddy Currents (1)

TECHNISCHE UNIVERSITÄT DARMSTADT







1. coupled flux

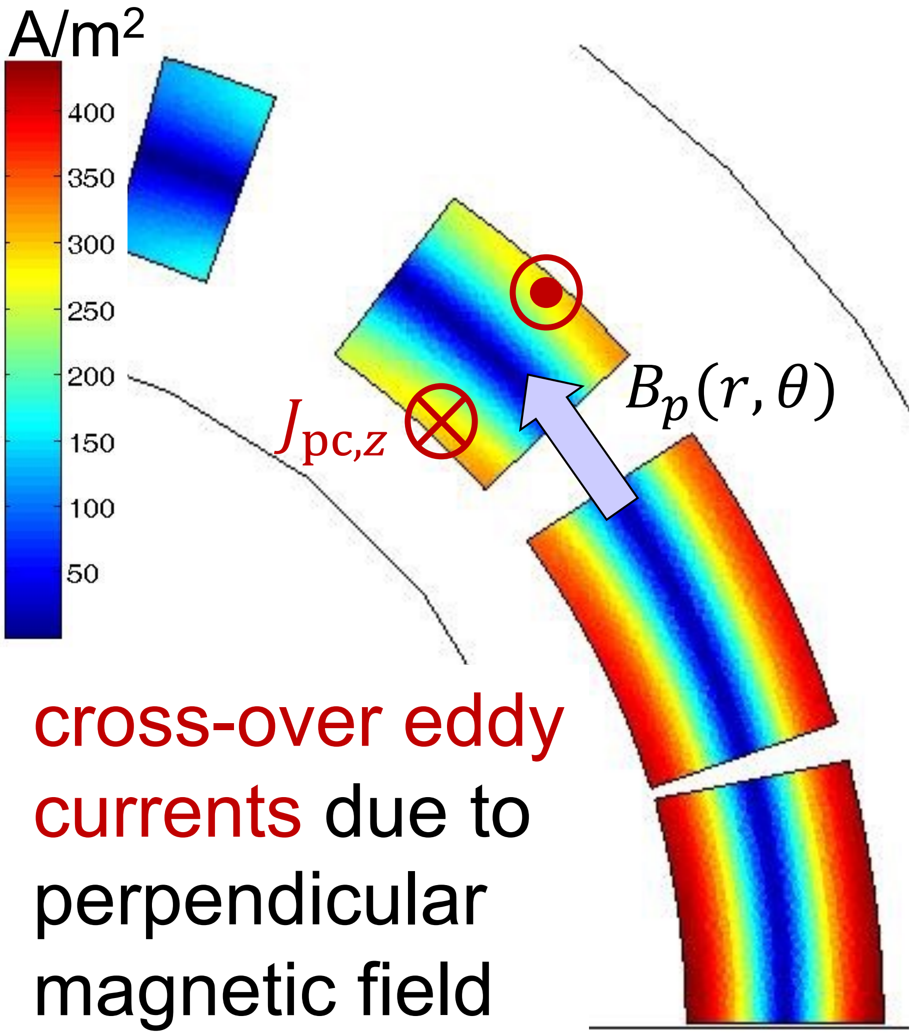
$$\phi_{\mathrm{p}}(\theta) = \ell_z\big(A_z(r_2,\theta) - A_z(r_1,\theta)\big)$$

2. magnetisation

$$\phi_{\mathrm{pc}}(\theta) = \tau_{\mathrm{pc}} \frac{\partial \phi_{\mathrm{p}}(\theta)}{\partial t}$$

time constant

Cross-over Eddy Currents (2)



cross-over eddy currents due to perpendicular magnetic field

cross-over magnetisation

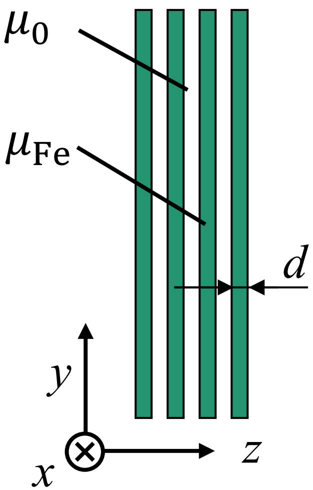
Overview

TECHNISCHE UNIVERSITÄT DARMSTADT

- Finite-element simulation
- Losses in accelerator magnets
- Modelling and simulation challenges
- Homogenisation of coils
- Homogenisation of lamination stacks

Homogenisation: Lamination Stacks

TECHNISCHE UNIVERSITÄT DARMSTADT



$$\nabla\times\left(\overleftrightarrow{\nu}\,\nabla\times\vec{A}\right)+\nabla\times\left(\overleftrightarrow{\tau}\,\nabla\times\frac{\partial\vec{A}}{\partial t}\right)+\overleftrightarrow{\sigma}\frac{\partial\vec{A}}{\partial t}=\vec{J}_s$$

homogenisation with fill factor γ

reluctivity

$$\overleftrightarrow{\nu}=\left(\frac{\gamma}{\mu_{\mathrm{Fe}}}+\frac{1-\gamma}{\mu_0}\right)\begin{bmatrix}1 & & \\ & 1 & \\ & & 0\end{bmatrix}+\frac{1}{\gamma\mu_{\mathrm{Fe}}+(1-\gamma)\mu_0}\begin{bmatrix}0 & & \\ & 0 & \\ & & 1\end{bmatrix}$$

in-plane flux

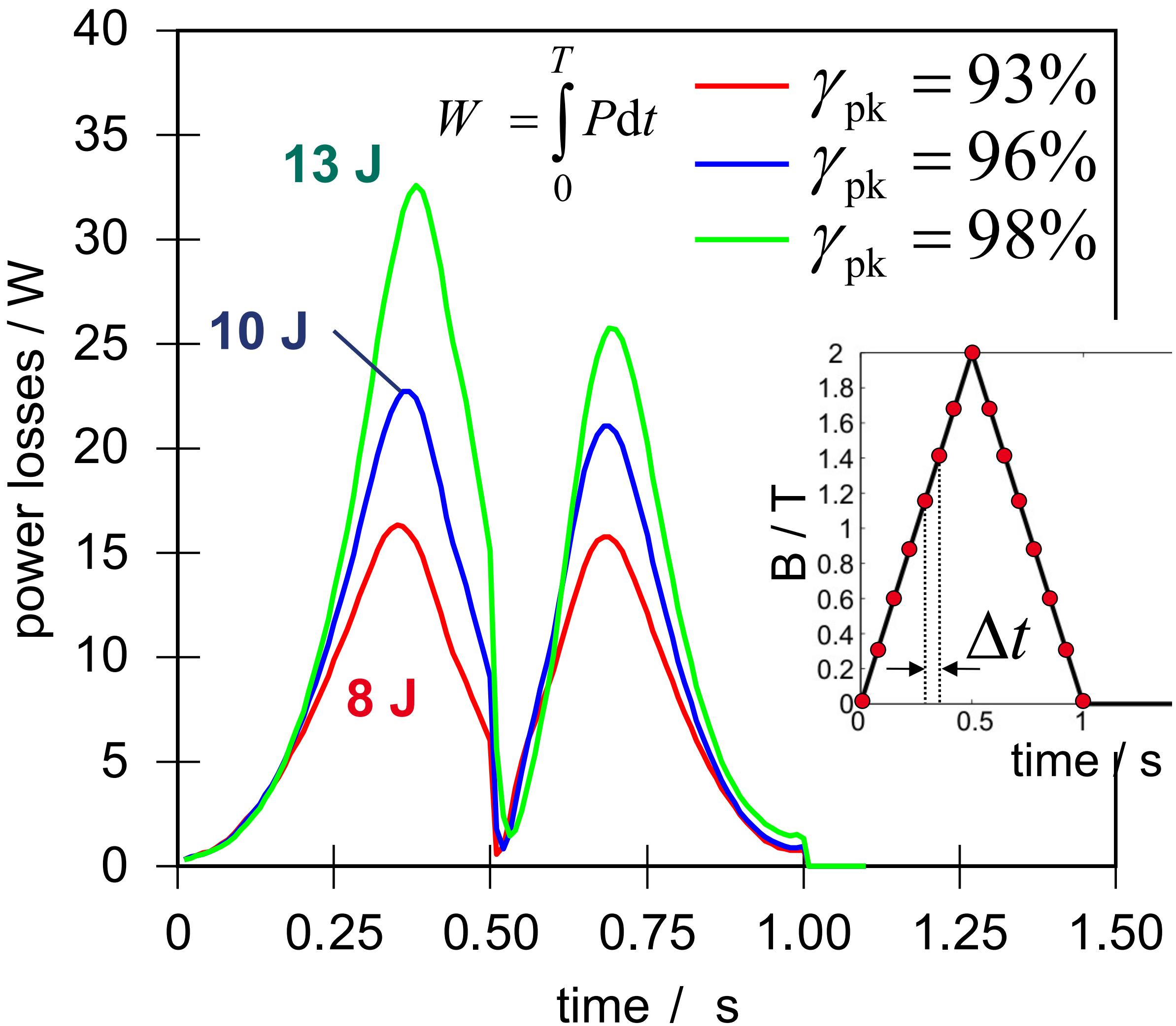
$$\overleftrightarrow{\tau}=\frac{1}{2}\gamma\sigma d^2\begin{bmatrix}1 & & \\ & 1 & \\ & & 0\end{bmatrix}$$

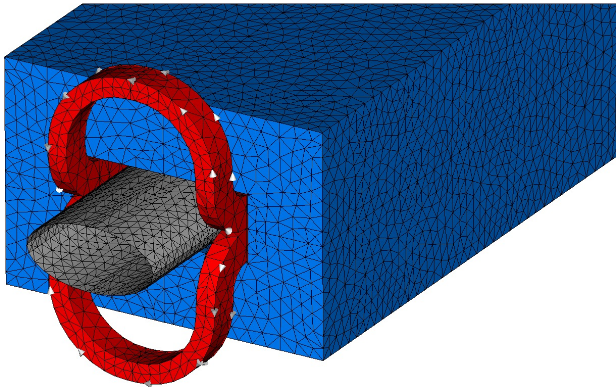
perpendicular flux

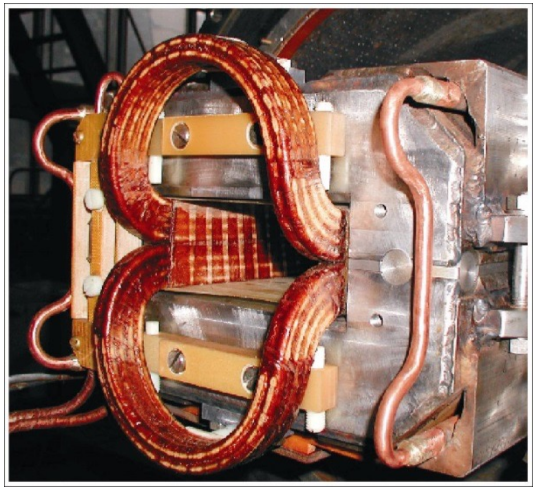
$$\overleftrightarrow{\sigma}=\gamma\sigma\begin{bmatrix}1 & & \\ & 1 & \\ & & 0\end{bmatrix}$$

S. Koch et al., 2008
J. Gyselinck et al. 1999

Example: FAIR SIS100 Magnet







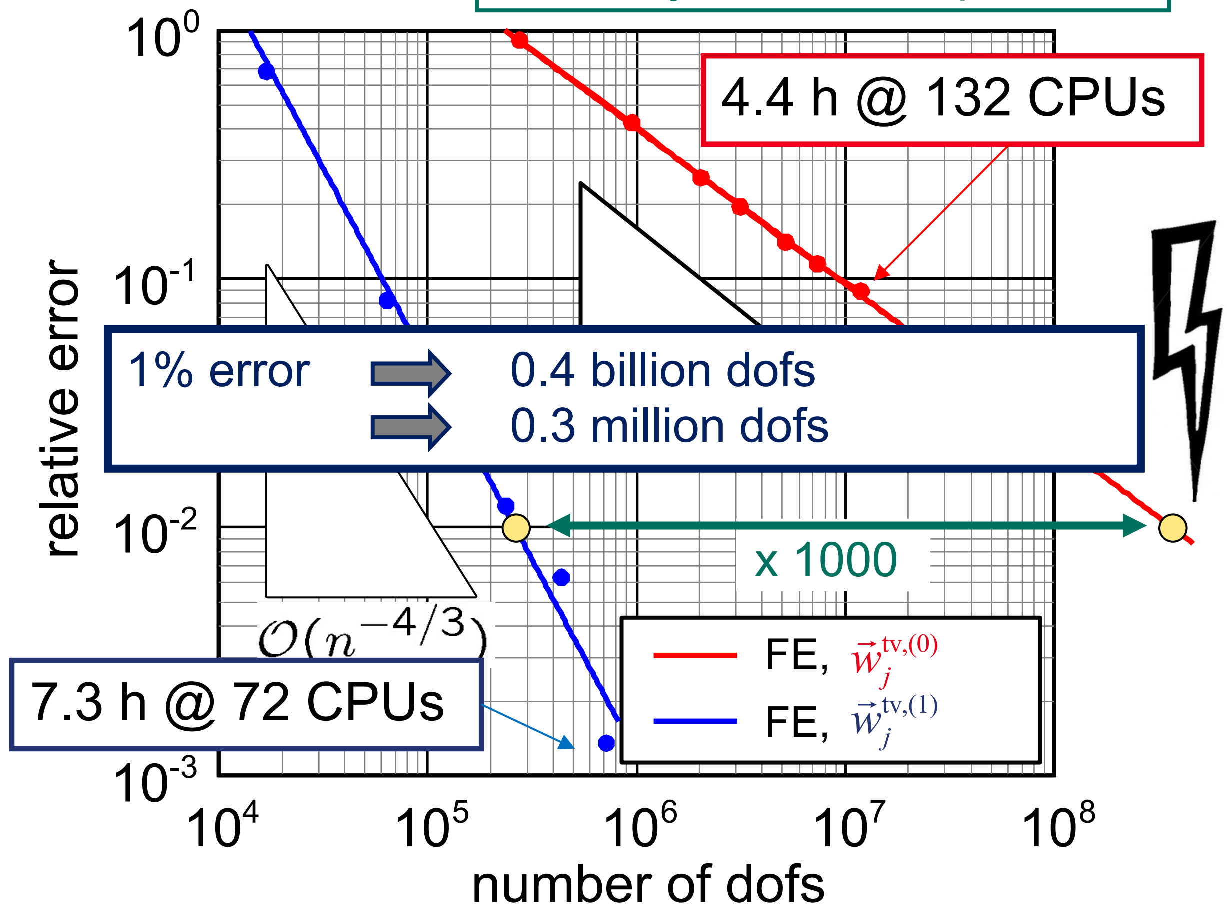
+ S. Koch, J. Trommler

TECHNISCHE UNIVERSITÄT DARMSTADT

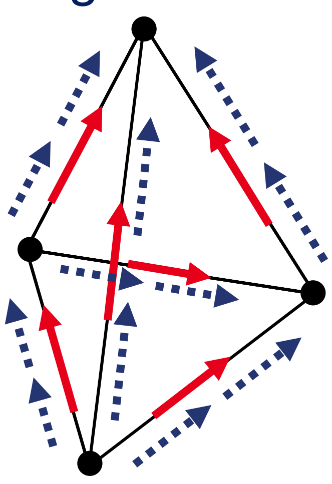
Example: FAIR SIS100 Magnet

solution of 110 x 4 = 440 linear systems of equations

relative error with respect to reference solution



degrees of freedom:



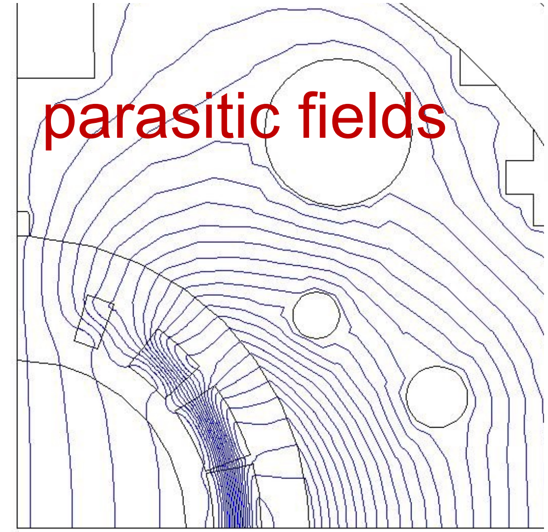
+ 2 per face

$\vec{w}_j^{\mathrm{tv},(0)}$

$\vec{w}_j^{\mathrm{tv},(1)}$

+ S. Koch, J. Trommler

Overview



- Finite-element simulation
- Losses in accelerator magnets
- Modelling and simulation challenges
- Homogenisation of coils
- Homogenisation of lamination stacks