CC, Continuous Charge Approach

Martin Dohlus, 21st September 2021
for DESY-TEMF-CANDLE meeting

CSR?

When we started to design BC chicanes for FELs in the 90s we recognized that the models

magnet optics,
geometrical and resistive wakes and
“space charge” (SC)

were not sufficient to describe the beam dynamics accurately enough.

As FEL processes were under discussion, the term

“coherent synchrotron radiation” (CSR)

was close at hand, and it was now commonly used for additional electromagnetic effects in BC chicanes elsewhere in beamlines.

The term got an additional scope. Worse, this term is now used for two things:

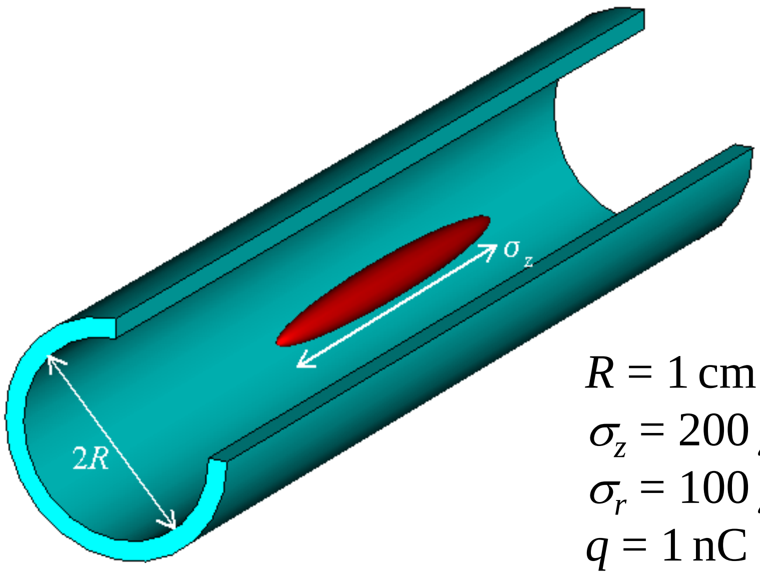
* in the original sense for coherent synchrotron radiation,
* for (non-specific) EM effects in beamlines which are not "wakes" or "SC".

My goal is not to calculate CSR, but to calculate beam dynamics more precisely. To avoid misunderstandings I will try to use the term CC for continuous charge and the resulting fields. The two Cs may also stand for continuous and coherent.

One effect that is definitely not explained by CSR is compressions work.

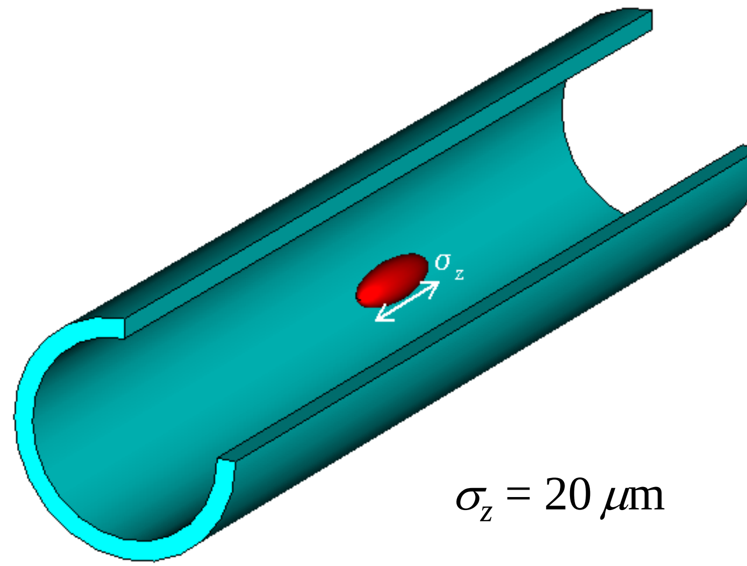
Compression Work

Gedankenexperiment: “if we could smoothly compress a bunch”
f.i. weak linear velocity chirp, shielding by small PEC pipe



$R = 1\,\mathrm{cm}$
$\sigma_z = 200\,\mu\mathrm{m}$
$\sigma_r = 100\,\mu\mathrm{m}$
$q = 1\,\mathrm{nC}$

$\Rightarrow$



$\sigma_z = 20\,\mu\mathrm{m}$

field energy = $W_{\mathrm{tot}} = 0.107\,\mathrm{mJ}$ $\Delta W_{\mathrm{tot}} = 0.958\,\mathrm{mJ}$ $W_{\mathrm{tot}} = 1.065\,\mathrm{mJ}$

CSR in the last arc of Zeuthen benchmark BC (steady state assumption):
$R_0 = 10\,\mathrm{m},\ \sigma_z = 20\,\mu\mathrm{m},\ L = 0.5\,\mathrm{m} \rightarrow P = 375\,\mathrm{kW},\ P\,L/c_0 = 0.625\,\mathrm{mJ}$

field energy + **kinetic energy** =const

$\rightarrow$ particles are decelerated/accelerated during compression/anti-compression
this effect is reversible, the effects is not small (for benchmark case)

SC?

in adiabatic compression, the effect seems to be a pure “SC” effect

compression in BCs is not adiabatic, build up time ~ $\gamma^2\sigma/2$ >> chicane dimensions

doubts about tracking programs tracking programs using Poisson approaches: even adiabatic processes can be calculatet incorrectly; compare: “Two Poisson Approaches” DESY-TEMF, Aug. 2018

Radiation and Space Charge Forces

PHYSICAL REVIEW ACCELERATORS AND BEAMS **24,** 020701 (2021)

Calculation of the wake due to radiation and space charge forces in relativistic beams

Gennady Stupakov and Jingyi Tang

SLAC National Accelerator Laboratory, Menlo Park 94025, California, USA

2D approach in accelerator coordinates

I would like to follow this approach and make it more generally usable.

Other Approaches

Methods with Lienard-Wiechert Solutions

it is a special case of retarded source solutions; ret. solutions naturally satisfy free space boundary conditions

not directly applicable to continuous distributions

problems with near fields (f.i. GPT)

problems with ultra-fast time dependency for point particles $\sim R/(c\gamma^3)$ at high energy

Wake-Like Approaches

it is a 2d- or 3d-generalization of the well known 1d CSR approach:

$$W_{x/y/s}(x,y,s,S) = \int K_{x/y/s}(x-x',y-y',s-s',S)\frac{\partial\lambda(x',y',s',S)}{\partial s'}dx'dy'ds'$$

core and source distribution are S dependent, but no retardation effects
↓
effects due to rapid change of bunch shape are not considered

Reference Methods ???

2D Approach in Accelerator Coordinates

2D approach

$$\rho(X,Y,Z,t) \to \rho(X,Y,t)\delta(Z)$$

$$\mathbf{v}\cdot\mathbf{e}_z = 0$$

cartesian coordinates and accelerator coordinates

$$\begin{pmatrix} X & Y & t \end{pmatrix} \leftrightarrow \begin{pmatrix} x & s & S \end{pmatrix}$$ time & space coordinates

$$p_x \quad p_y \qquad \eta = \frac{E - E_\mathrm{r}}{E_\mathrm{r}} \qquad x' = \frac{dx}{dS}$$ dynamic coordinates

trajectory of reference particle

$$\begin{bmatrix} X_r \\ Y_r \end{bmatrix} = \begin{bmatrix} X_\mathrm{r}(S) \\ Y_\mathrm{r}(S) \end{bmatrix}$$

$$t_r = \frac{S}{v_\mathrm{r}}$$

trajectory of arbitrary particle

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X_\mathrm{r}(S) \\ Y_\mathrm{r}(S) \end{bmatrix} + \begin{bmatrix} -Y_\mathrm{r}'(S) \\ X_\mathrm{r}'(S) \end{bmatrix} x$$

$$t = \frac{S - s}{v_\mathrm{r}}$$

phase space coordinates of particles $\mathbf{X} = \begin{bmatrix} x & x' & s & \eta \end{bmatrix}^t$ (long.) bunch coordinate

$\mathbf{X} = \mathbf{X}(S)$ beamline coordinate

continuous phase space distribution $f\left(\left[x, x', s, \eta\right]', S\right)$

linear optics without self effects $f\left(\mathbf{X}_S, S\right) = f_0\left(\mathbf{T}_{0\leftarrow S}\mathbf{X}_S\right)$

this is the basic assumption !!!
(for the beginning I want to estimate self-effects by perturbation theory)

initial gaussian distribution $f_0\left(\mathbf{X}\right) \sim \exp\left(-\frac{1}{2}\left(\mathbf{X}-\mathbf{X}_\mathrm{i}\right)^t \mathbf{C}_\mathrm{i}^{-1}\left(\mathbf{X}-\mathbf{X}_\mathrm{i}\right)\right)$

with the initial offset $\mathbf{X}_\mathrm{i}$ and the initial correlation matrix $\mathbf{C}_\mathrm{i}$

static magnetic field $B_\mathrm{z}(X,Y) \rightarrow$ reference trajectory $\begin{bmatrix} X_r \\ Y_r \end{bmatrix} = \begin{bmatrix} X_\mathrm{r}\left(S\right) \\ Y_\mathrm{r}\left(S\right) \end{bmatrix}$

transport matrix $\mathbf{T}_{S\leftarrow 0}$ $\mathbf{T}_{0\leftarrow S} = \mathbf{T}_{S\leftarrow 0}^{-1}$

to calculate EM fields we need the charge- and current density in space and time

$$\left.\begin{matrix} \rho\left(X,Y,t\right) \\ \mathbf{J}\left(X,Y,t\right) \end{matrix}\right\} \leftarrow f\left(\left[x,x',s,\eta\right]', S\right)$$

this step involves a 2d integration ($dx'd\eta$)
and a coordinate transformation ($X,Y,t \leftrightarrow x,s,S$)

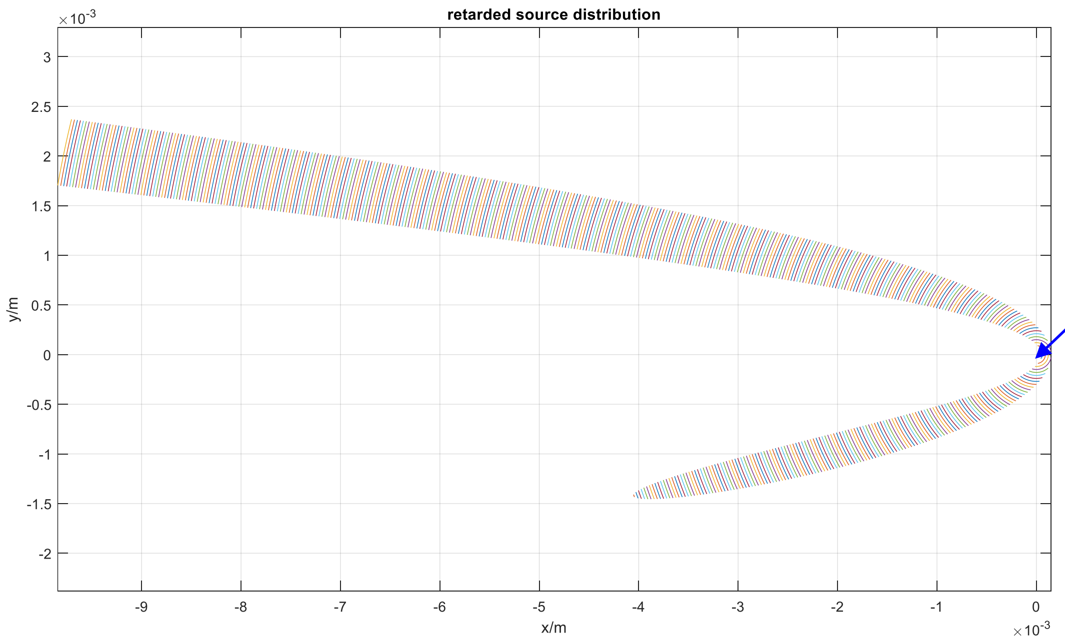
Retarded Potentials

$$V(X,Y,t)=\frac{1}{4\pi\varepsilon}\int\frac{\rho(X',Y',t')}{\sqrt{(X-X')^2+(Y-Y')^2}}dX'dY'=\frac{1}{4\pi\varepsilon}\int\frac{\rho(X-X',Y-Y',t')}{R=\sqrt{X'^2+Y'^2}}dX'dY'$$

$$=\frac{1}{4\pi\varepsilon}\int\rho(X+R\cos\varphi,Y+R\sin\varphi,t-R/c)\,dR\,d\varphi$$

direct integration in polar coordinates

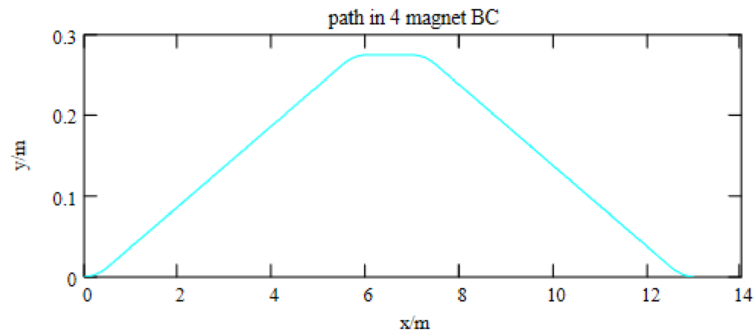
distribution of retarded sources for a particular observer (X,Y,t)



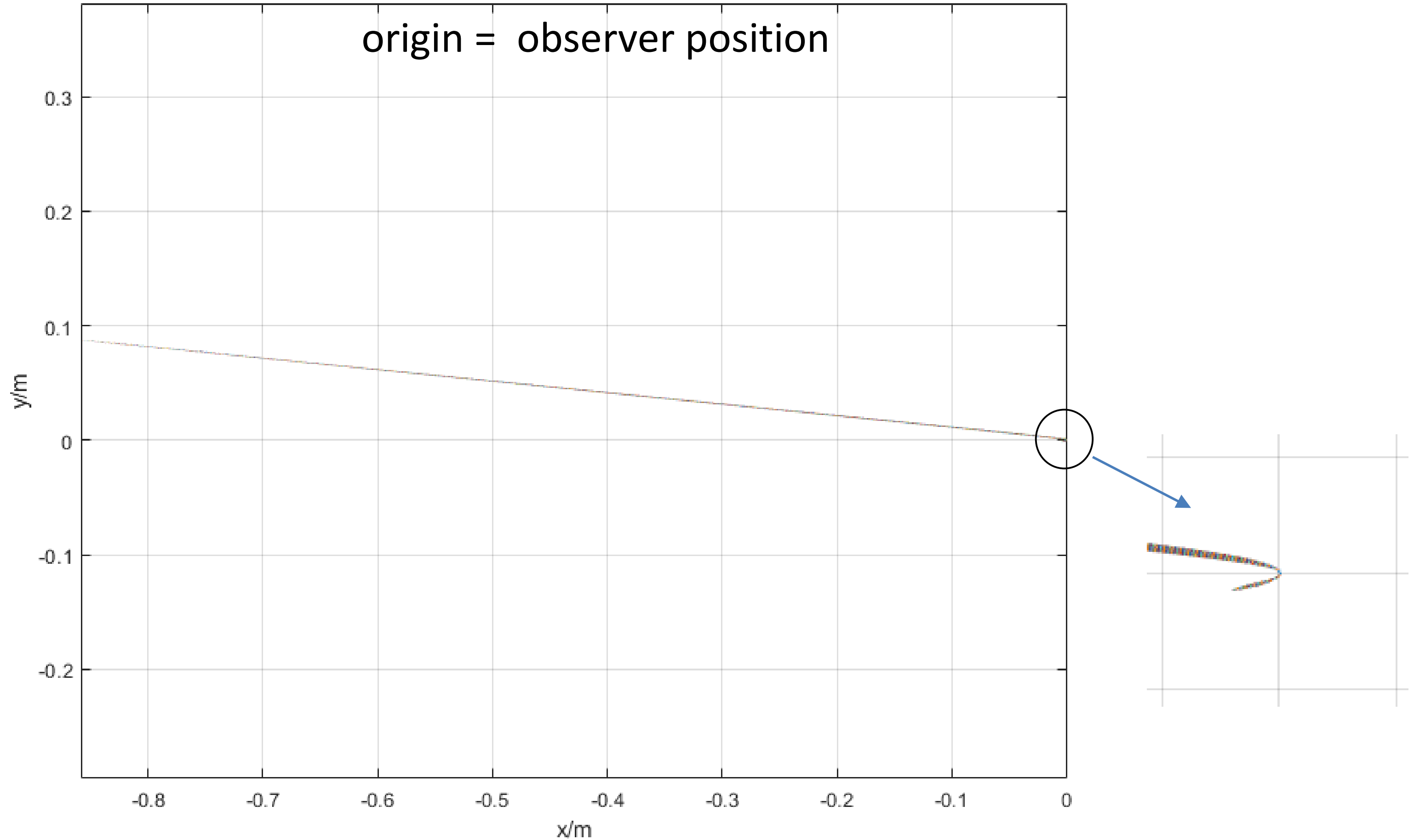
gaussian source distribution
5 σ integration range

observer is in the origin
(outside of the distribution)

Example: Zeuthen benchmark chicane: (2002)

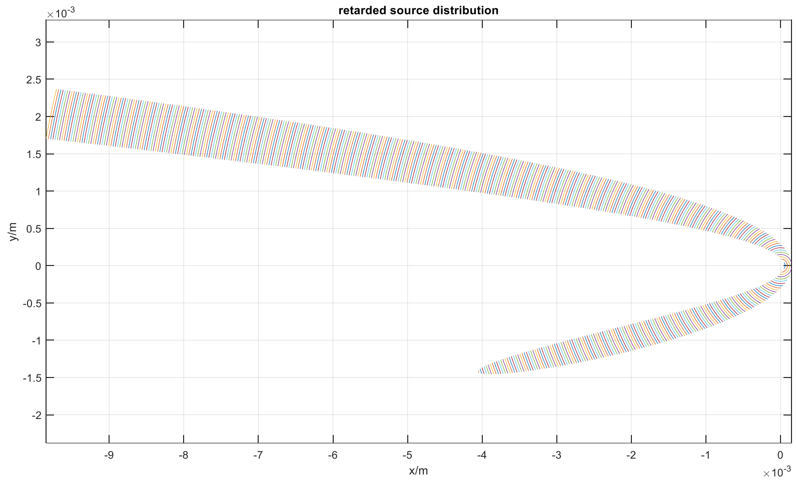


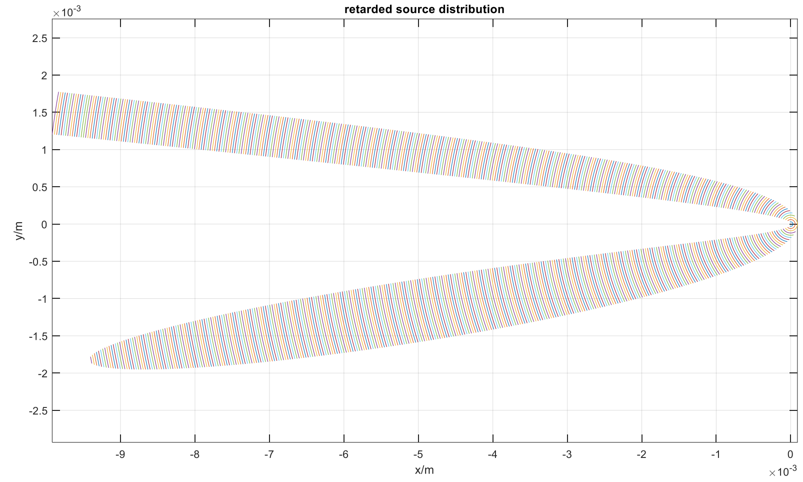
retarded source distribution:

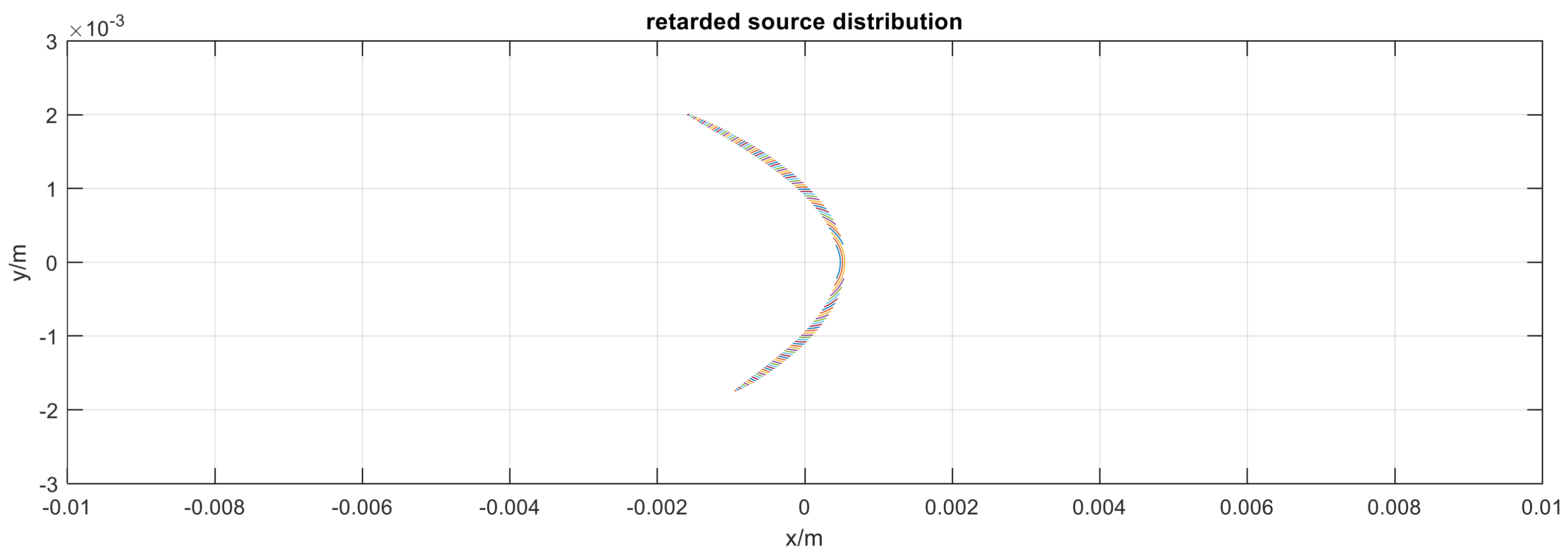


most of the bunch looks like a line charge - mostly

shape of retarded source depends on observer!
f.i. same observer time, but different observer position:







coordinate system always centered to observer

Retarded Source Integral

derivatives of V and $\mathbf{A}$ to calculate $\mathbf{E}$ and $\mathbf{B}$

$$\nabla V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int \nabla \frac{\rho\left(\mathbf{r}', t - \|\mathbf{r}-\mathbf{r}'\|/c\right)}{\|\mathbf{r}-\mathbf{r}'\|} dV' \rightarrow$$

Lienard-Wiechert like kernel functions

$$\nabla V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int \nabla \frac{\rho\left(\mathbf{r}-\mathbf{r}', t - \|\mathbf{r}'\|/c\right)}{\|\mathbf{r}'\|} dV' \rightarrow$$

apply derivatives to source functions

$$\left.\begin{matrix}\rho(X,Y,t)\\ \mathbf{J}(X,Y,t)\end{matrix}\right\} \leftarrow f\left(\left[x,x',s,\eta\right]', S\right)$$

mathematical transformations (dummy parameters)

full range

$$V(X,Y,t) = \frac{1}{4\pi\varepsilon}\int \rho\left(X + R\cos\varphi, Y + R\sin\varphi, t - R/c\right) dR\, d\varphi$$

= my actual implementation

adaptive step width control

Retarded Source Integral

derivatives of V and $\mathbf{A}$ to calculate $\mathbf{E}$ and $\mathbf{B}$

$$\nabla V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int \nabla \frac{\rho\left(\mathbf{r}', t - \|\mathbf{r}-\mathbf{r}'\|/c\right)}{\|\mathbf{r}-\mathbf{r}'\|} dV' \rightarrow$$

Lienard-Wiechert like kernel functions

$$\nabla V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int \nabla \frac{\rho\left(\mathbf{r}-\mathbf{r}', t - \|\mathbf{r}'\|/c\right)}{\|\mathbf{r}'\|} dV' \rightarrow$$

apply derivatives to source functions

$$\left.\begin{matrix}\rho(X,Y,t)\\ \mathbf{J}(X,Y,t)\end{matrix}\right\} \leftarrow f\left(\left[x, x', s, \eta\right]', S\right)$$

mathematical transformations (dummy parameters)

split integration range

near range
$$\left\{ V(X,Y,t) = \frac{1}{4\pi\varepsilon}\int \rho\left(X + R\cos\varphi, Y + R\sin\varphi, t - R/c\right) dR\, d\varphi \right.$$

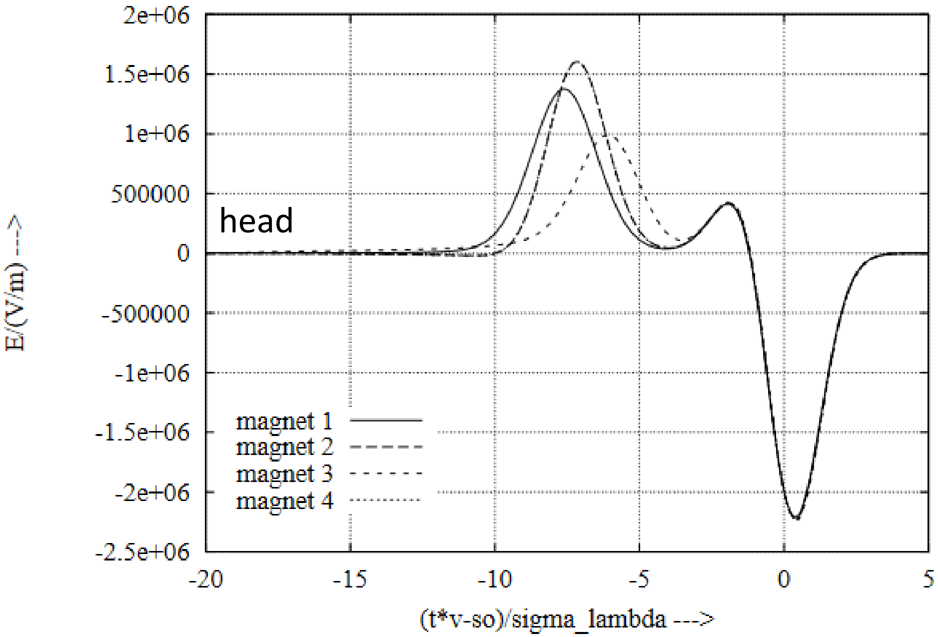
middle range
$$\left\{ \begin{aligned} &\rho(X,Y,t)\,dX\,dY = R\left(x, s = S - tv_{\mathrm{r}}, S\right) dx\,dS \\ &V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int \frac{R\left(x, S - tv_{\mathrm{r}} - \|\mathbf{r}-\mathbf{r}'(x,S)\|\beta_{\mathrm{r}}, S\right)}{\|\mathbf{r}-\mathbf{r}'(x,S)\|} dx\,dS \end{aligned} \right.$$

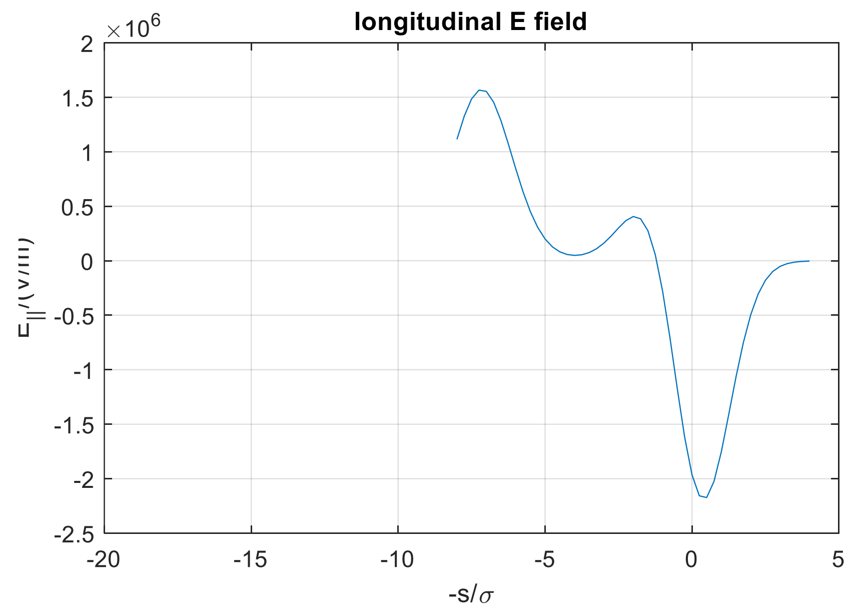
far range: analytic approximation for $\int \ldots dx$ integration $\rightarrow$ 1d integral

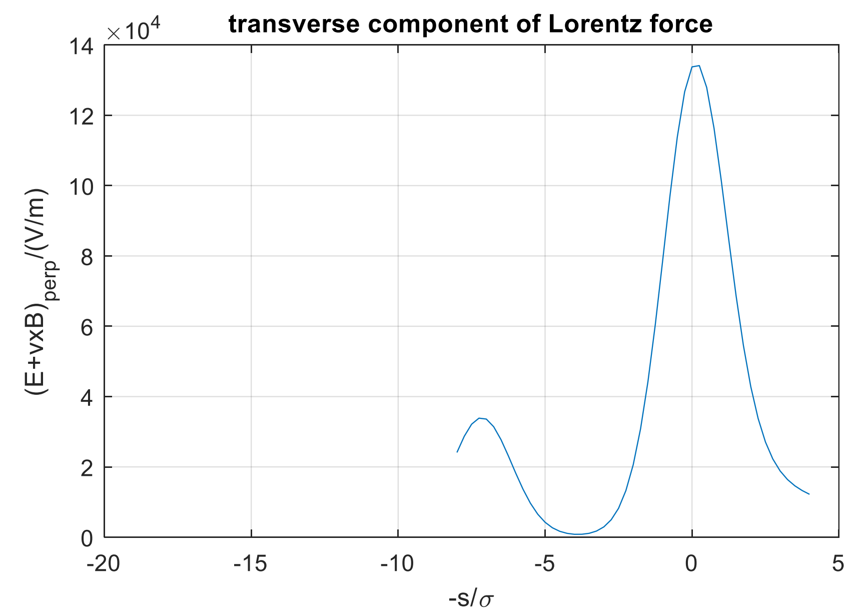
use accelerator coordinates

adaptive step width control

1^{st} Test transient longitudinal field: comparison with an older calculation







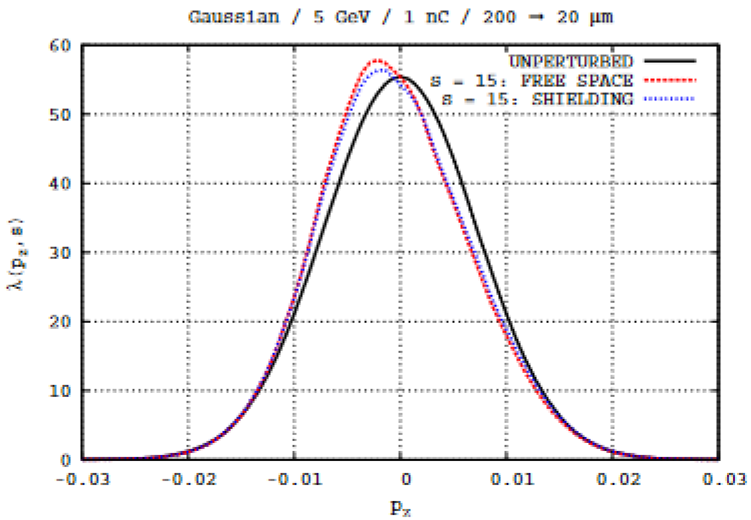
TESLA-FEL-2003-05

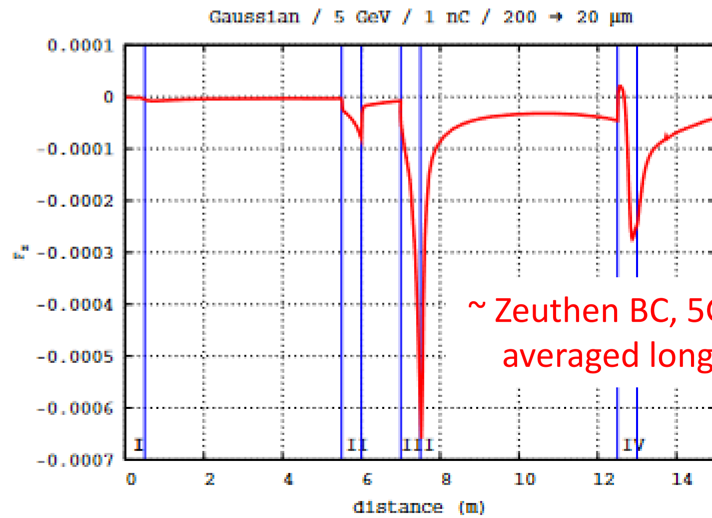
Two Methods for the Calculation of CSR Fields

M. Dohlus
Deutsches Elektronen Synchrotron, DESY, Hamburg, Germany

CSR EFFECTS IN A BUNCH COMPRESSOR: INFLUENCE OF THE TRANSVERSE FORCE AND SHIELDING *

G. Bassi[†], J. A. Ellison, K. Heinemann,
Department of Mathematics and Statistics, UNM, Albuquerque, NM 87131





~ Zeuthen BC, 5GeV, free space
averaged longitudinal field