Energy-Binning Fast Multipole Method for Electron Injector Simulations

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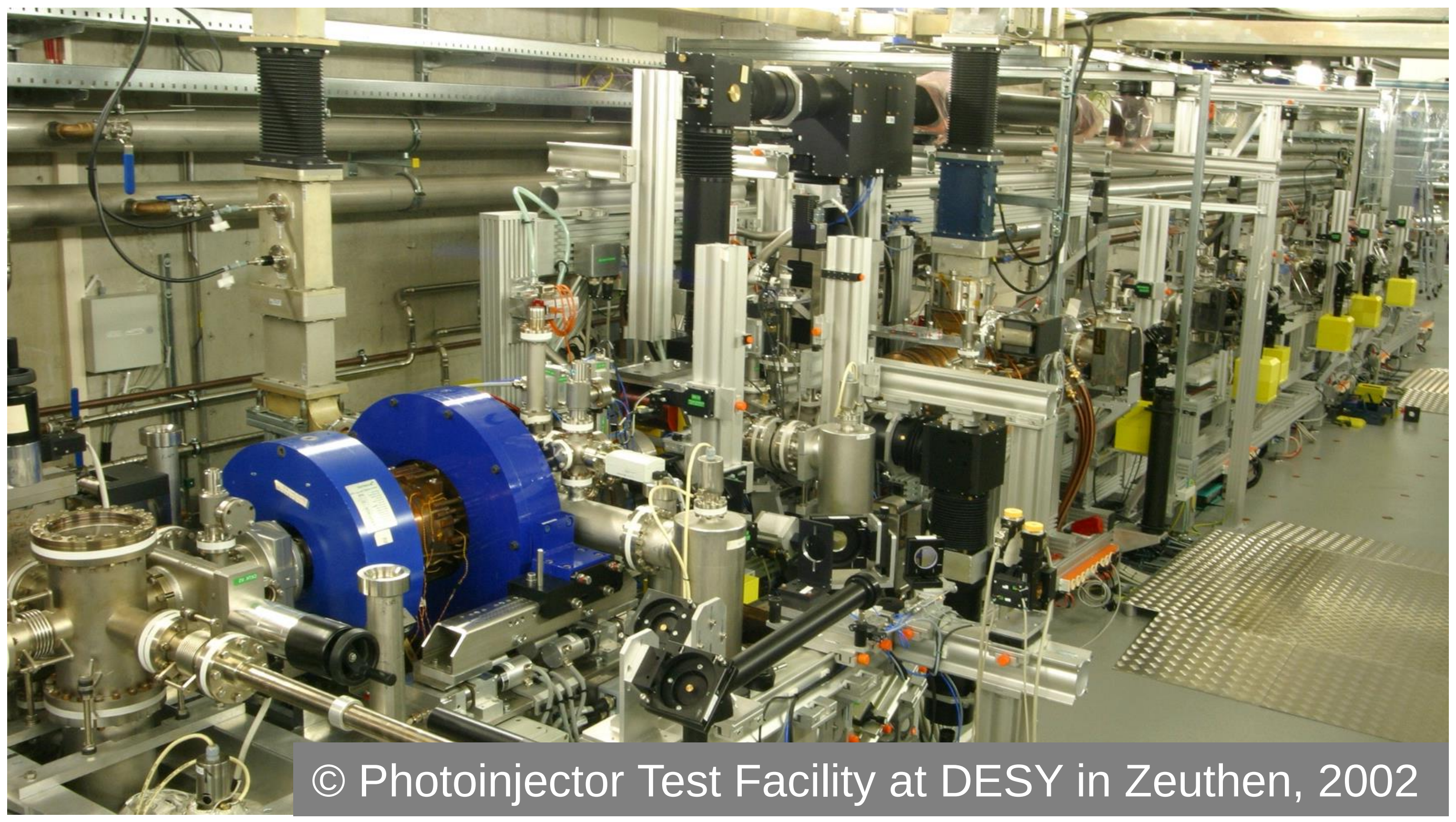
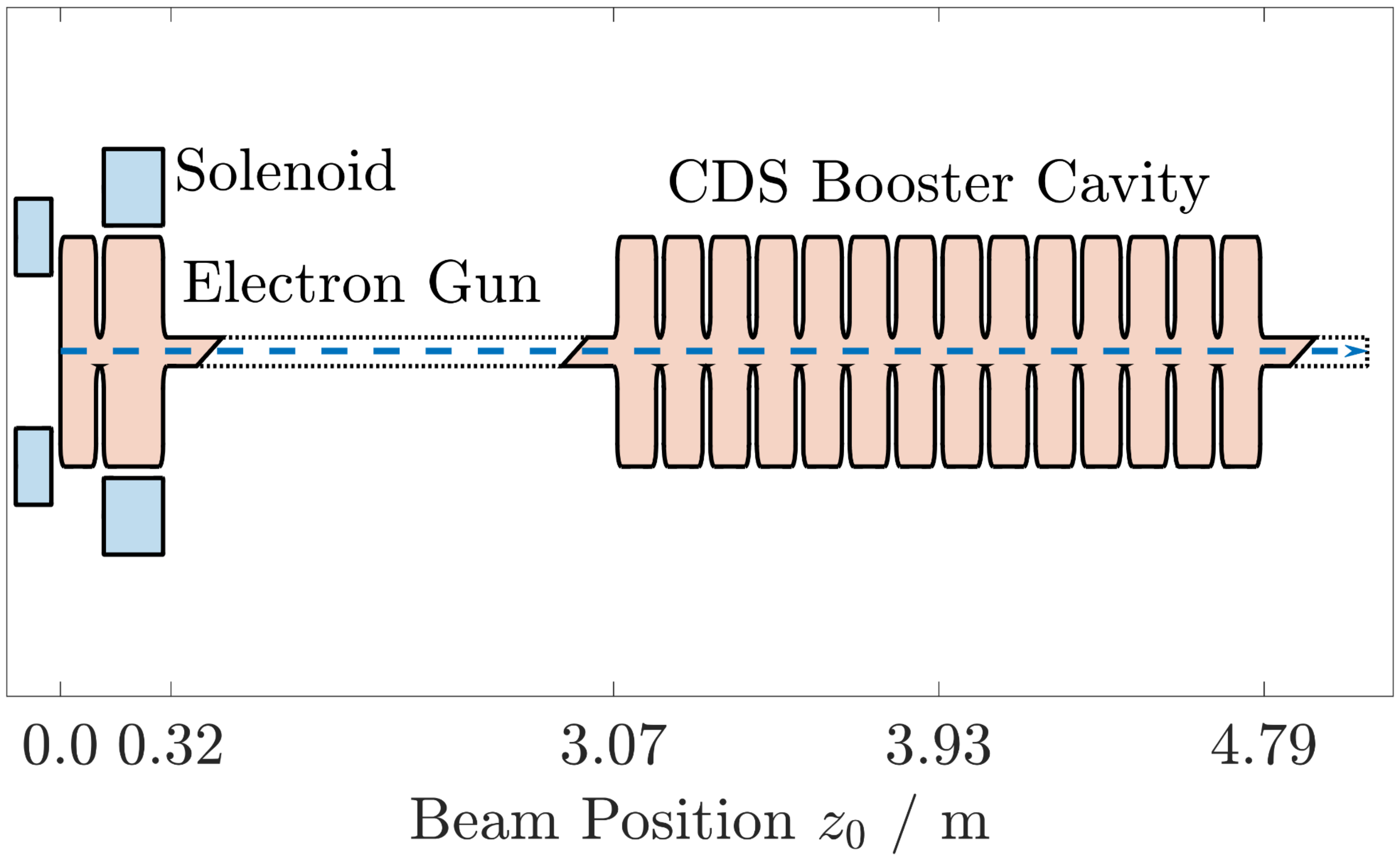
Modeling the DESY-PITZ Injector



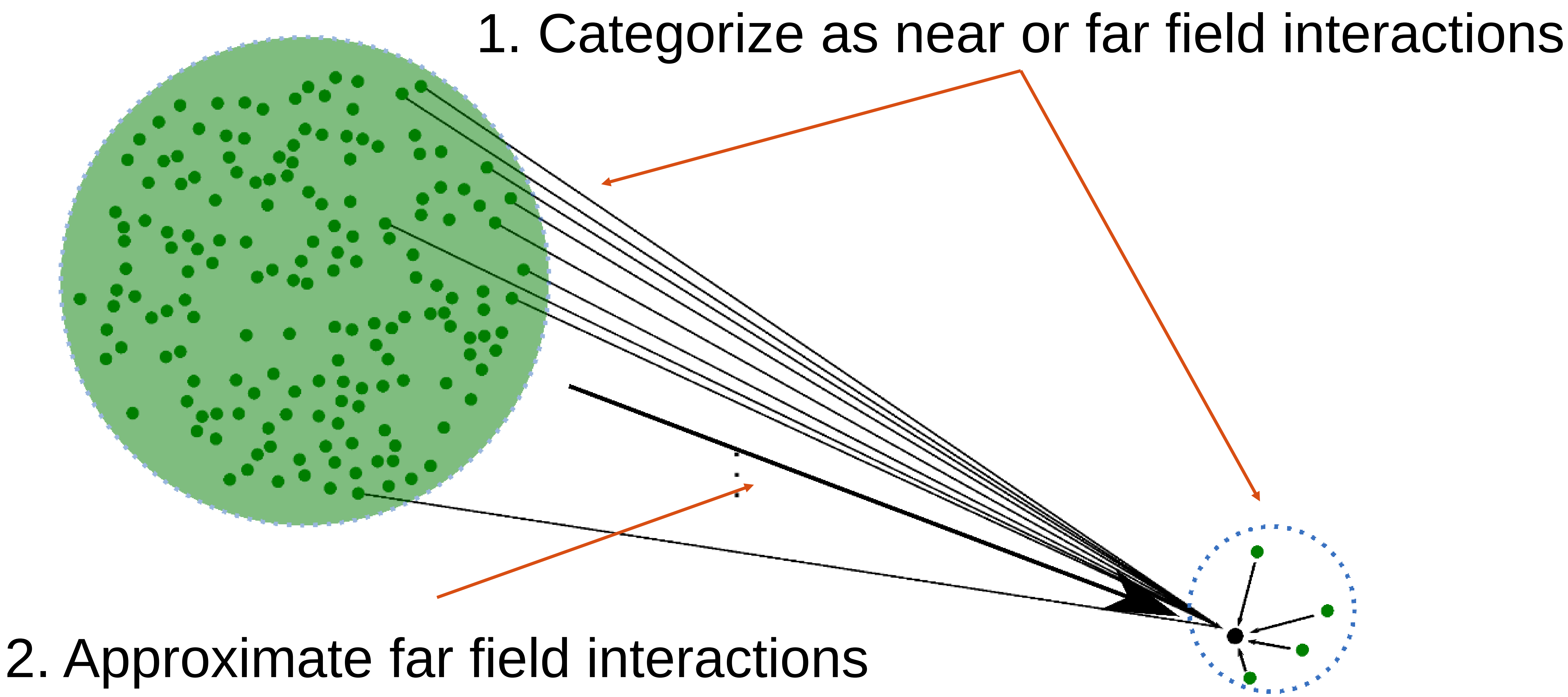


- Photoinjector: 1.6 cells, $\mathrm{E_G} = 60.58\,\mathrm{MV/m}$ field amplitude at $\mathrm{f_G} = 1.3\ \mathrm{GHz}$
- Electron beam: $Q_0 = 1\mathrm{nC}$ bunch charge, $E_0 = 17.1\ \mathrm{MeV}$ injection energy

⇒ **Requires space charge model for beam with energy dispersion**

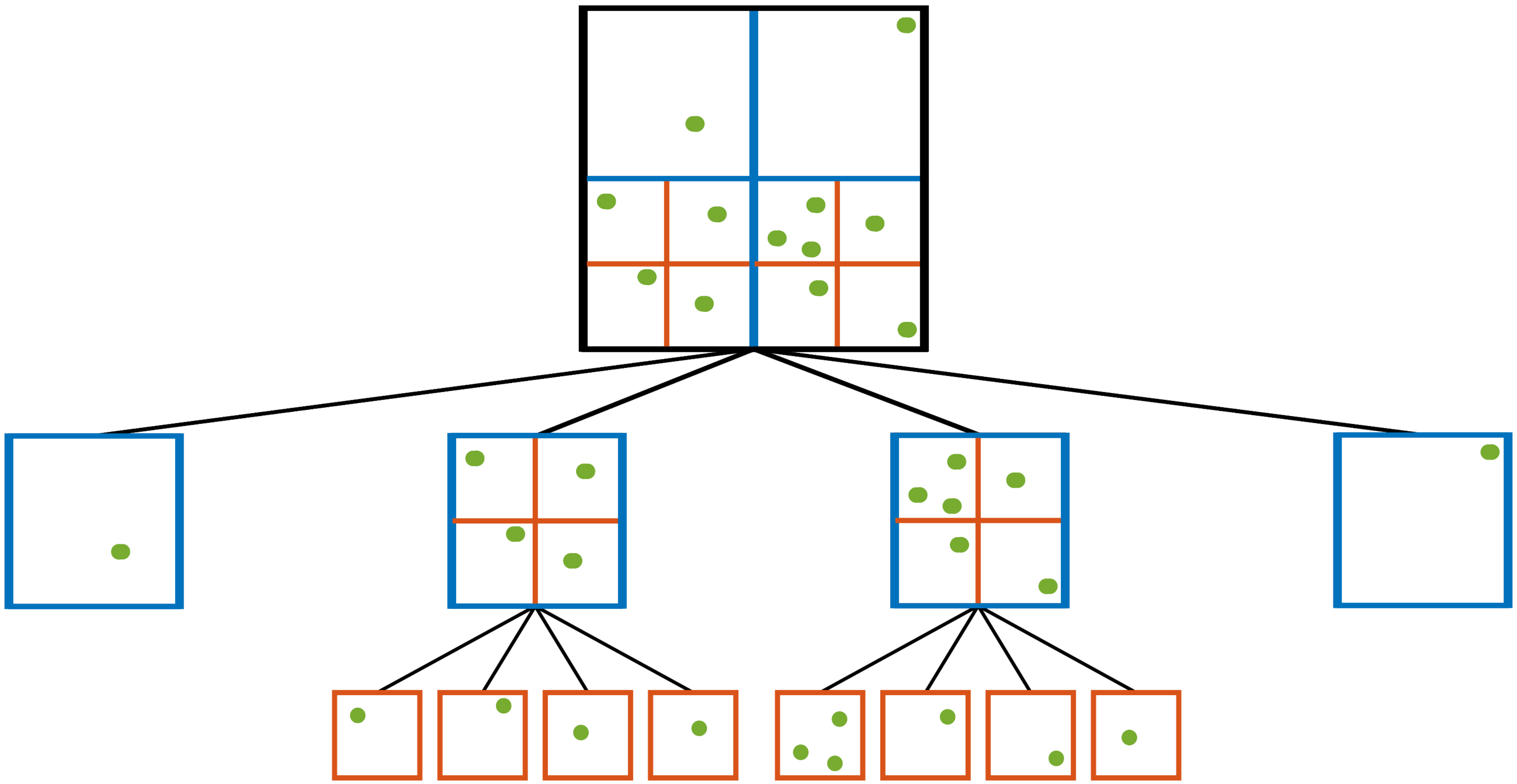
Reminder: Fast Multipole Method

- Particle-particle method (PPM) computationally inefficient
- Collective evaluation improves efficiency



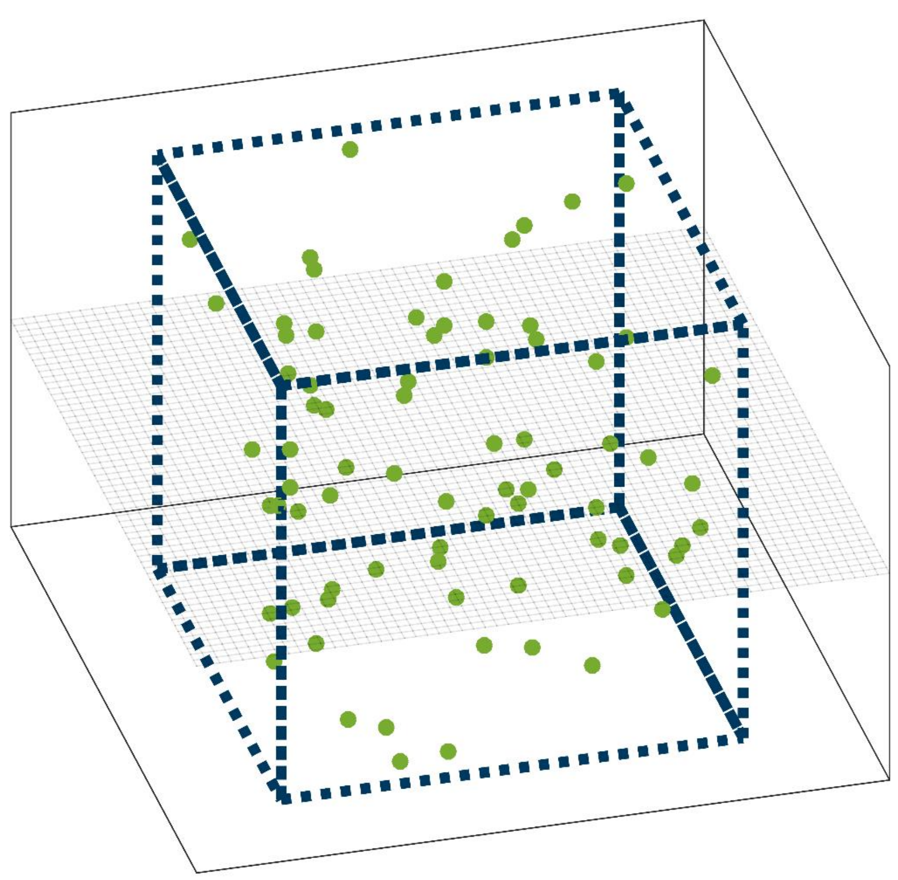
Reminder: Fast Multipole Method

- Categorization of interactions with tree structure
  - Domain decomposition into nodes based on particle density
  - Successive refinement until $N_n < N_0$ particles per leaf node

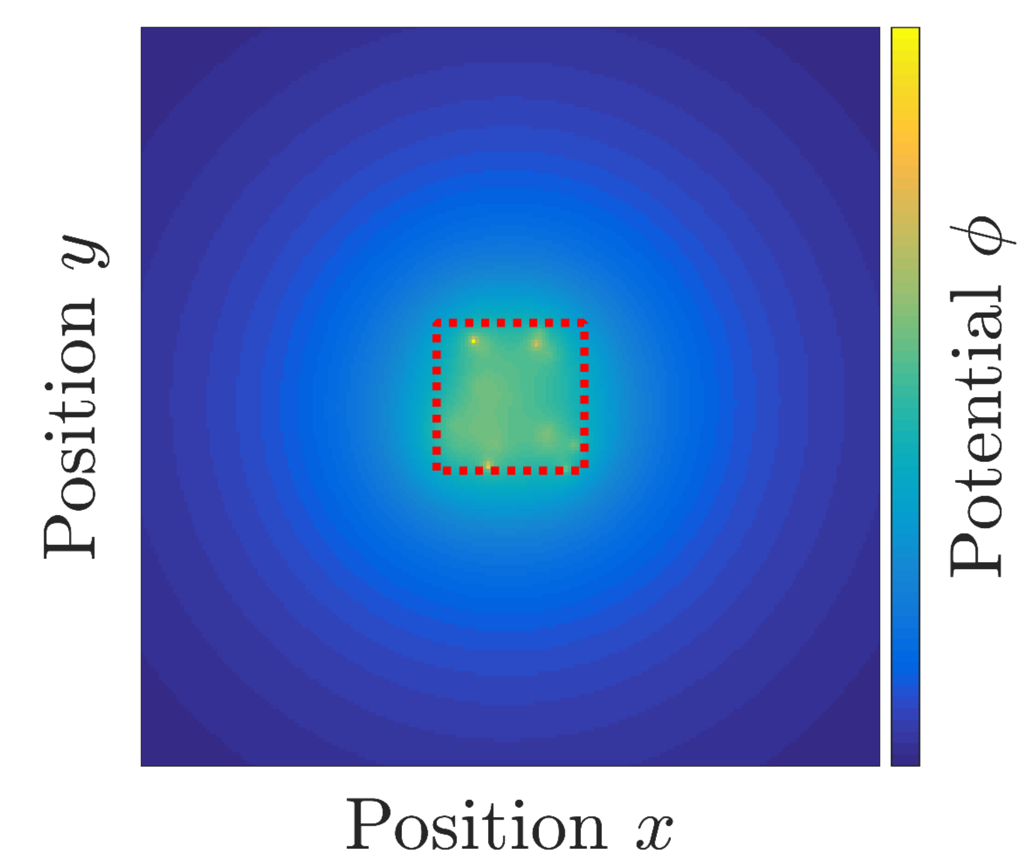


Reminder: Fast Multipole Method

- Far field approximation of node n with multipole expansion of order l_0
  - Spherical moments $M^n_{l,m} = \sum_j \frac{q_j}{4\pi\epsilon_0} F_{l,m}(\boldsymbol{x}_n - \boldsymbol{x}_j)$
  - Solid harmonics $F_{l,m}(\boldsymbol{x}) \propto r^l P_l^m(\cos\vartheta)\, e^{im\varphi}$



Maximum order $l_0 = 0$

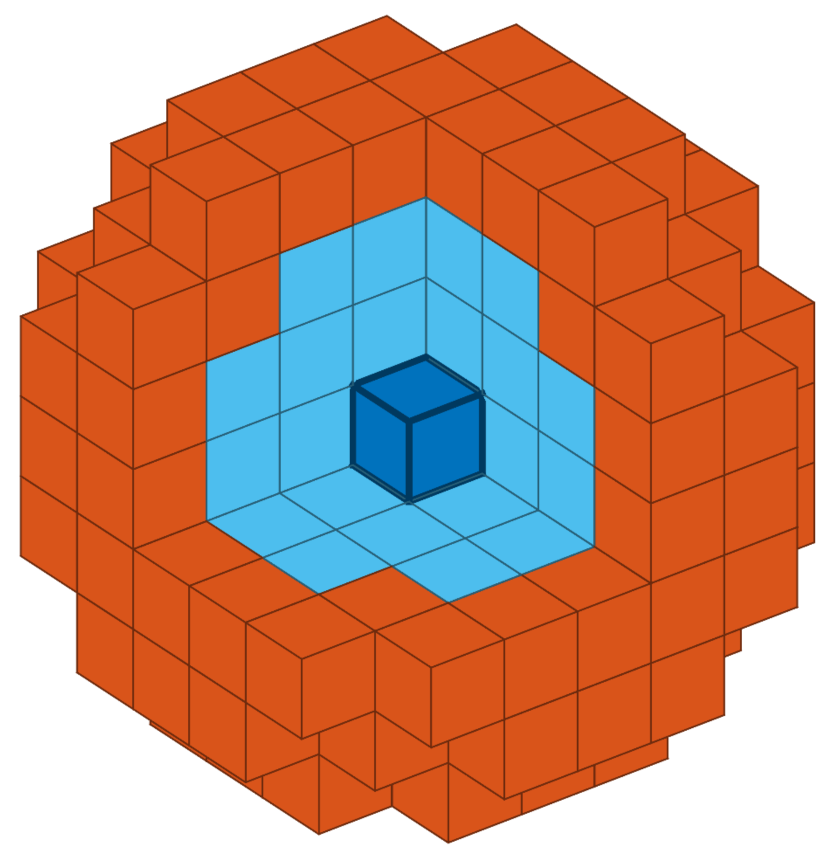


Particles in Tree Node

Potential of Tree Node
Deviation of Approximation

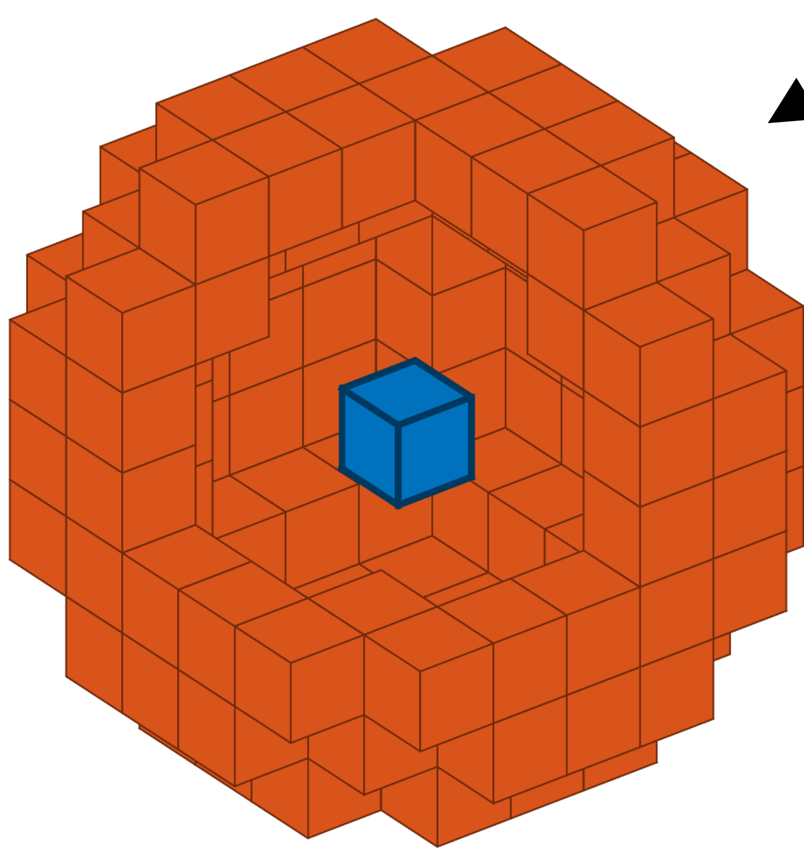
Reminder: Fast Multipole Method

- Check for each node n interactions with surrounding nodes n'



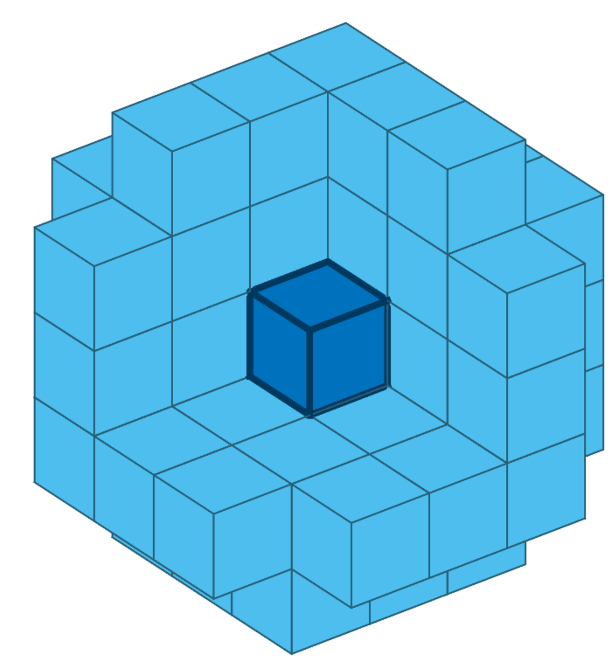
$\theta_{n,n'} \leq \theta_0$

$\theta_{n,n'} > \theta_0$



$$\theta_{n,n'} \equiv \frac{r_n + r_{n'}}{|\boldsymbol{x}_n - \boldsymbol{x}_{n'}|}$$

(node radius r_n and center $\boldsymbol{x}_n$)

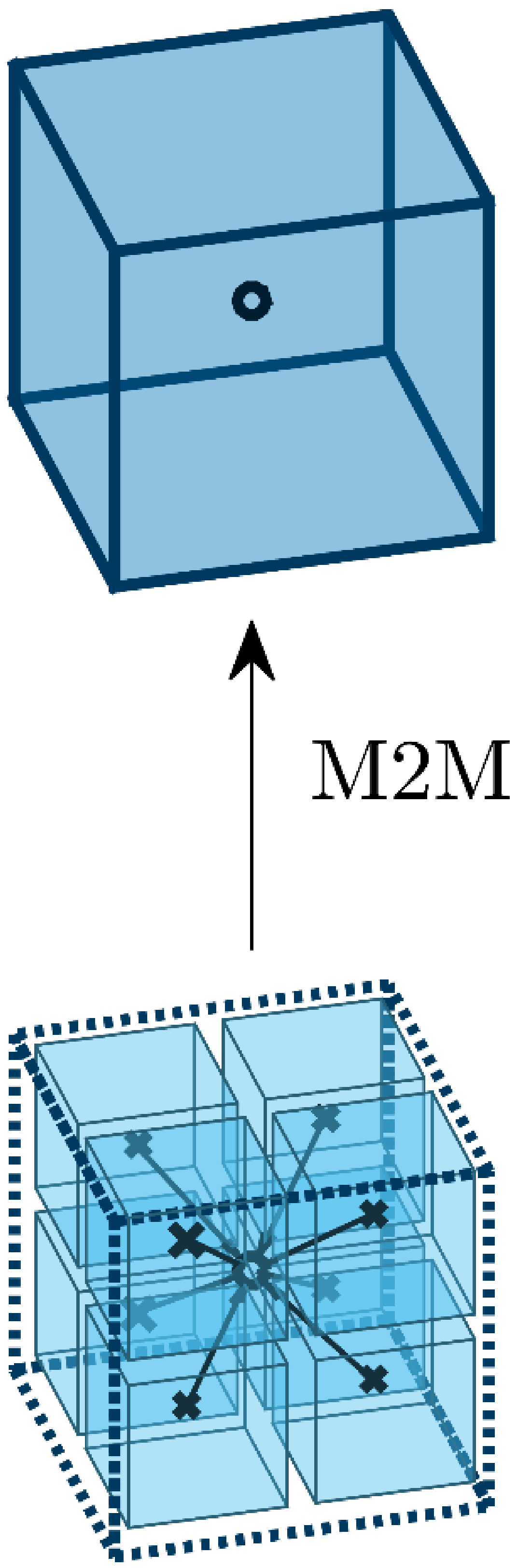


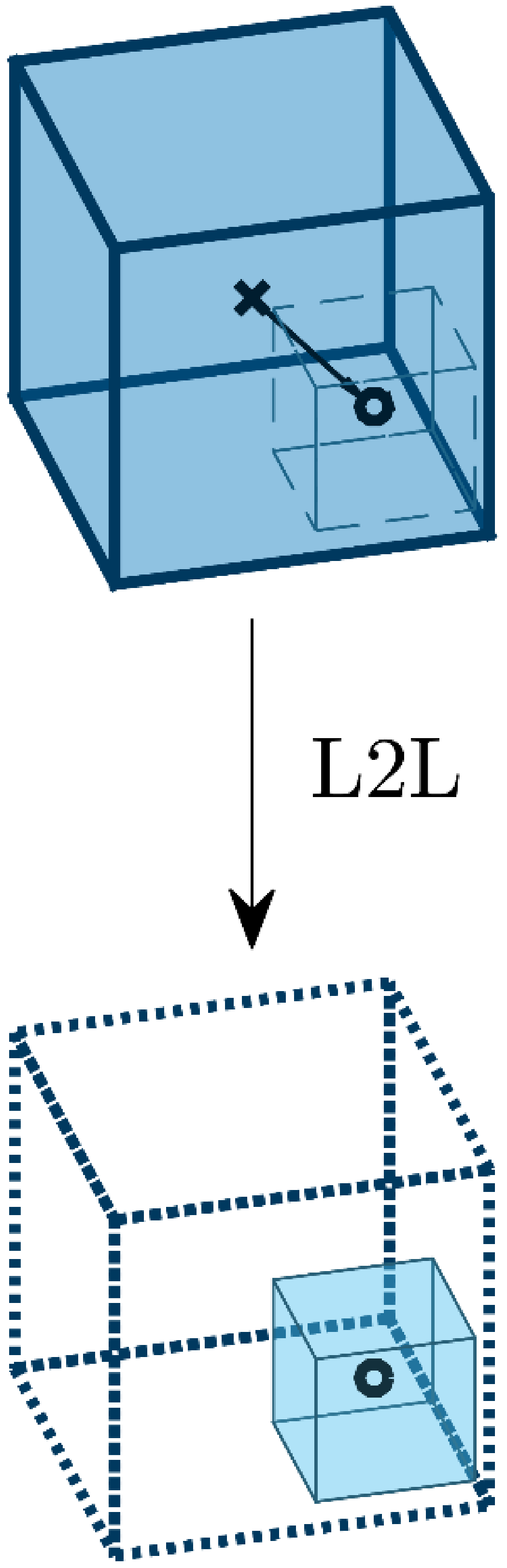
Multipole to Local (M2L)
Field Approximation

Particle to Particle (P2P)
or Level Refinement

Reminder: Fast Multipole Method

- Communication of multipole moments between tree levels

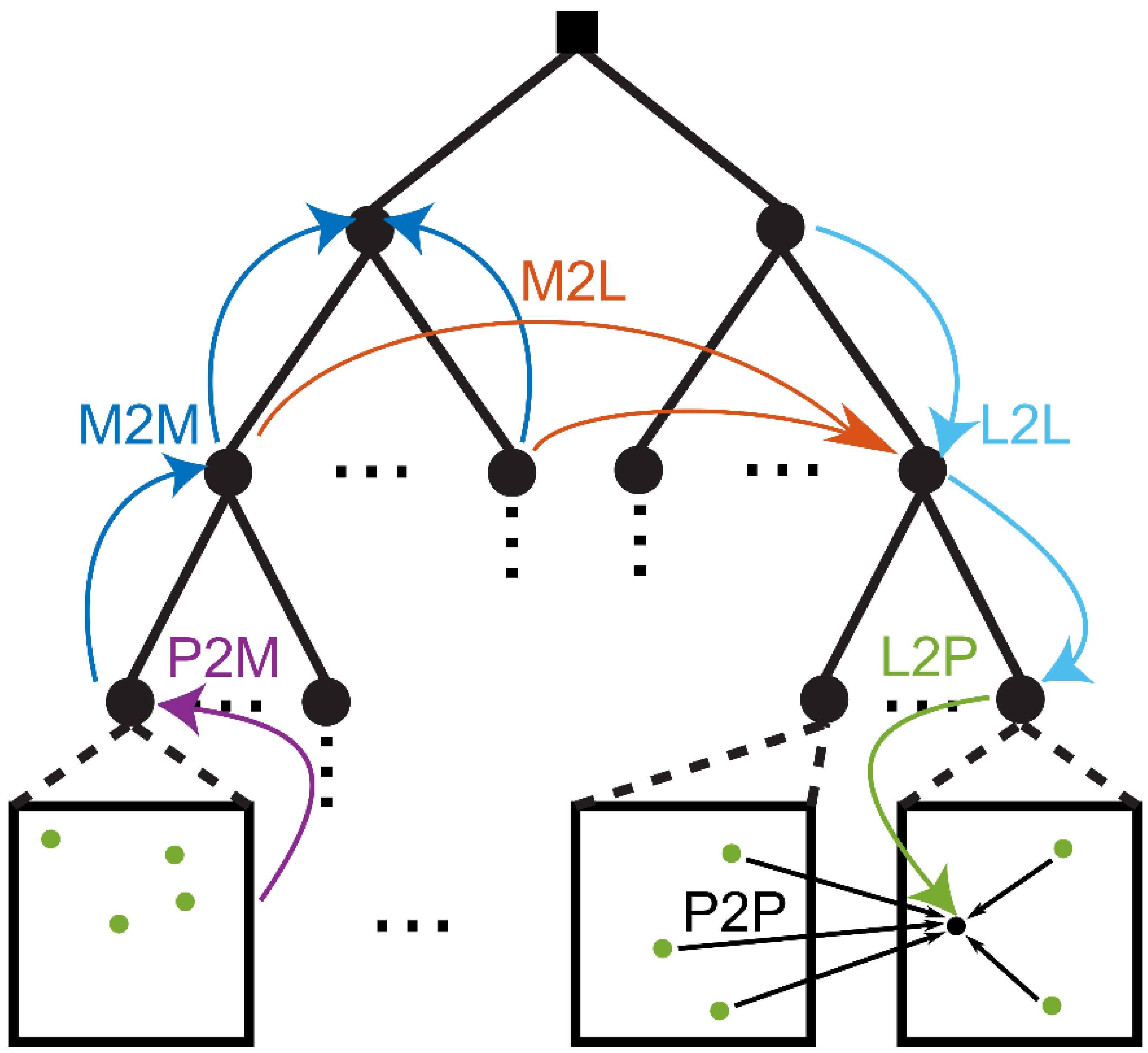


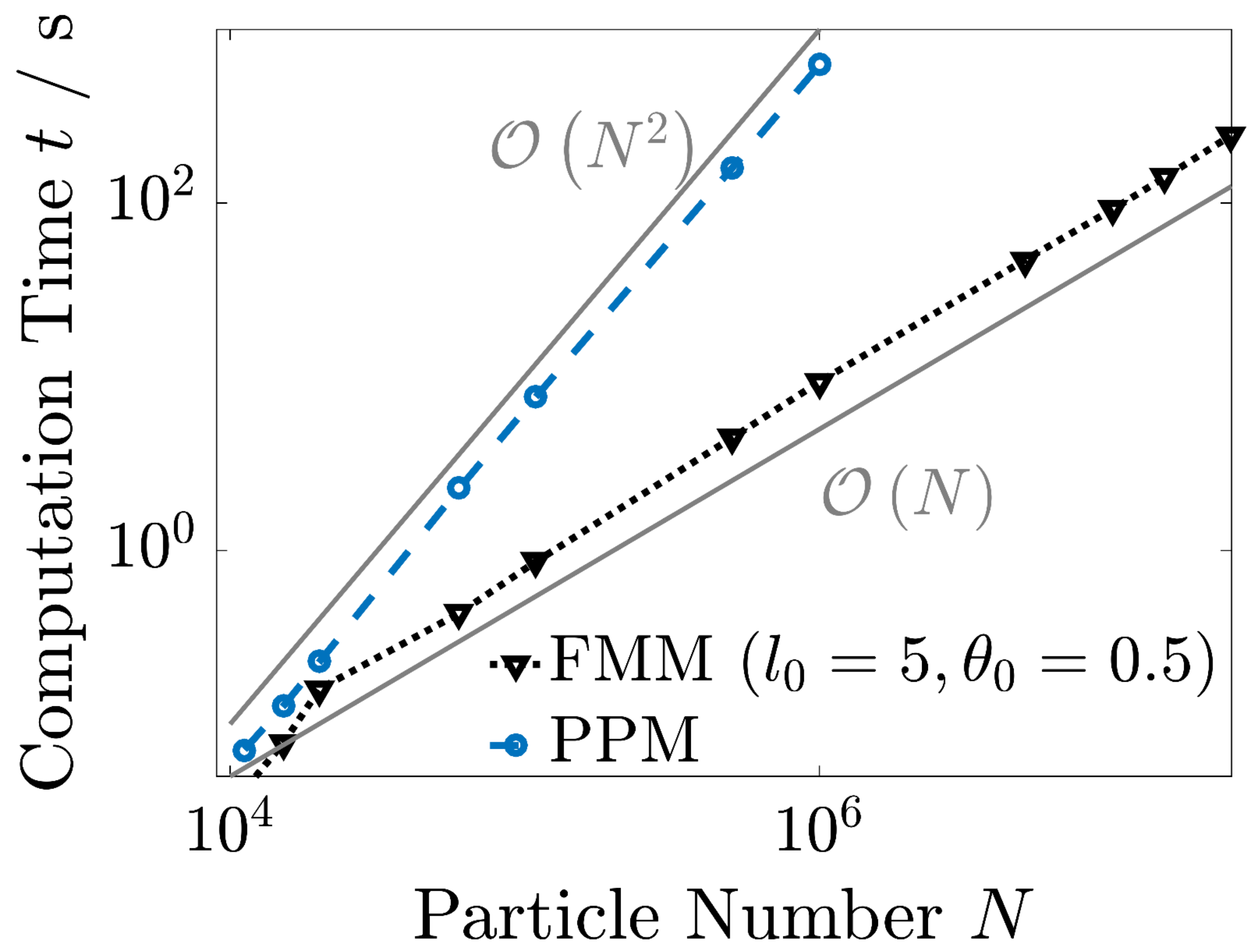


Towards Parent Nodes (M2M) **Towards Child Nodes (L2L)**

Reminder: Fast Multipole Method

- Runtime $\propto \underbrace{O(l_0^2\,N)}_{\text{P2M \& L2P}} + \underbrace{O(l_0^4\,N/N_0)}_{\text{M2M \& L2L}} + \underbrace{O(l_0^4\,N/N_0)}_{\text{M2L}} + \underbrace{O(N_0\,N)}_{\text{P2P}} \Rightarrow O(N)$





Runtime FMM Program

Space Charge Interaction Model

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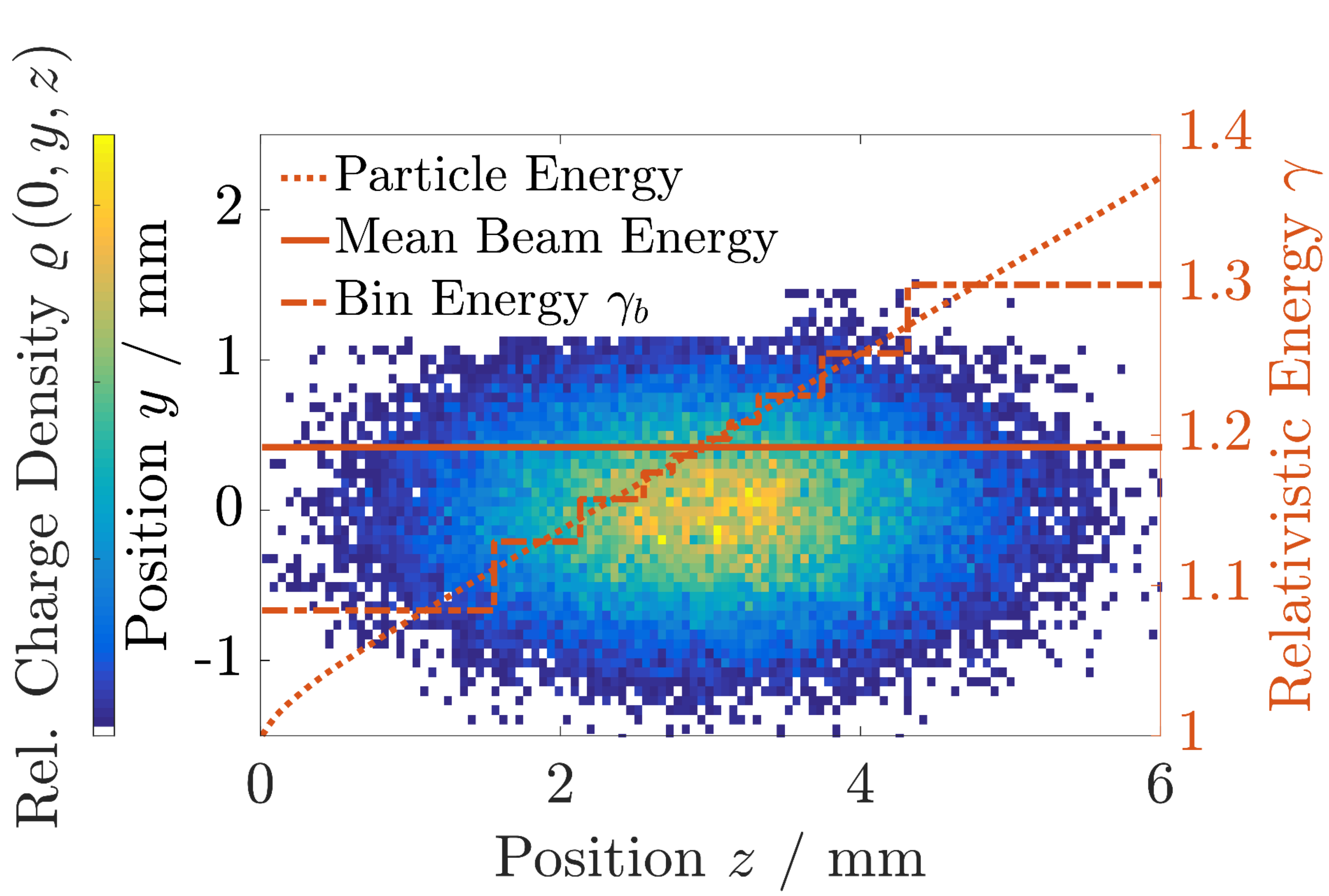
- Quasi electrostatic (QES) model for beams with $\boldsymbol{\beta}'(t') \approx \boldsymbol{\beta}_0$
  - Transformation to co-moving coordinates $(\mathrm{c}_0\tilde{t}, \tilde{\boldsymbol{x}})$ of relativistic beam

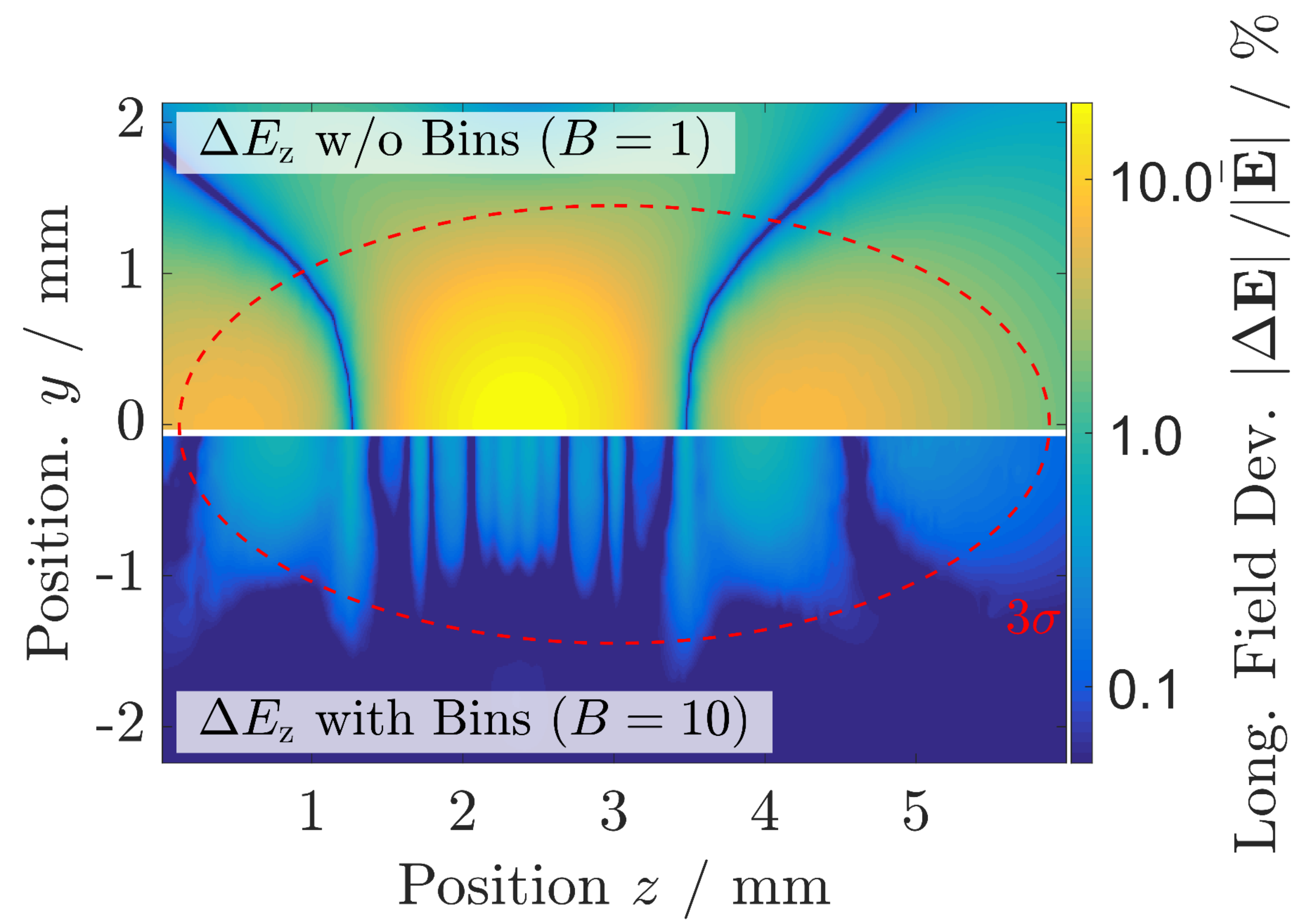
$\varrho(x, y, z)$ $\qquad$ $\tilde{\varrho}(\tilde{x}, \tilde{y}, \tilde{z}) = \varrho(x, y, \gamma_0\, z)/\gamma_0$

  - Transformation to laboratory coordinates $(\mathrm{c}_0 t, \boldsymbol{x})$

$$\boldsymbol{E}(\boldsymbol{x}) = \gamma_0\, \widetilde{\boldsymbol{E}}(\boldsymbol{x}) - \frac{\gamma_0^2\, [\boldsymbol{\beta}_0\, \widetilde{\boldsymbol{E}}(\boldsymbol{x})]}{\gamma_0 + 1}\, \boldsymbol{\beta}_0 \quad \text{and} \quad \boldsymbol{B}(\boldsymbol{x}) = \boldsymbol{\beta}_0 / c_0 \times \boldsymbol{E}(\boldsymbol{x})$$

Space Charge Interaction Model



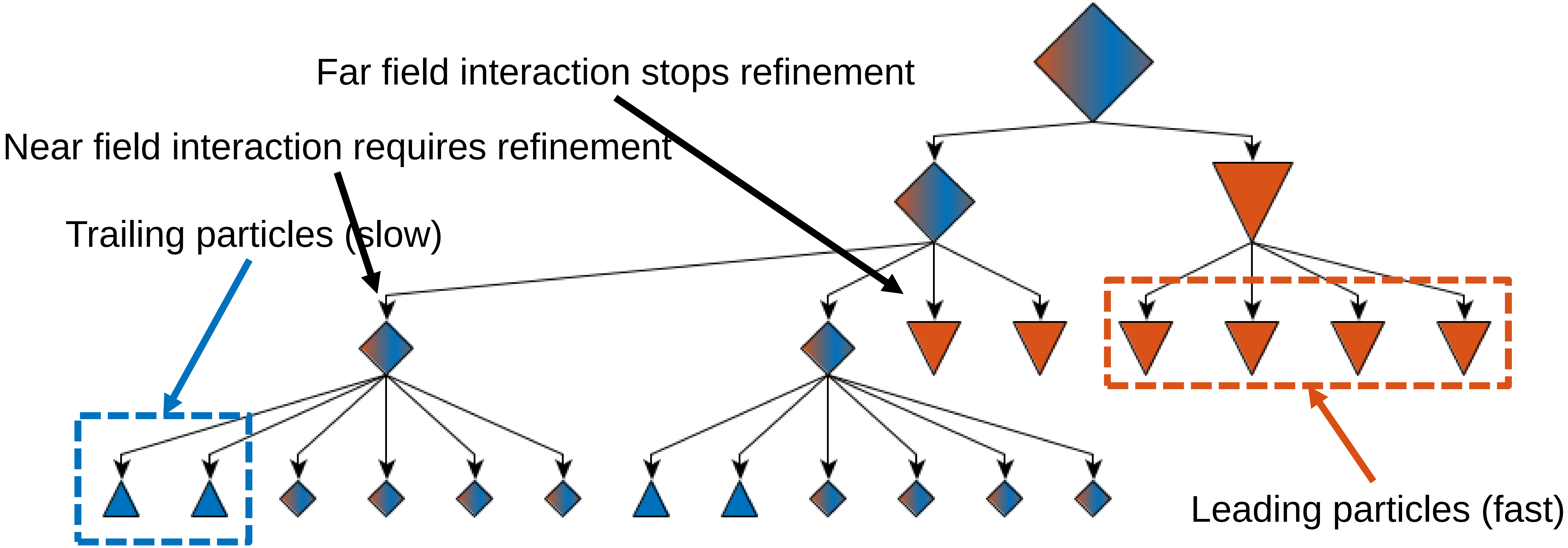


Electron Beam at Photocathode **Field Deviation at Photocathode**

- Systematic field error during beam generation
  - Single particle energy γ significantly different from mean energy γ_0
  - Quasi-electrostatic interaction model inappropriate if $\gamma \neq \gamma_0$
- Energy binning with $\gamma_b = \gamma_1, \ldots, \gamma_B$ compensates systematic error

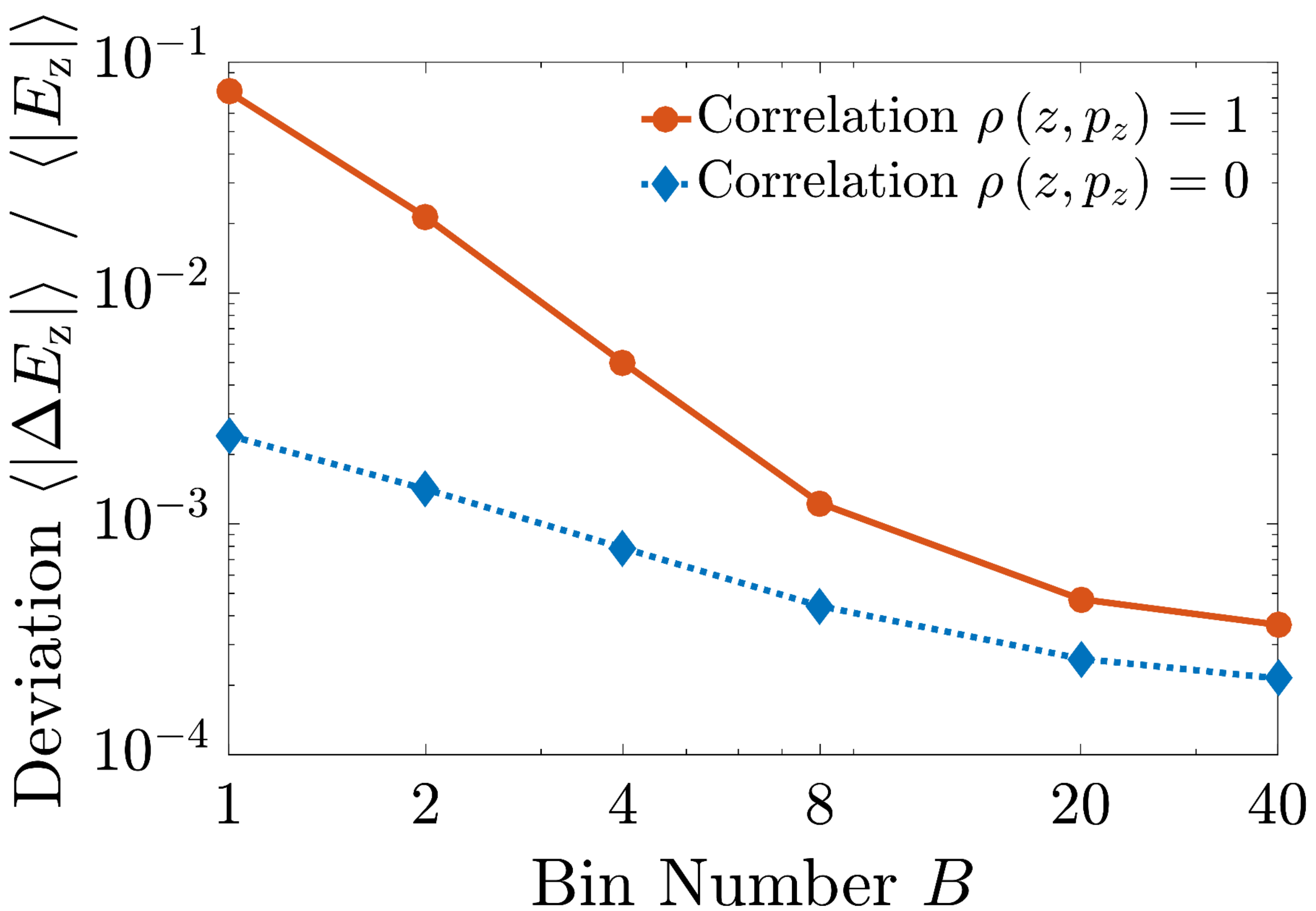
Energy-Binning Fast Multipole Method

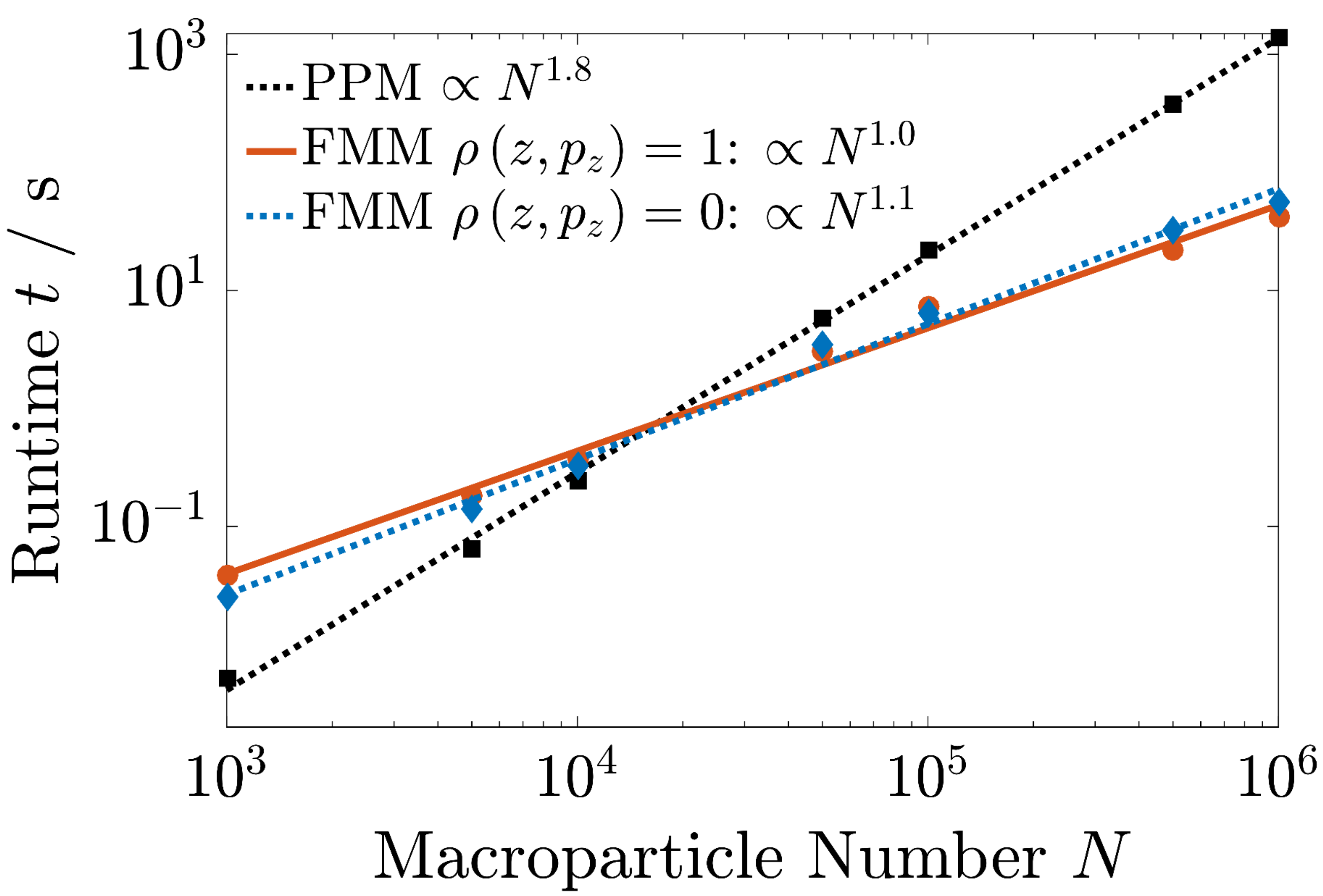
- Separate tree structure for each energy bin γ_b
  - Nodes for **source** $(\gamma \approx \gamma_1)$ and **target** $(\gamma_2 \leq \gamma \leq \gamma_\mathrm{B})$ particles
  - Modified refinement criterion $N_0' = N_0/\sqrt{B}$ reduces near field operations
  - Branch filtering reduces far field operations



Energy-Binning Fast Multipole Method

- Field approximation converges with bin number B
  - Larger effect for beam with phase space correlation $\rho(z, p_z) = 1$
  - Moderate number of energy bins $B \approx 6$ sufficient
- Energy-binning FMM conserves $O(N)$ complexity





Energy-Binning Fast Multipole Method

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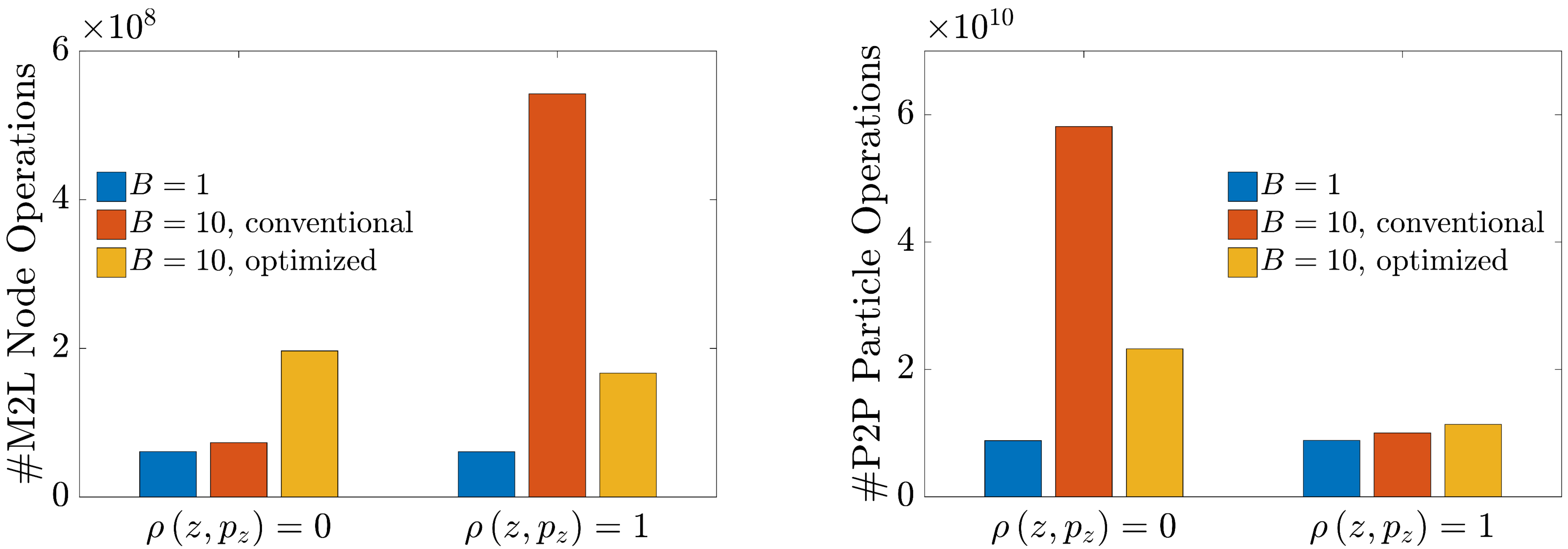
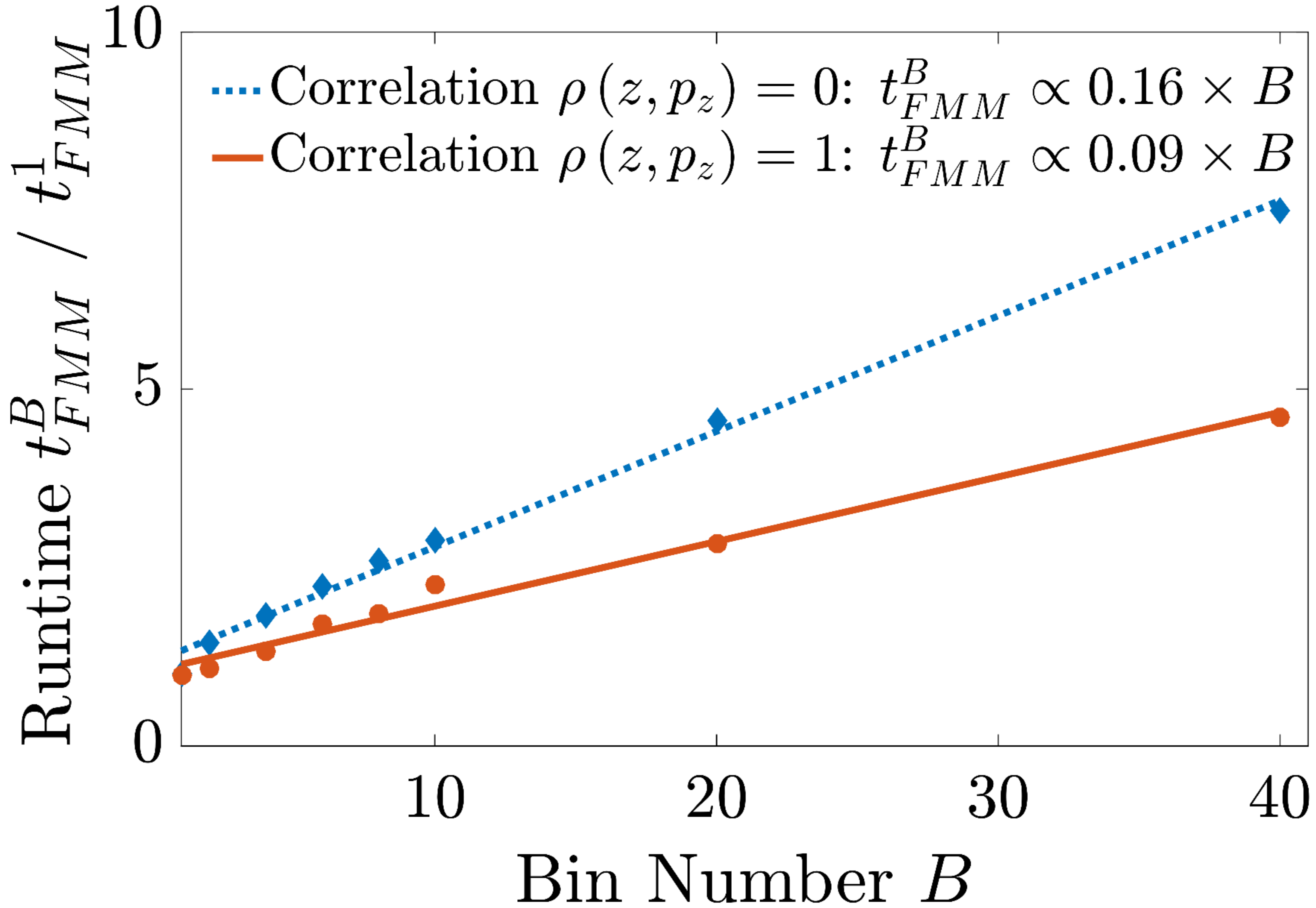
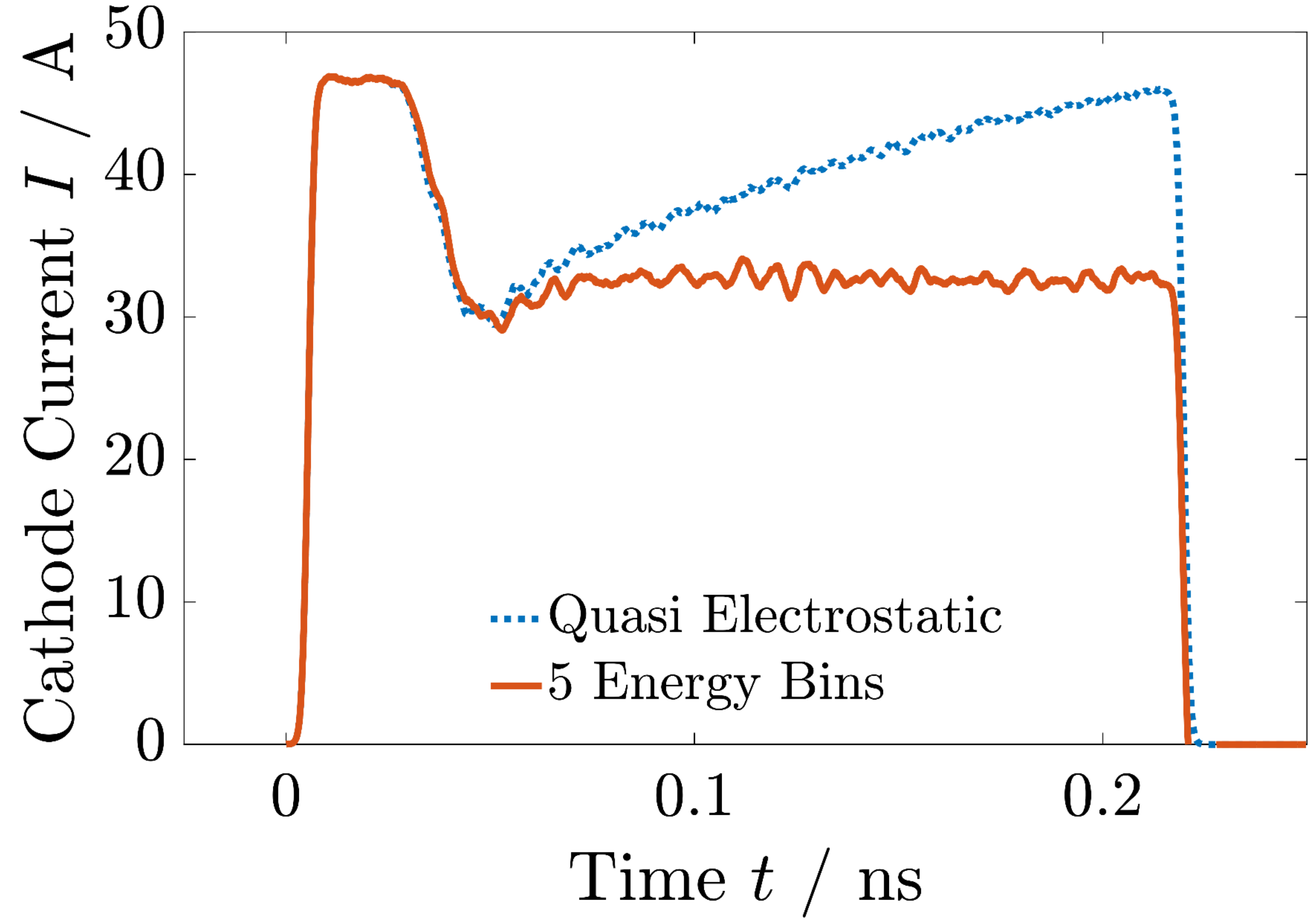
- Optimized tree structure improves numerical efficiency
  - Large $\#M2L$ for $\rho(z, p_z) = 1$ and conventional tree
  - Large $\#P2P$ for $\rho(z, p_z) = 0$ and conventional tree
  - Optimized tree structure balances $\#M2L$ and $\#P2P$
  - Energy-binning FMM particularly suitable for injector simulation ($\rho \approx 1$)



Energy-Binning Fast Multipole Method



Energy Binning FMM

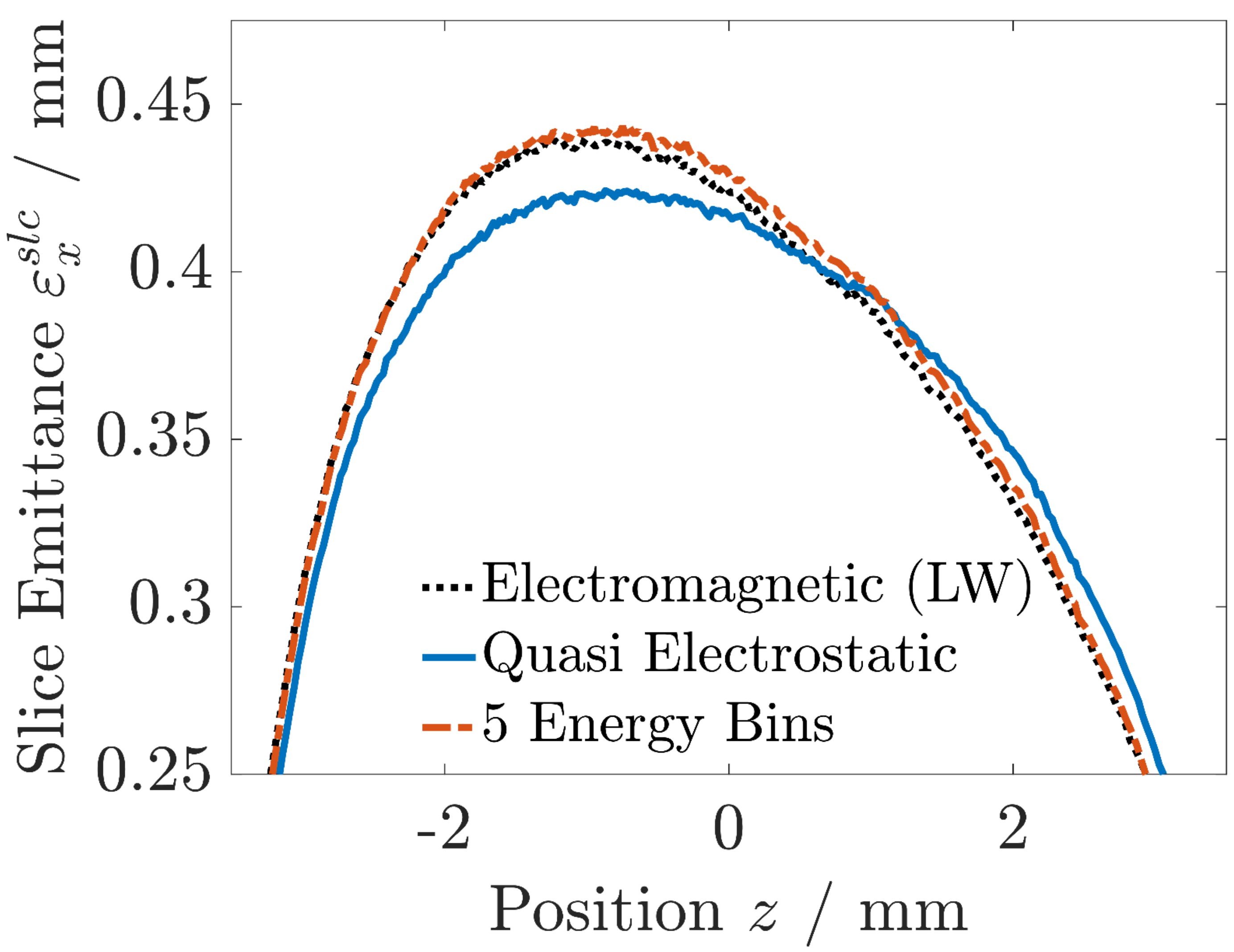


Space Charge Limited DC Injector

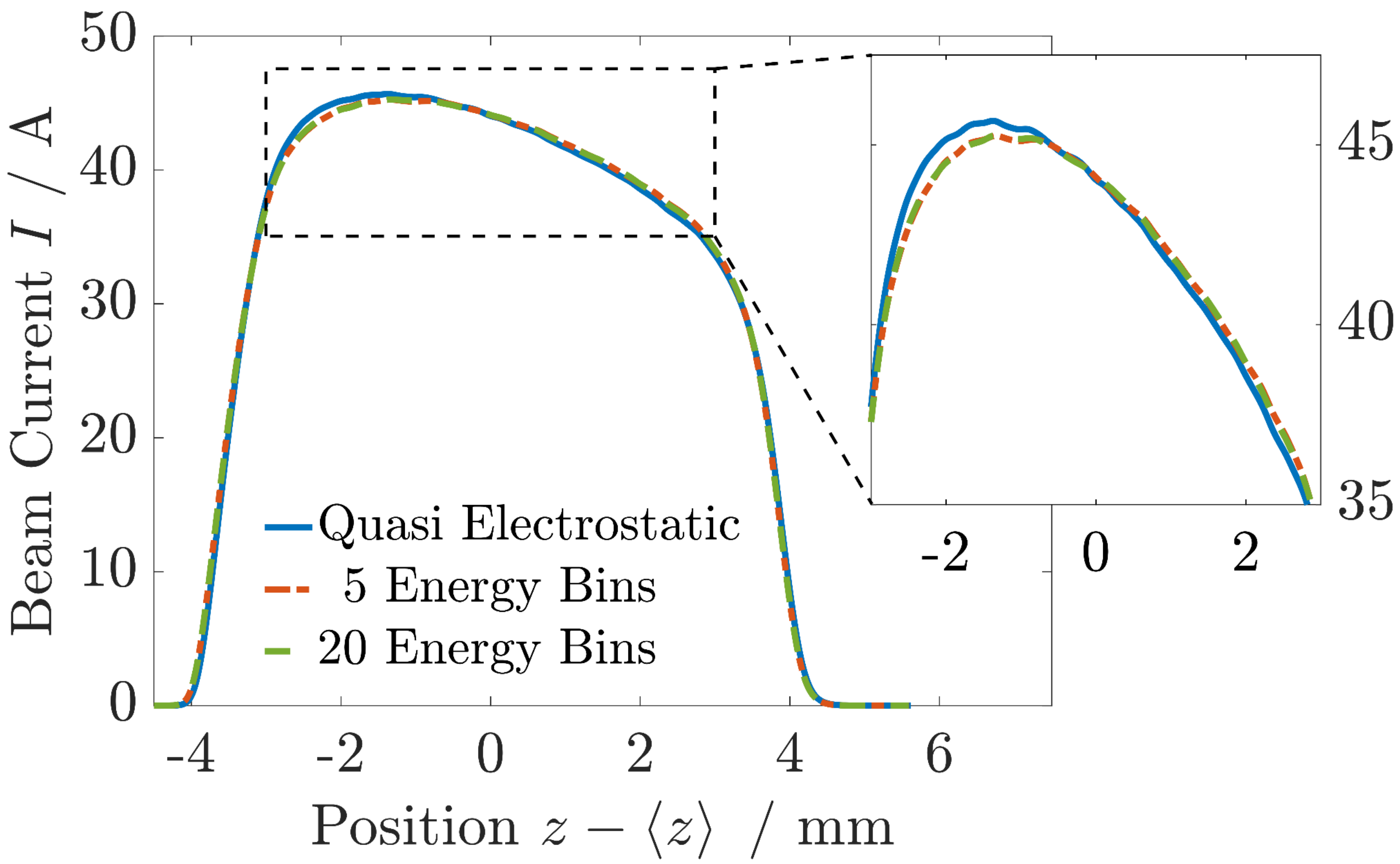
- Numerical properties
  - Runtime $t^B_{\mathrm{FMM}} \propto k\,B$ and optimized tree facilitates $k < 1$
  - Spatial decoupling of trailing and leading particles reduces runtime

- Small number of energy bins sufficient to compensate systematic error (e.g. $B = 5$ suppresses artificial cathode current growth in DC injector)

Modeling the DESY-PITZ Injector



Validation Study



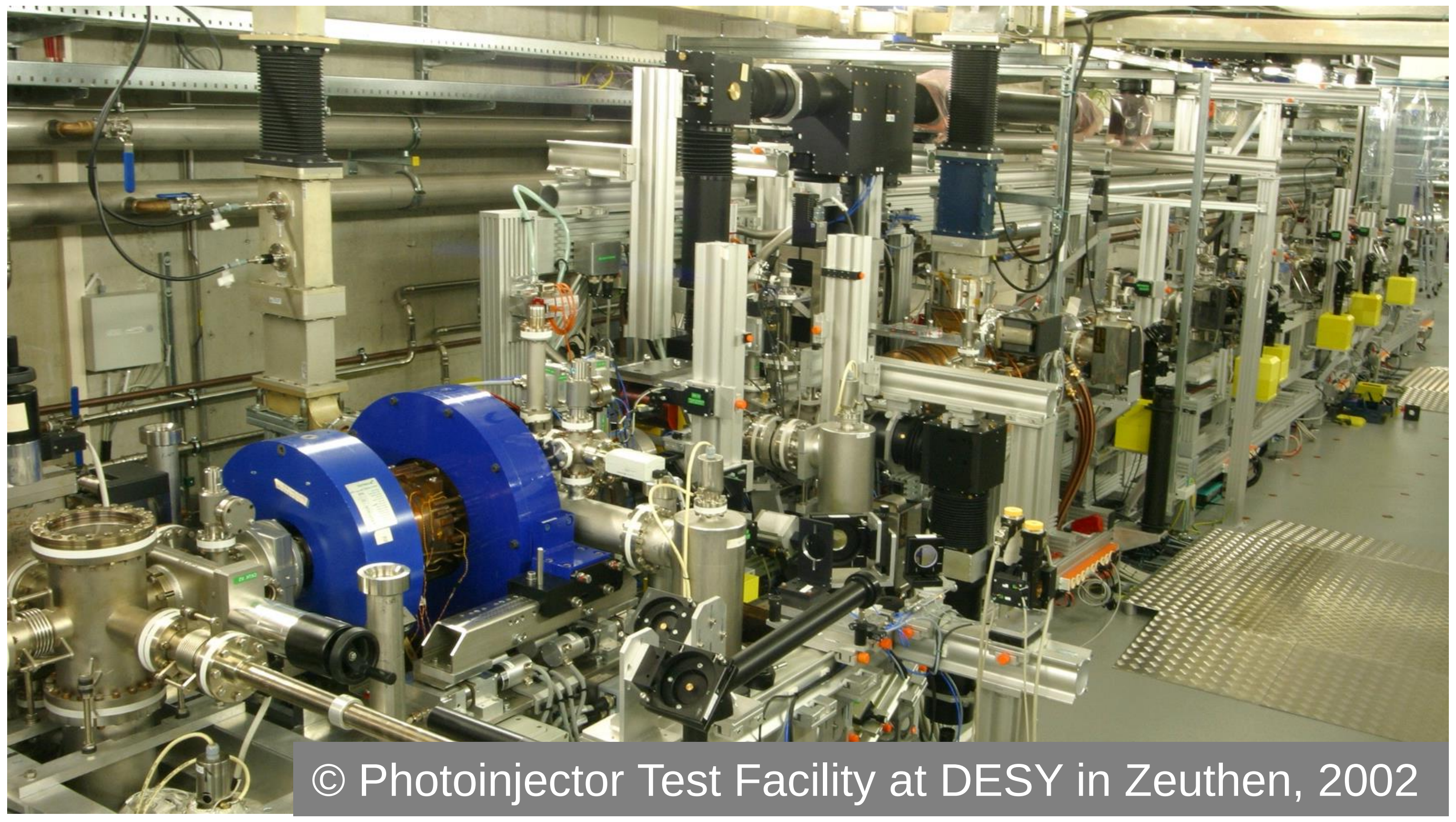
Application for Injector Modeling

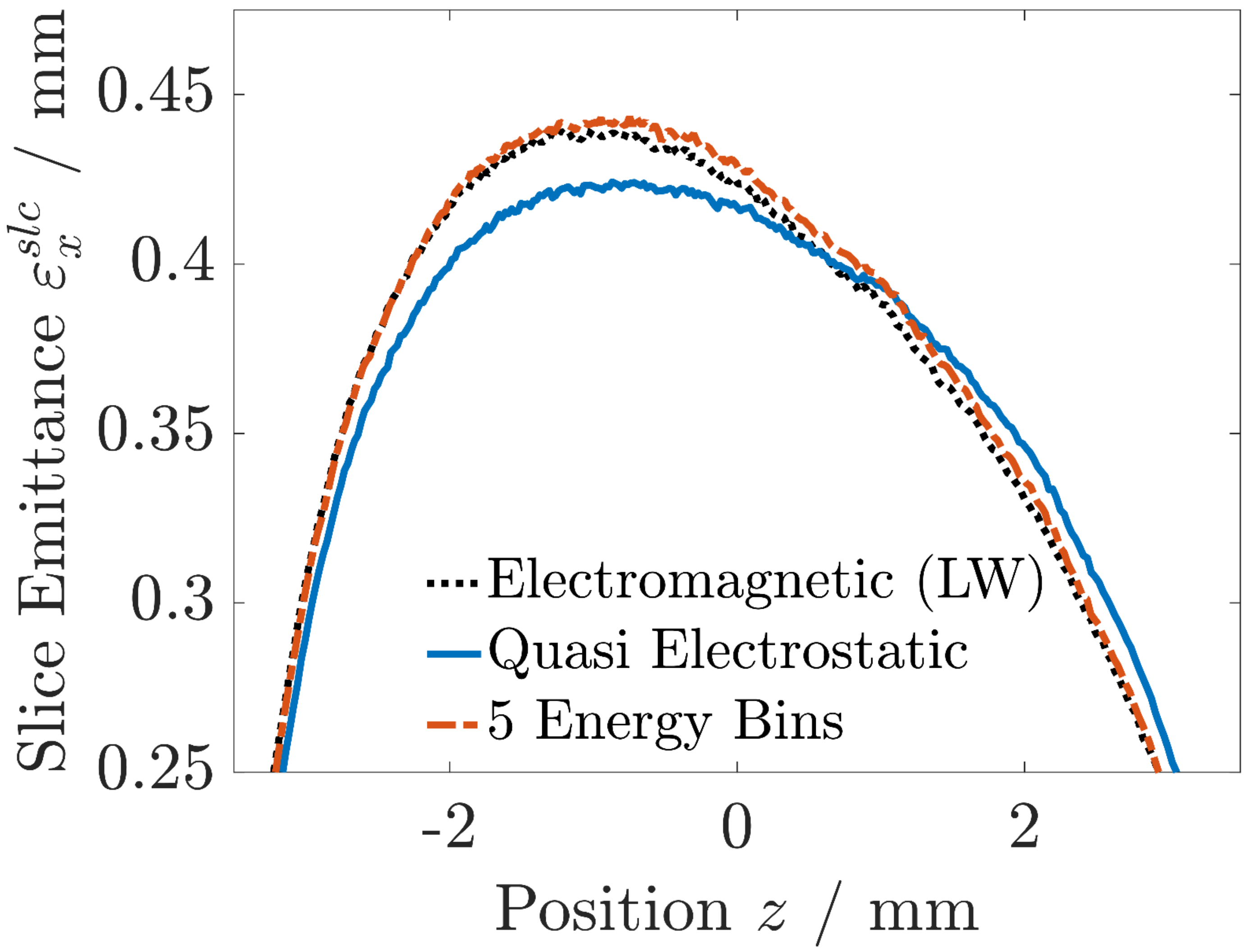
- Validation of energy binning FMM simulation
  - $B = 5$ energy bins reproduce results of electromagnetic LW model
  - Space charge interaction increases slice emittance at rear end of beam
- Numerically efficient FMM applicable for large N simulation studies

S. A. Schmid, et al., „Energy-Binning […] Injector Simulations", submitted in Proceedings of the 12th IPAC (2021)

Conclusions

- Energy-Binning FMM for Beam Dynamics Simulation





- Meshfree modeling for large macroparticle ensembles $N \sim 10^7$
- Energy-binning improves QES space charge interaction model
- QES model underestimates interactions between the trailing particles
- Energy-binning FMM with $B = 5$ approximates Liénard-Wiechert results

References

1. L. Greengard and V. Rokhlin, “A Fast Algorithm for Particle Simulations",
   in J. Comp. Phys. **73**.2 (1987)
2. G. Fubiani, et al., “Space Charge Modeling of Dense Electron Beams with Large Energy Spreads“,
   in PRAB **9**.6 (2006)