Surface Effects

Part 1: Estimations for Petra 4

the simple NTW model
round beam pipes
transformation of surface- to beam-impedance
with Petra 4 numbers

Part 2: General Approach

wavelength regimes
local condition $\rightarrow$ surface impedance
1d roughness
- PEC: rectangular surfaces $\rightarrow$ NTW model and shallow regime
  - shallow sinusoidal corrugations
  - general equation for 2d shallow regime
- empirical approach (with losses) $\rightarrow$ "measure" surface impedance
  - examples

2d steep regime
- PEC: "measure" surface impedance with periodic eigenmode solver

Summary/Conclusion

Part 1: Estimations for Petra 4

the simple NTW model

THE SURFACE ROUGHNESS WAKEFIELD EFFECT

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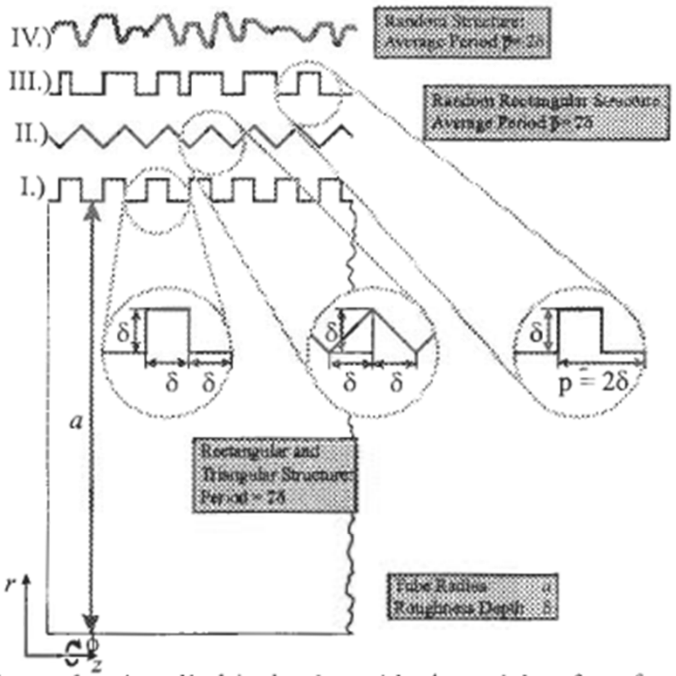


Figure 1: A cylindrical tube with 4 models of surface roughness: I.) periodically rectangular, II.) periodically triangular, III.) random with rectangular shape, IV.) random in longitudinal and radial direction. The tube radius is a, the depth of the roughness is δ, the period (I.), II.)) or average period (III.), IV.)) respectively is $2 \cdot \delta$

replace the rough surface by a dielectric layer (same thickness, effective permittivity)

a roughness depth $\delta = 100\mu$m, the permittivity of the dielectric layer is found to be around $\varepsilon \approx 1.9$ in cases Fig. 1.I.) and Fig. 1.III.) and $\varepsilon \approx 1.4$ in case Fig. 1.II.). Note that the equivalent permittivity depends on different parameters as e.g. the roughness shape.

round beam-pipes: surface and beam impedance

field in any homogeneous layer

$$H_\varphi = -K\left\{CJ_0'(Kr) + DY_0'(Kr)\right\}\exp(-jk_z z)$$

$$E_r = -\frac{k_z K}{\omega\varepsilon}\left\{CJ_0'(Kr) + DY_0'(Kr)\right\}\exp(-jk_z z)$$

$$E_z = \frac{K^2}{j\omega\varepsilon}\left\{CJ_0(Kr) + DY_0(Kr)\right\}\exp(-jk_z z)$$

$$K^2 = \omega^2\varepsilon\mu - k_z^2$$

field matching of tangential components

$$H_\varphi(R-0) = H_\varphi(R+0)$$

$$E_z(R-0) = E_z(R+0)$$

transformer equation for for v $\rightarrow$c

$$Z_\mathrm{b}' = \frac{Z_\mathrm{s}}{2\pi R}\frac{1}{1 + j\dfrac{\omega}{c}\dfrac{R}{2}\dfrac{Z_\mathrm{s}}{Z_0}}$$

with surface impedance $Z_\mathrm{s} = -\dfrac{E_z(R)}{H_\varphi(R)}$

and beam impedance (per length) $Z_\mathrm{b}' = -\dfrac{E_z(0)}{I_\mathrm{beam}}$

PEC surface impedance $Z_{\mathrm{s,PEC}} = 0$

surface impedance of conducting half space $Z_{\mathrm{s,h}} = -j\dfrac{w_{\mathrm{h}}}{\omega\varepsilon_{\mathrm{h}}}$

with $\varepsilon_{\mathrm{h}} = \varepsilon'_{\mathrm{h}} + \dfrac{\kappa_{\mathrm{h}}}{j\omega},\ \mu_{\mathrm{h}},\ w_{\mathrm{h}} = \sqrt{k_z^2 - \omega^2\varepsilon_{\mathrm{h}}\mu_{\mathrm{h}}}$

surface impedance of dielectric layer (above conducting half space) $Z_{\mathrm{s,d}} = Z_{\mathrm{d}}\dfrac{e^{w_{\mathrm{d}}\Delta} - re^{-w_{\mathrm{d}}\Delta}}{e^{w_{\mathrm{d}}\Delta} + re^{-w_{\mathrm{d}}\Delta}}$

with $\varepsilon_{\mathrm{d}},\ \mu_{\mathrm{d}},\ \Delta,\ w_{\mathrm{d}} = \sqrt{k_z^2 - \omega^2\varepsilon_{\mathrm{d}}\mu_{\mathrm{d}}},\ Z_{\mathrm{d}} = -j\dfrac{w_{\mathrm{d}}}{\omega\varepsilon_{\mathrm{d}}} \rightarrow r = \dfrac{Z_{\mathrm{d}} - Z_{\mathrm{s,h}}}{Z_{\mathrm{d}} + Z_{\mathrm{s,h}}}$

the layer can by thick compared to the wavelength, but it has to be thin compared to the curvature (planar model)

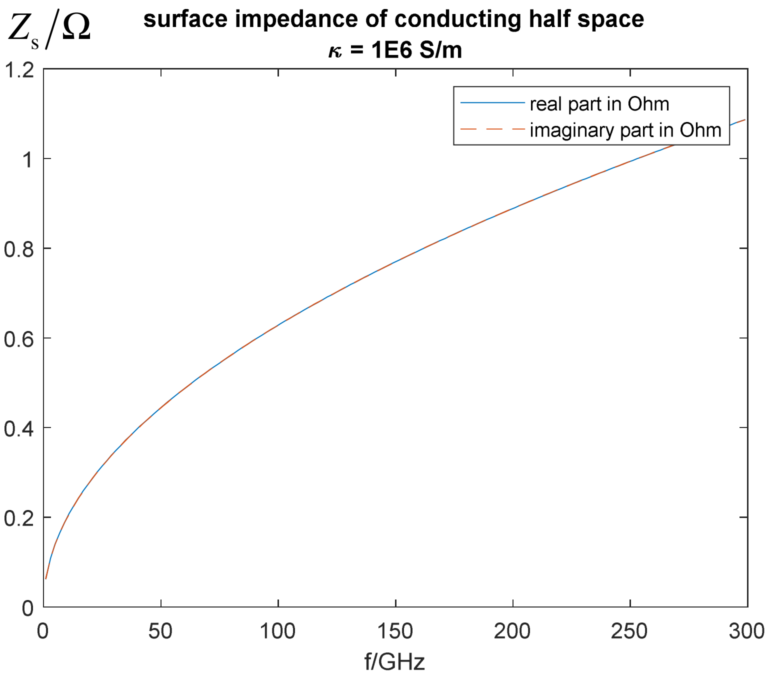
thin dielectric layer (above conducting half space) $Z_{\mathrm{s,d}} \approx Z_{\mathrm{s,h}} + j\omega\underbrace{\left(\mu_{\mathrm{d}} - \dfrac{k_z^2}{\varepsilon_{\mathrm{d}}\omega^2}\right)\Delta}_{L_{\mathrm{s}}}$

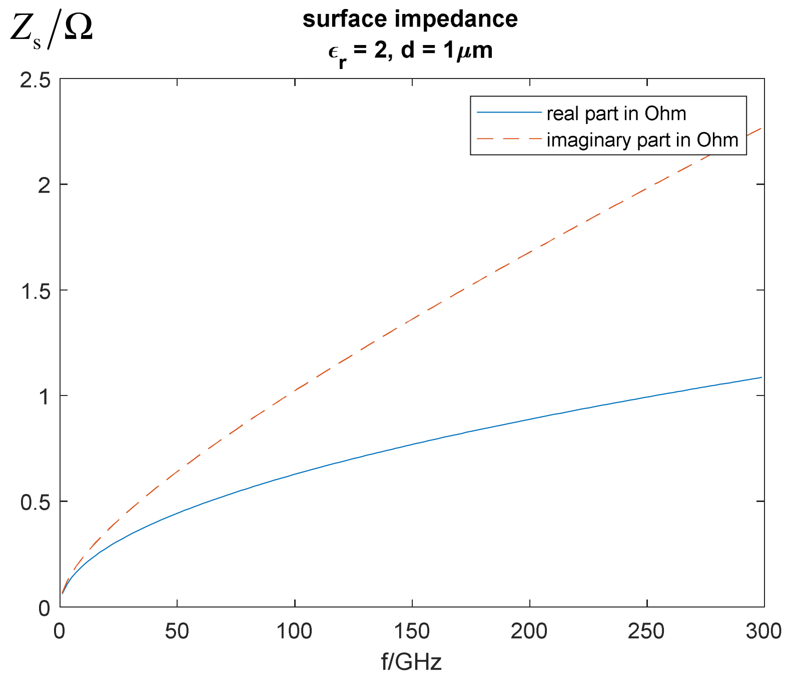
it is the impedance of the half space plus an inductive part

any multi layer structure …

assumption for Petra 4: conductivity of NEC coating ~ 1E6 S/m

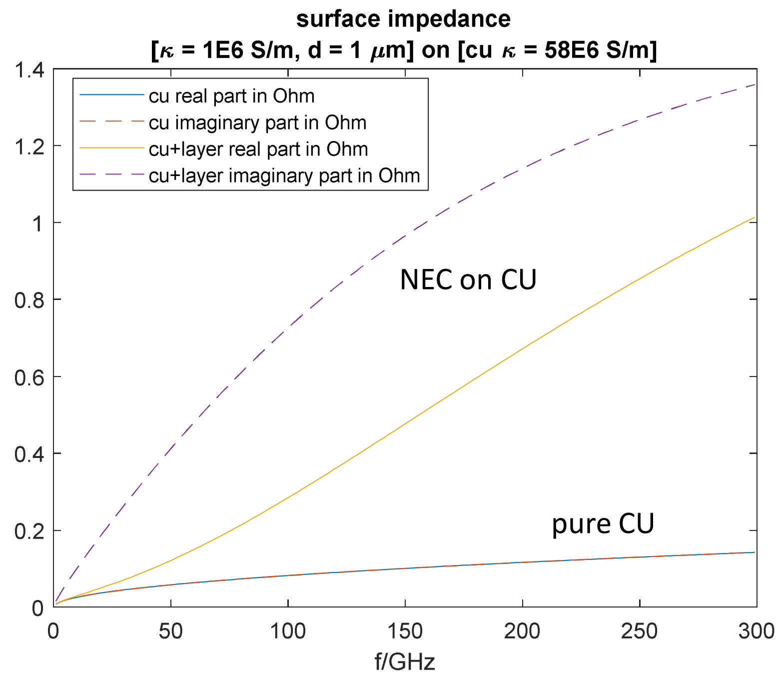
effect of dielectric layer

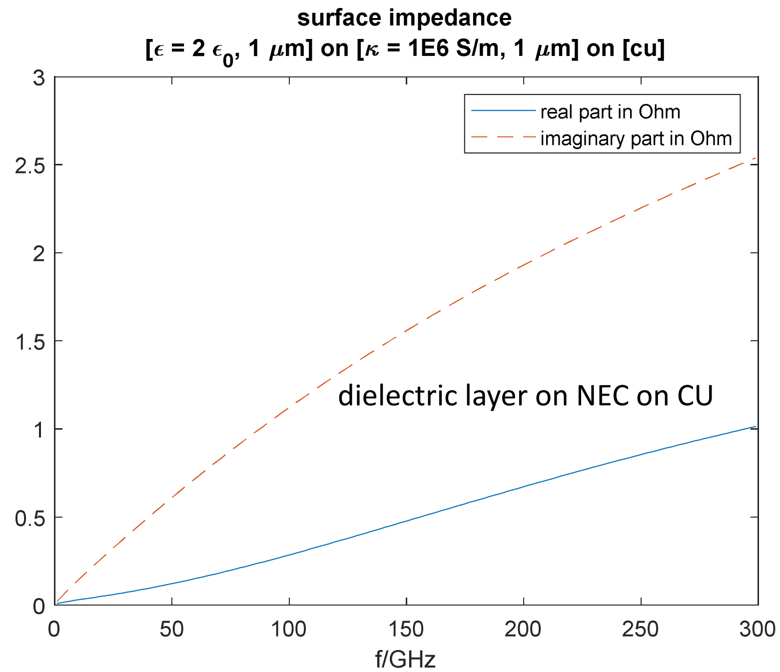




nearly no effect to real part

assumption for Petra 4: 1 um NEC layer above copper

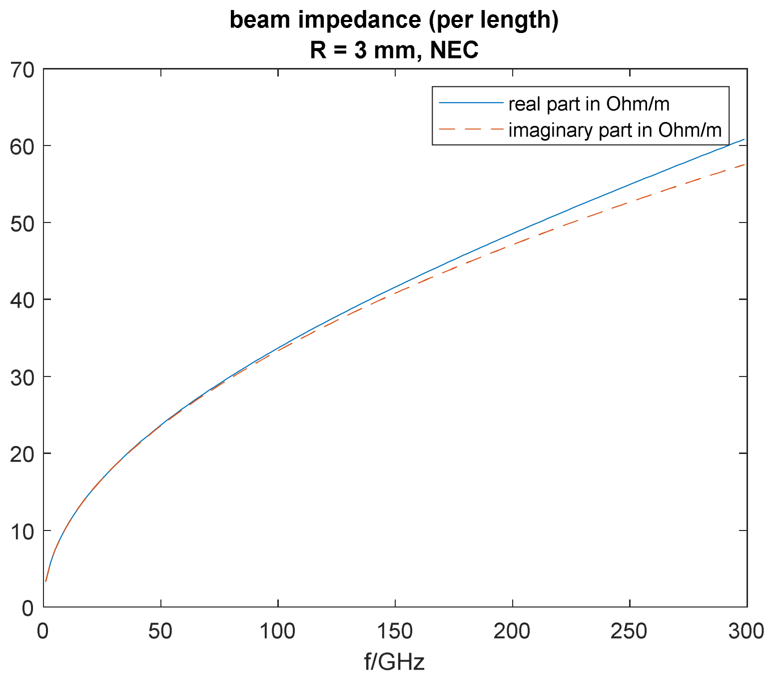


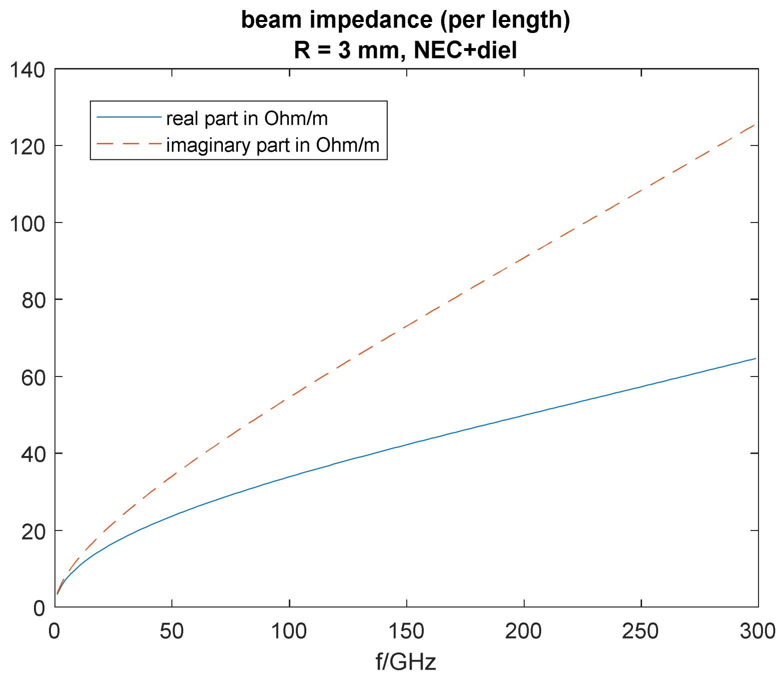


beam impedance $Z_b' = \frac{Z_s}{2\pi R} \frac{1}{1 + j\frac{\omega}{c}\frac{R}{2}\frac{Z_s}{Z_0}}$

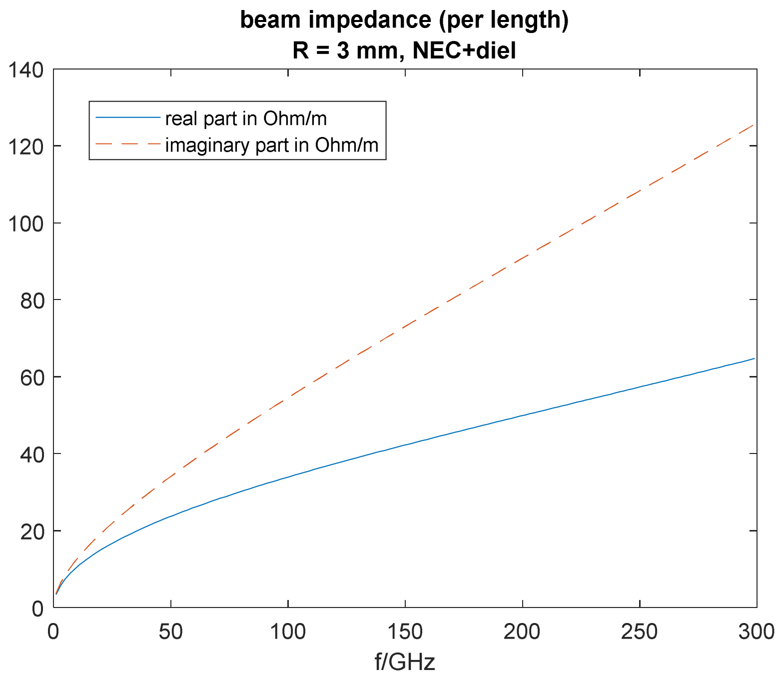
assumption for Petra 4: R = 3 mm

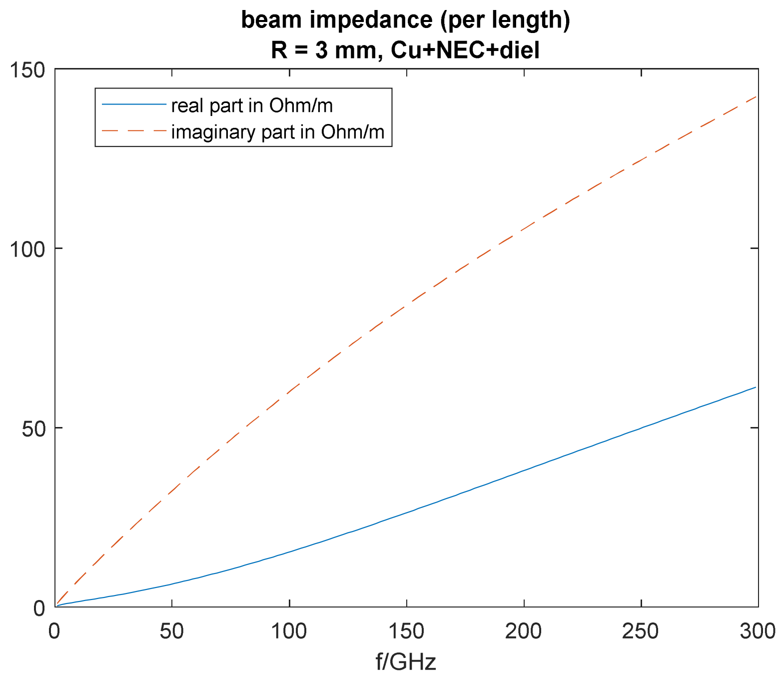
NEC on CU





weak influence from the second term, therefore $Z_b' \approx \frac{Z_s}{2\pi R} \approx \frac{53 Z_s}{\mathrm{m}}$





weak influence from the second term, … where is the resonance?

estimation of frequency of synchronous mode (for perfect conductivity)

$$Z_s \rightarrow j\omega\mu_2 \frac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2}\Delta$$

$$Z_b' \rightarrow \frac{Z_s}{2\pi R} \frac{1}{\underbrace{1 + j\frac{\omega}{c}\frac{R}{2}\frac{1}{Z_0} j\omega\mu_2 \frac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2}\Delta}_{\downarrow \atop 0}}$$

$$\rightarrow \frac{\omega_r}{c} = \sqrt{\frac{2}{R\Delta}\frac{\mu_0}{\mu_2}\frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_0}} = \frac{2}{\sqrt{R\Delta}}$$

$f_r \approx 1.7$ THz for $\varepsilon_2 = 2\varepsilon_0$

$\mu_2 = \mu_0$

$R = 3$ mm

$\Delta = 1\ \mu$m

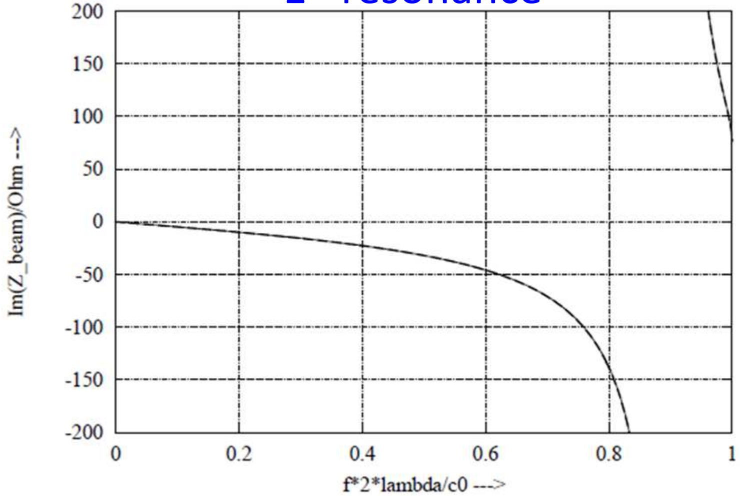
multiple propagating modes $\longrightarrow$ **if** there are higher spatial harmonics with $k_{r,n} \in$ real

$$k_{r,n} = \sqrt{k_0^2 - \left(k_0 + n\frac{\Lambda}{2\pi}\right)^2}$$

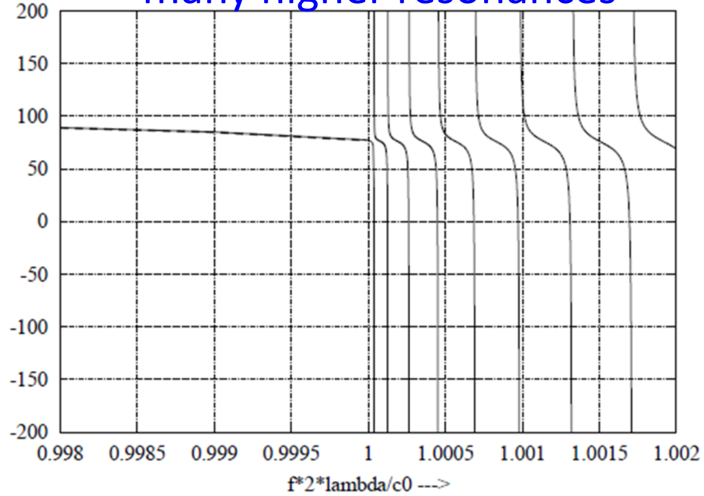
Λ is periodicity

$$k_0 \approx \frac{2}{\sqrt{R\Delta}}$$

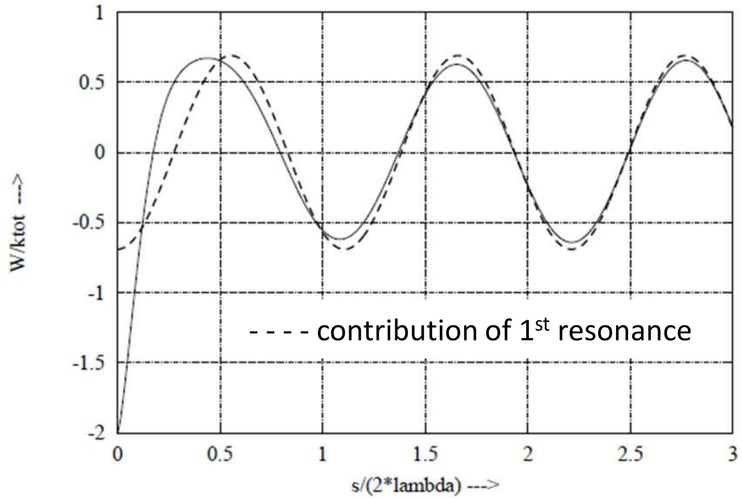
1st resonance



many higher resonances



wake



$$\sum k_{\mathrm{loss},\nu} = \frac{1}{2\pi\varepsilon_0 R_0^2}$$

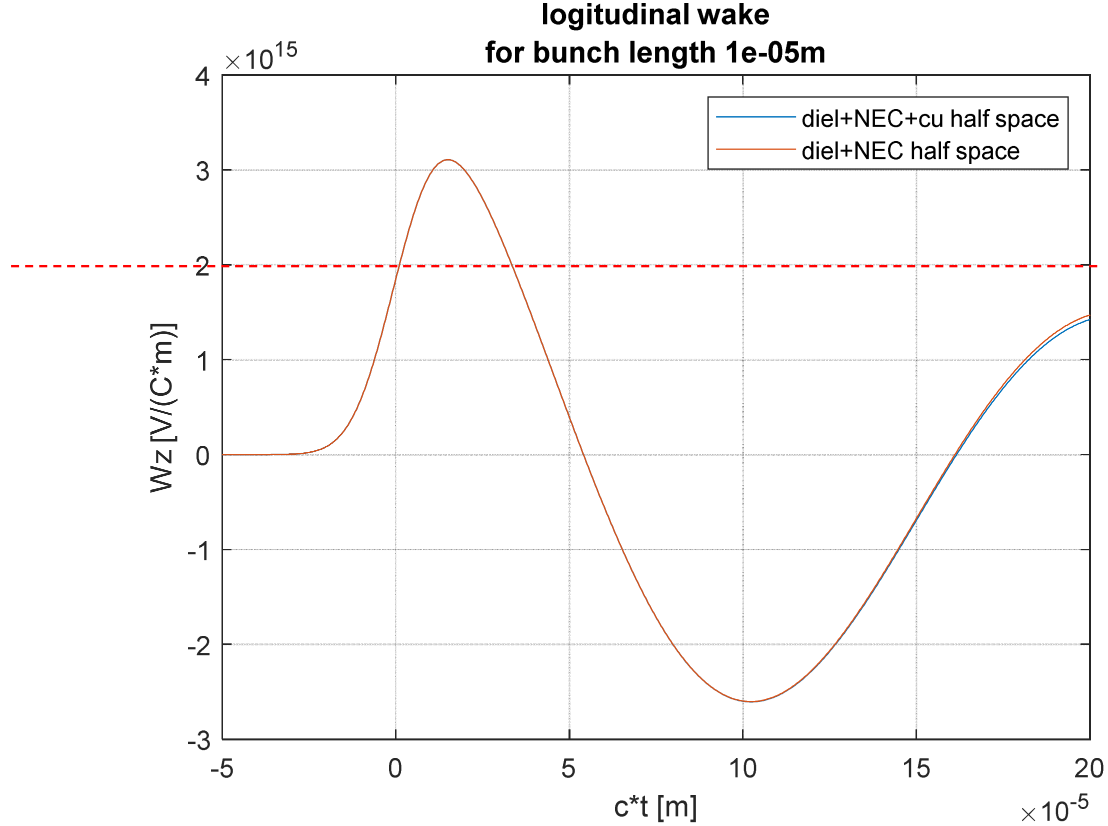
loss factor is distributed to many modes

"incoherent behavior" of higher modes

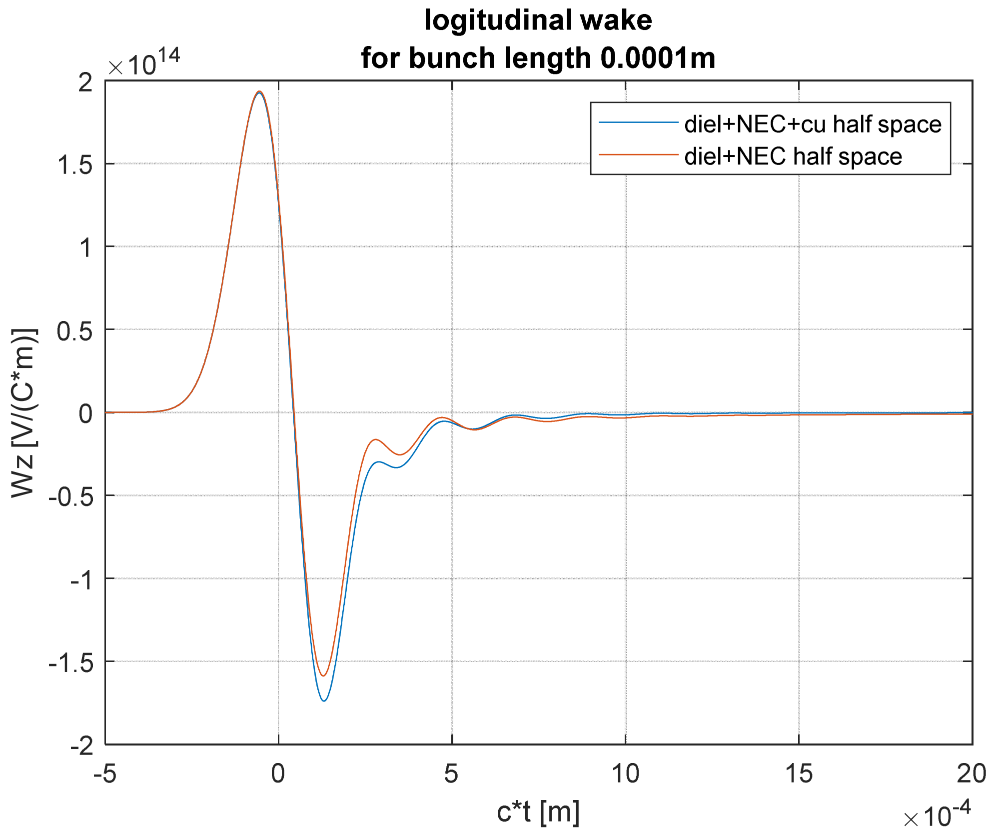
longitudinal wake for short Gaussian bunches

effect of the copper below the NEC and the dielectric layer

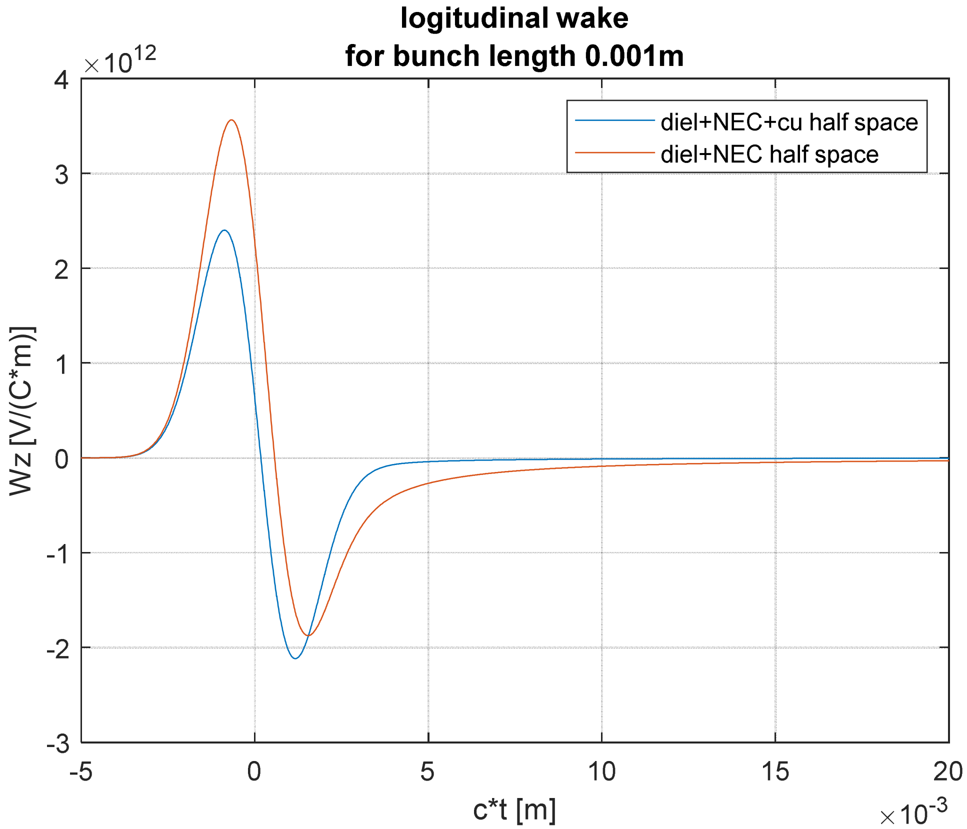
$$\frac{1}{2\pi\varepsilon_0 R^2}$$



Gaussian bunch, “bunch length” is rms bunch length



Gaussian bunch, “bunch length” is rms bunch length



Gaussian bunch, “bunch length” is rms bunch length

Part 2: General Approach

wavelength regimes
local condition $\rightarrow$ surface impedance
1d roughness
- PEC: rectangular surfaces $\rightarrow$ NTW model and shallow regime
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2d steep regime
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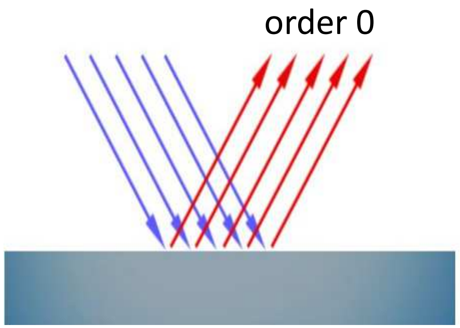
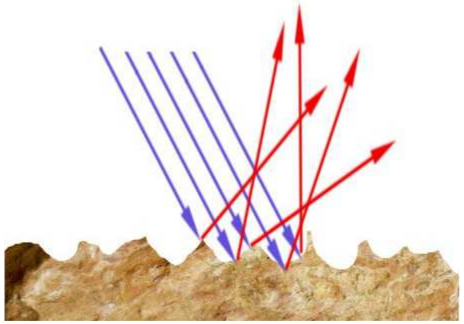
wavelength regimes

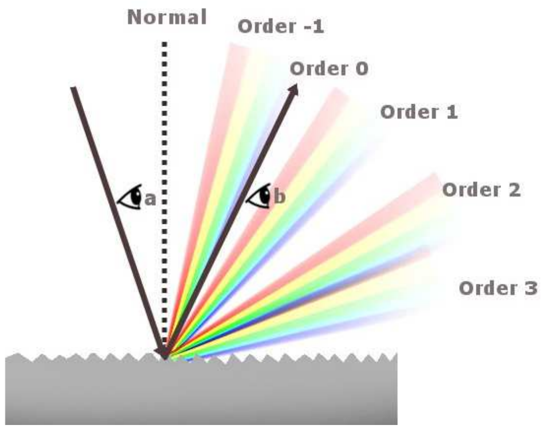
short wavelength:

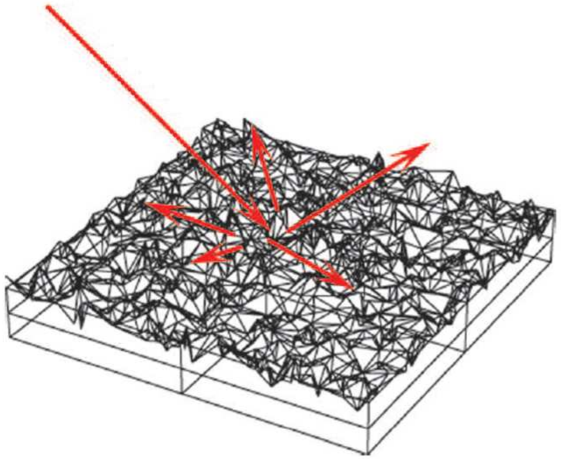
specular reflection



diffuse reflection



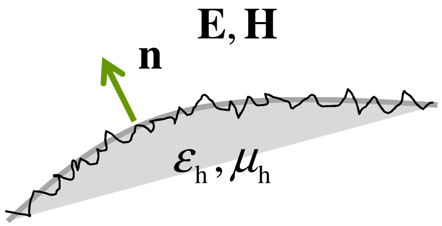




Patrik Hermansson, Göran Forssell, Jan Fagerström
A Review of Models for Scattering from Rough Surfaces

long wavelength: only specular reflection … find **local condition**

Local Condition $\rightarrow$ Surface Impedance



use boundary condition to model roughness effects?

PEC like surfaces: $\mathbf{n}\times\mathbf{E} \leftarrow f\{\mathbf{n}\times\mathbf{H}\}$

classical scalar surface impedance $\mathbf{n}\times\mathbf{E} = Z_\mathrm{s}\;\mathbf{n}\times\mathbf{n}\times\mathbf{H}$

limitations: f.i. thin dielectric layer on bulk conductor

$$Z_\mathrm{s} \approx Z_\mathrm{s,h}(k_z) + j\omega\left(\mu_\mathrm{d} - \frac{k_z^2}{\varepsilon_\mathrm{d}\omega^2}\right)\Delta \qquad Z_\mathrm{s,h} = -j\frac{\sqrt{k_z^2 - \omega^2\varepsilon_\mathrm{h}\mu_\mathrm{h}}}{\omega\varepsilon_\mathrm{h}} \approx \sqrt{\frac{\mu_\mathrm{h}}{\varepsilon_\mathrm{h}}} - k_z^2\frac{1}{2\omega^2\varepsilon_\mathrm{h}\mu_\mathrm{h}}\sqrt{\frac{\mu_\mathrm{h}}{\varepsilon_\mathrm{h}}}$$

$$E_z = -Z_\mathrm{s}H_\varphi \quad \text{with} \quad k_z^2 \rightarrow -\frac{\partial^2}{\partial z^2}$$

$$E_z = -\left(\sqrt{\frac{\mu_\mathrm{h}}{\varepsilon_\mathrm{h}}} + j\omega\mu_\mathrm{d}\Delta\right)H_\varphi + \left(\frac{-1}{2\omega^2\varepsilon_\mathrm{h}\mu_\mathrm{h}}\sqrt{\frac{\mu_\mathrm{h}}{\varepsilon_\mathrm{h}}} + \frac{\Delta}{j\omega\varepsilon_\mathrm{d}}\right)\frac{\partial^2 H_\varphi}{\partial z^2}$$

usual implementations do not consider derivatives:

method 1: neglect k_z

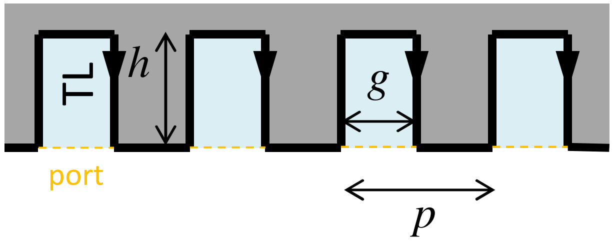
method 2: set $k_z=\omega/v$

1D Roughness

PEC: rectangular surfaces $\rightarrow$ NTW model and shallow regime
shallow sinusoidal corrugations
general equation for 2d shallow regime
empirical approach (with losses) $\rightarrow$ "measure" surface impedance
examples

Transmission Line Models for Rectangular Surfaces

$$h >\approx g$$



neglect higher spatial harmonics

$$Z_s = jZ_0 \tan\left(\frac{\omega}{c} h\right)\frac{g}{p}$$

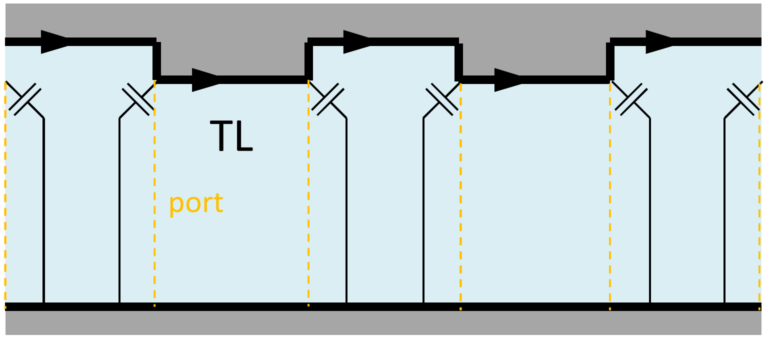
$$\downarrow$$

Bane/Stupakov: $Z_s \approx jZ_0 \frac{\omega}{c}\frac{hg}{p}$

$$\downarrow$$

NTW model with $\varepsilon = 2\varepsilon_0$

$$g >> h$$



$Z_s \approx jZ_0 \frac{\omega}{c}\frac{h^2}{p} f$ with $f \approx 1$

"shallow" regime

Shallow Sinusoidal Corrugations

Stupakov:

$\varphi = 0$

$$Z_{s,\mathrm{PEC,plane}}(j\omega) = jk_0 Z_0 \frac{(ak_1)^2}{4}\left(\frac{j}{k_{r,-1}} + \frac{j}{k_{r,1}}\right)$$

with $k_{r,\pm1} = \sqrt{k_0^2 - \left(k_0 \pm \frac{p}{2\pi}\right)^2}$

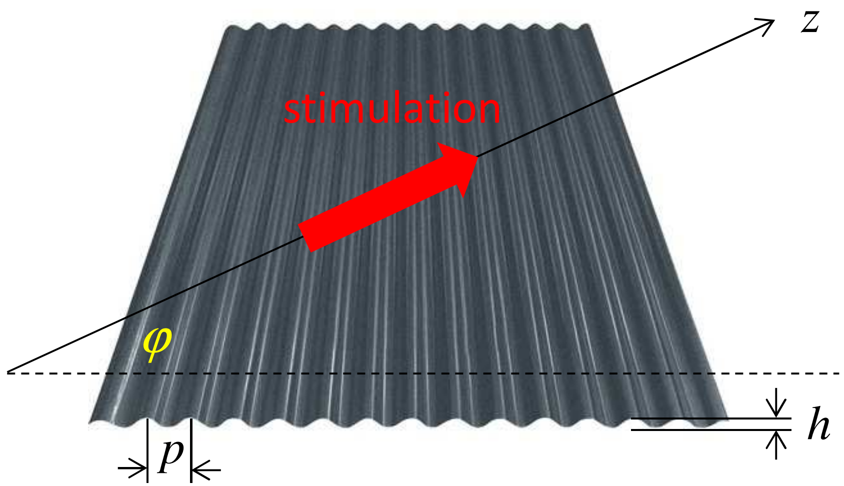
for $\frac{2\pi}{p} >> k_0$

$$Z_{s,\mathrm{PEC,plane}} \approx jZ_0 \frac{\omega}{c}\frac{h^2}{p}\frac{\pi}{4}$$ with $h = 2a$

↕

$$Z_s \approx jZ_0 \frac{\omega}{c}\frac{h^2}{p} f$$ rectangular shape

$\varphi \neq 0$



$$Z_{s,\mathrm{PEC,plane}} \approx jZ_0 \frac{\omega}{c}\frac{h^2}{p}\frac{\pi}{4}\cos^2\varphi$$

General Equation for 2D Shallow Regime

G. V. Stupakov: Impedance of small obstacles and rough surfaces
https://journals.aps.org/prab/pdf/10.1103/PhysRevSTAB.1.064401

$$Z_b' = \frac{Z_r}{2\pi R} \quad \text{with} \quad Z_r = j\omega\mu_0 \int R(\kappa_z, \kappa_x) \frac{\kappa_z^2}{\kappa} d\kappa_x d\kappa_z$$

$$R(\kappa_x, \kappa_z) = \frac{1}{(2\pi)^2} \iint K(x, z) \exp(-j\kappa_x x - j\kappa_z z) dxdy$$

$$K(\tilde{x}, \tilde{y}) = \left\langle h(x-\tilde{x}, y-\tilde{y}) h(x, y) \right\rangle$$

$h(x, y)$ random surface function

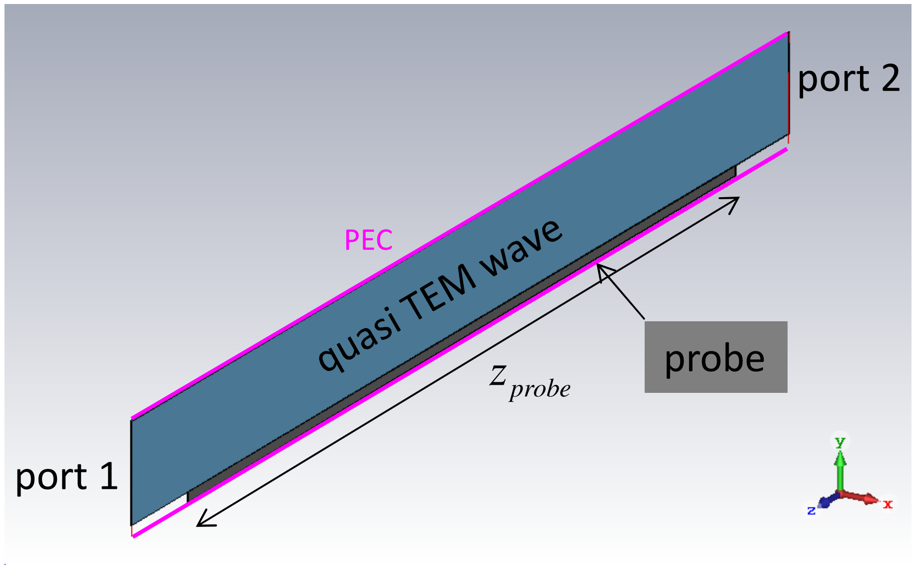
This is the low frequency approximation of the beam impedance. Z_r has the meaning of a surface impedance. It is purely inductive as dielectric layers.
Extrapolation:

$$Z_b' = \frac{j\omega L_r}{2\pi R} \frac{1}{1 + j\frac{\omega}{c}\frac{R}{2}\frac{1}{Z_0} j\omega L_r} \quad \text{with} \quad L_r = \mu_0 \int R(\kappa_z, \kappa_x) \frac{\kappa_z^2}{\kappa} d\kappa_x d\kappa_z$$

Empirical Approach → "Measure" Surface Impedance

although we do not know if something like surface impedance exists, we assume that it does; we measure and use it;

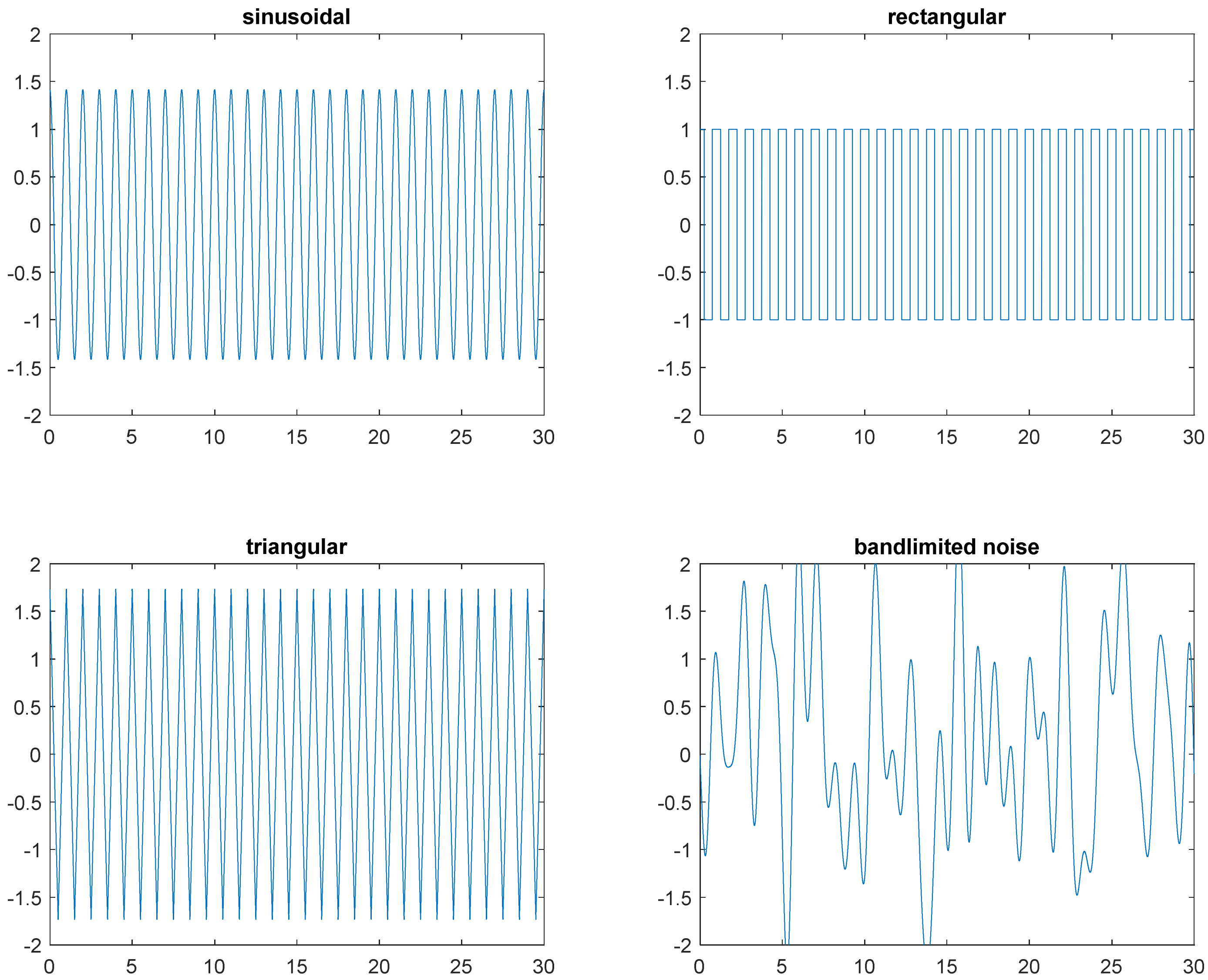
a possible setup to "measure" Z_s with help of a MWS-CST simulation:



field does not depend on x, therefore only a thin slice is modeled

resolution is high enough to resolve roughness and (is necessary) skin depth
f.i. roughness/skin depth/resolution = 1 µm/0.2 µm/0.02 µm
height of port 1 = 50 µm, zprobe= 4000 µm

normalized surface shapes $p = 1 \quad x_{\mathrm{rms}} = 1$



random surface = white, band limited, shortest period = p

surface impedance

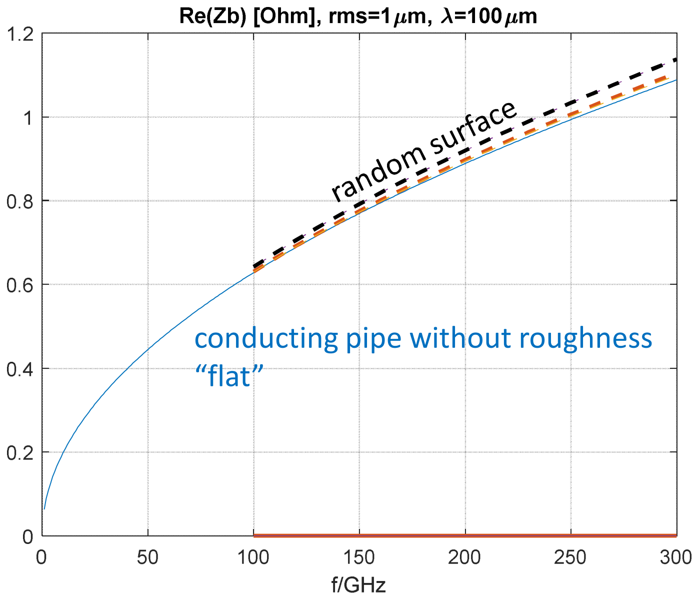
$p = 100\ \mu\text{m}$

$x_{\text{rms}} = 1\ \mu\text{m}$

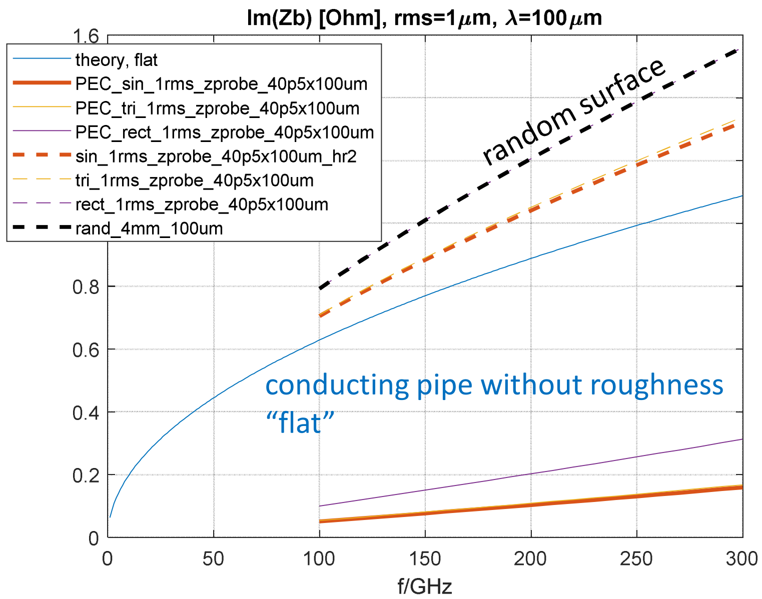
$\max\{x'_{\sin}\} = \sqrt{2}x_{\text{rms}}\dfrac{2\pi}{p} \approx 0.1$

shallow regime

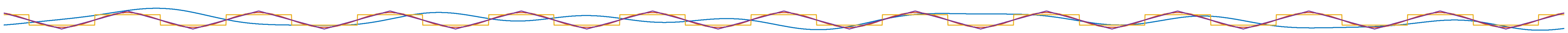
solid for PEC surfaces
dashed for κ = 1E6 S/m



real part:
{rough+conductivity}
≈{flat+conductivity}



imaginary part:
{rough+conductivity} >
{rough+PEC}+{flat+conductivity}



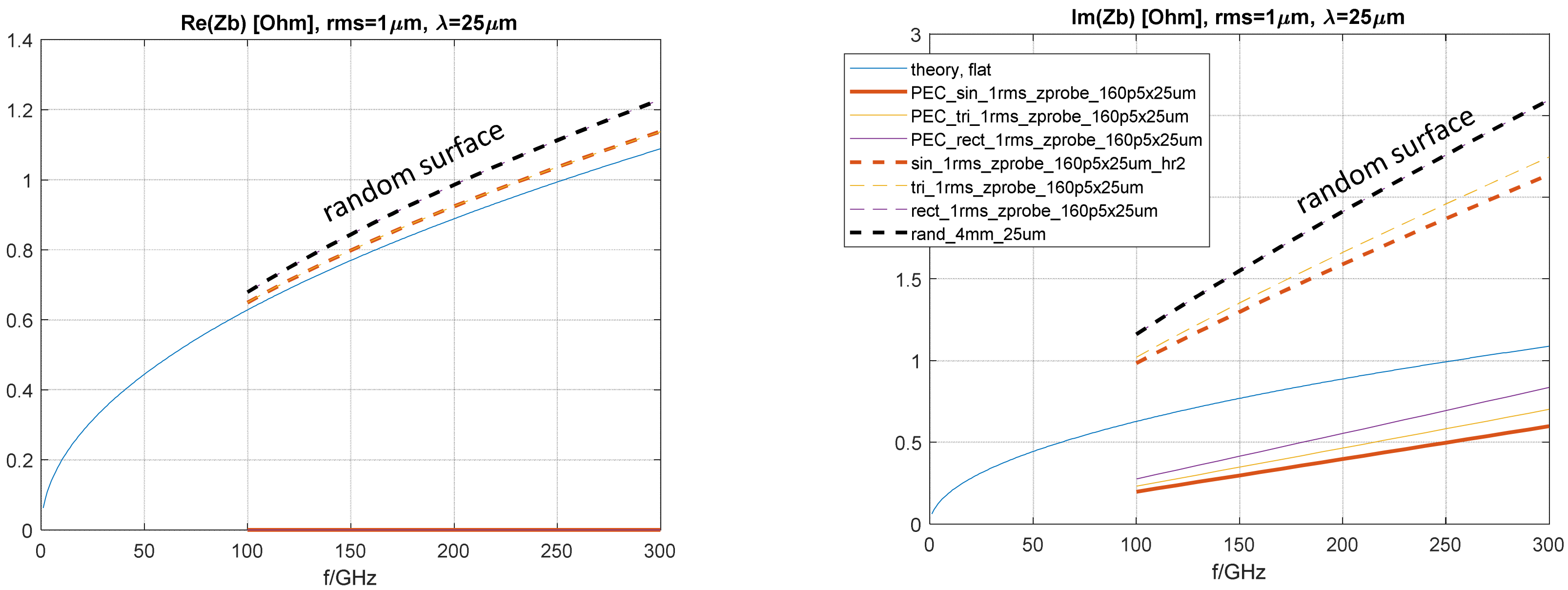
surface impedance

$p = 25\ \mu\text{m}$

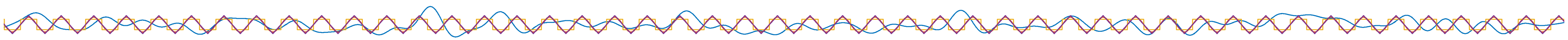
$x_{\text{rms}} = 1\ \mu\text{m}$

$\max\{x'_{\sin}\} = \sqrt{2}x_{\text{rms}}\dfrac{2\pi}{p} \approx 0.4$ not shallow!

solid for PEC surfaces
dashed for κ = 1E6 S/m



these numbers are comparable to my estimations with NTW model



surface impedance

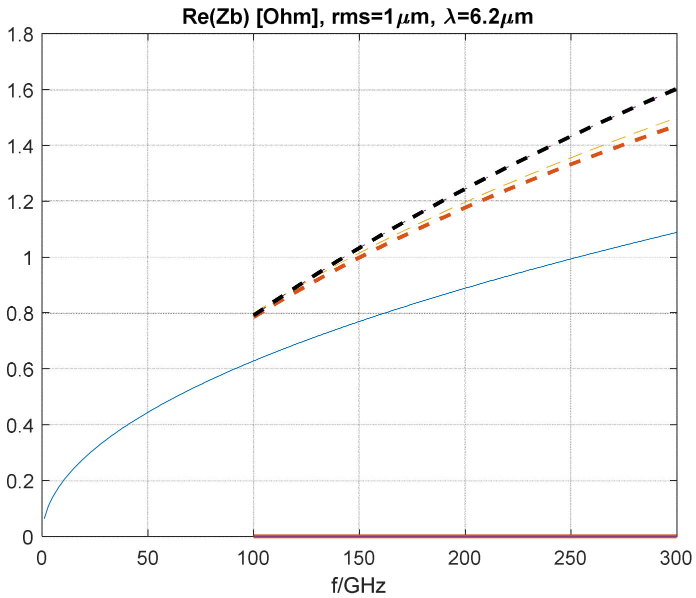
$$p = 6.2\ \mu\mathrm{m}$$

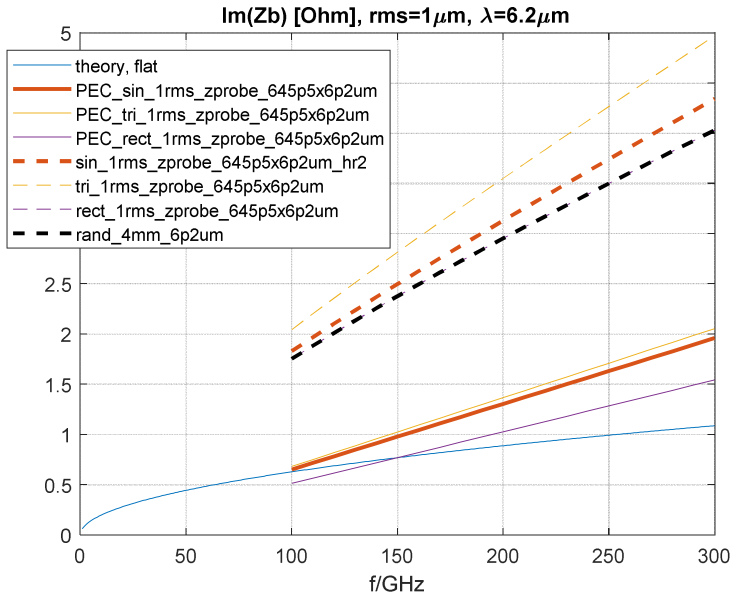
$$x_{\mathrm{rms}} = 1\ \mu\mathrm{m}$$

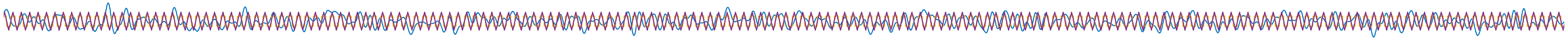
$$\max\{x'_{\sin}\} = \sqrt{2}x_{\mathrm{rms}}\frac{2\pi}{p} \approx 1.4$$

steep

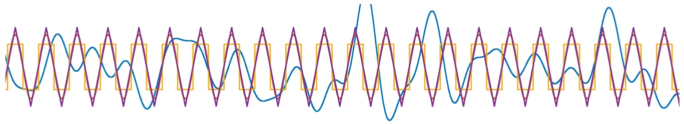
solid for PEC surfaces
dashed for κ = 1E6 S/m







surface impedance



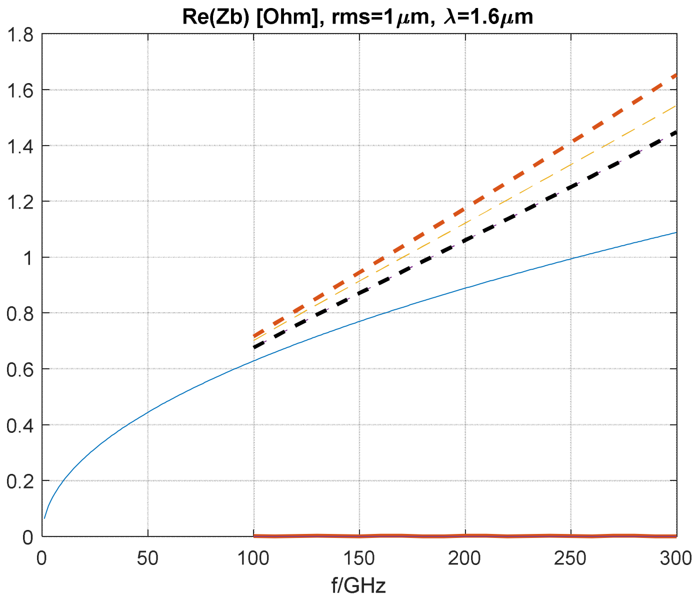
$p = 1.6\ \mu\text{m}$

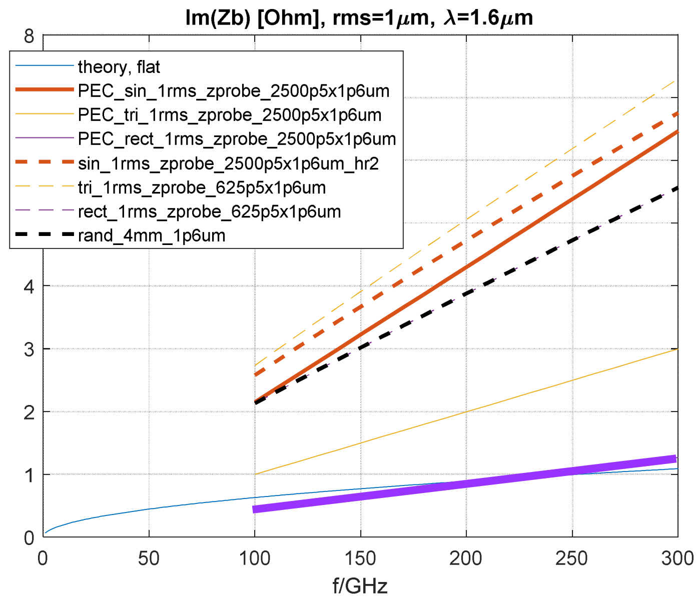
$x_{\text{rms}} = 1\ \mu\text{m}$

$\max\{x'_{\sin}\} = \sqrt{2}x_{\text{rms}}\dfrac{2\pi}{p} \approx 5$

very steep

solid for PEC surfaces
dashed for κ = 1E6 S/m



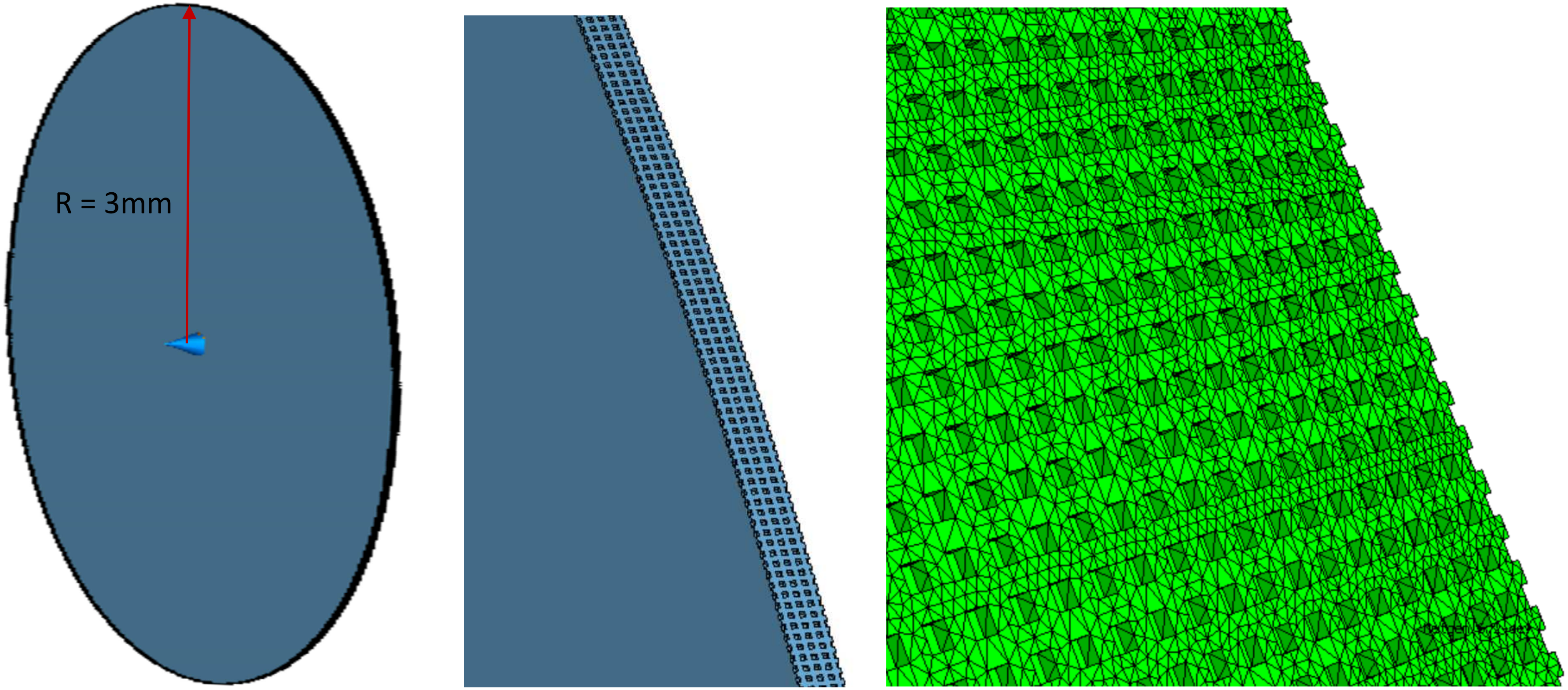


real part:
<< imaginary part

imaginary part:
rect.+PEC underestimates effects

2D Steep Regime

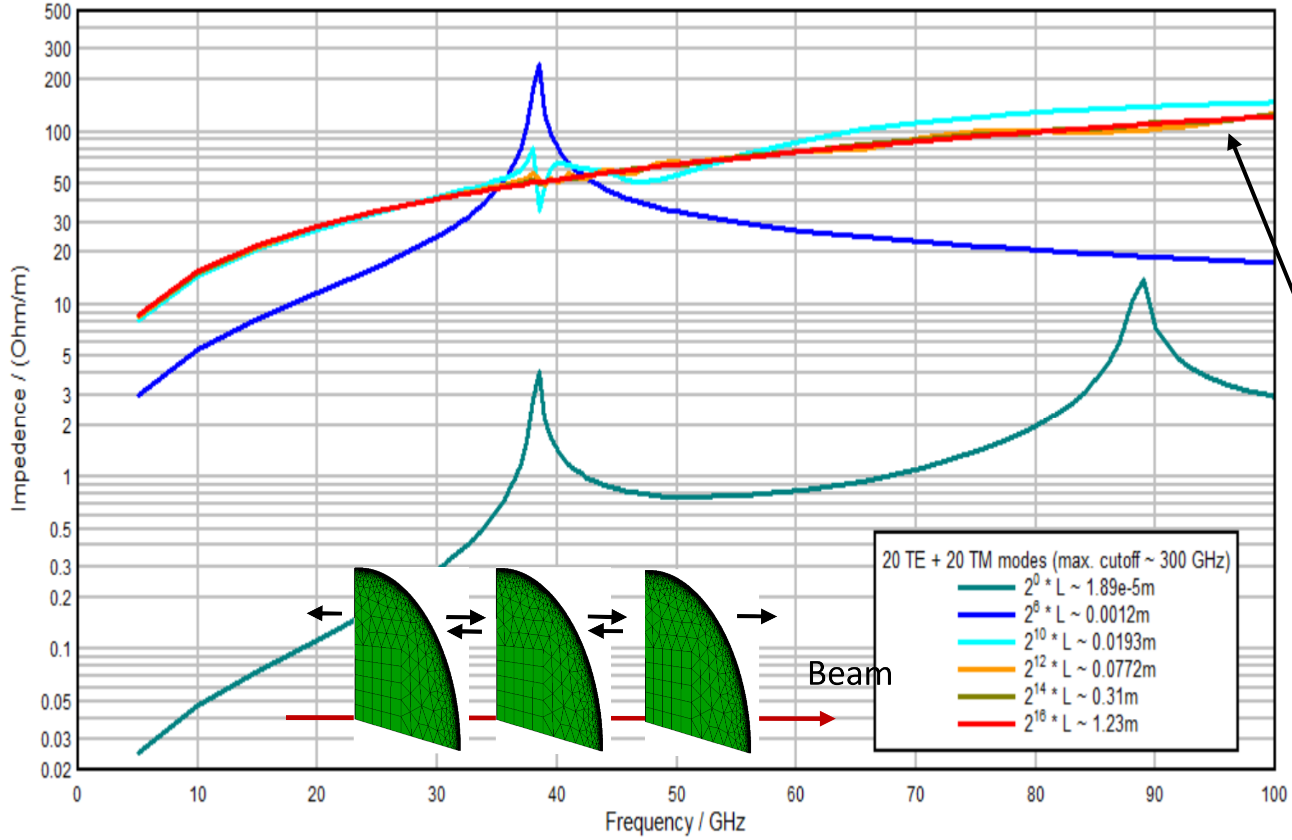
Erion Gjonaj: 3D geometry

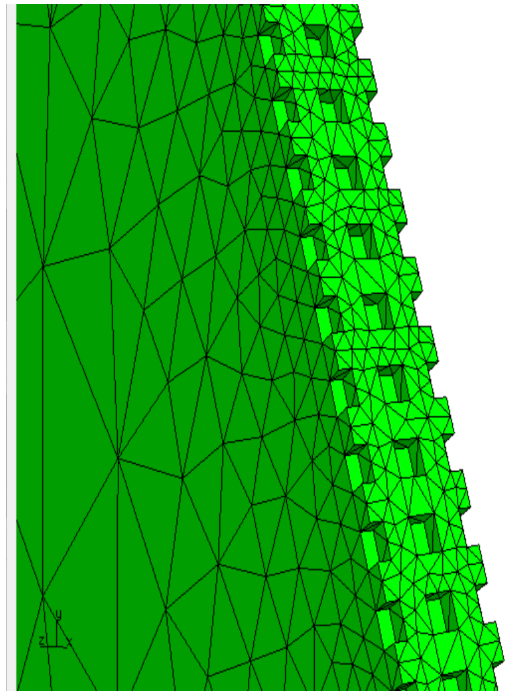


~6.5x10^6 elements (hex, tet, prism)

Impedance of rough resistive pipe

- Beam impedance: convergence to steady state



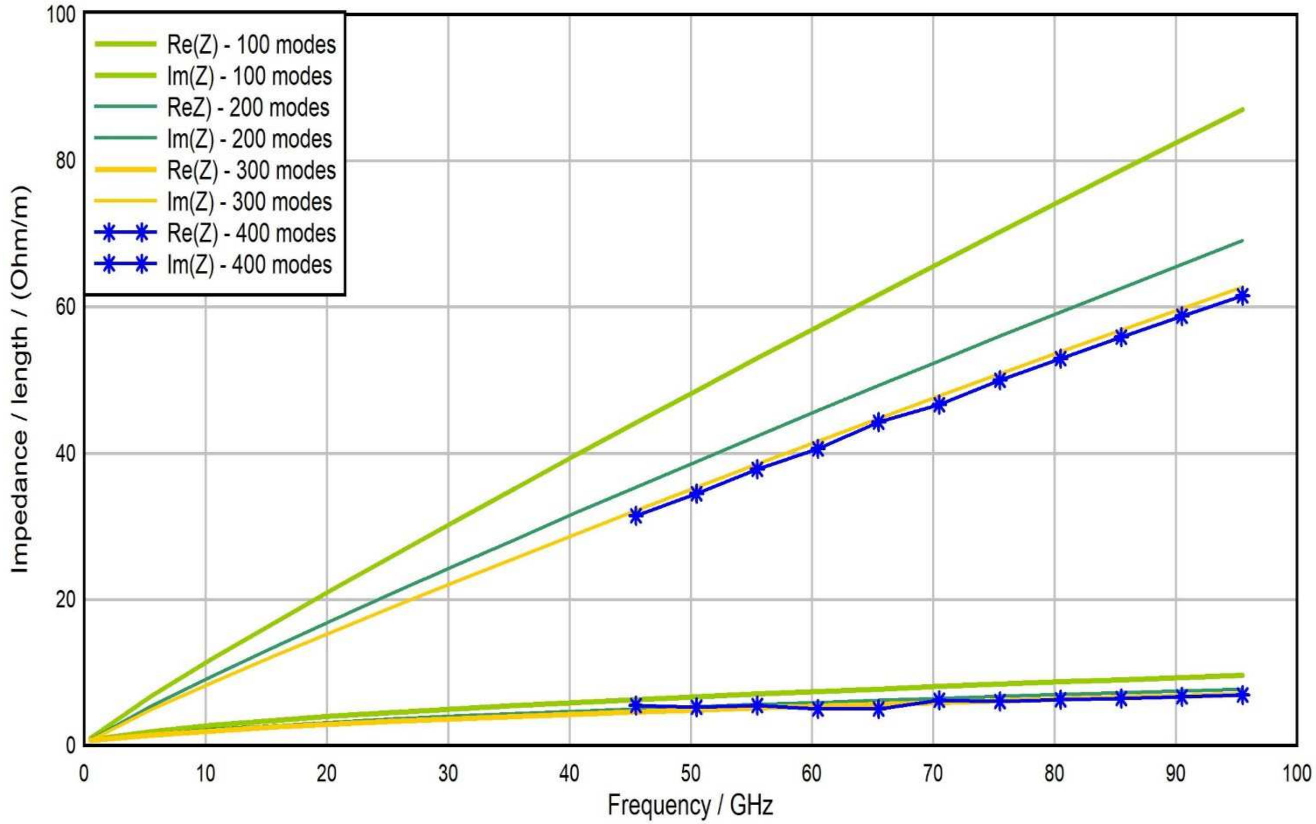


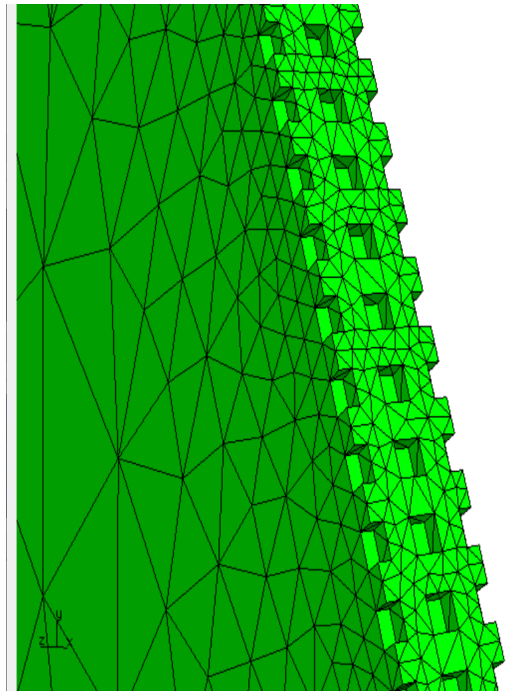
steady state at ~ 1m

40 coupling modes / port
highest cutoff frequency
~ 300GHz

Impedance of rough resistive pipe

- Beam impedance: convergence w.r.t. coupling modes

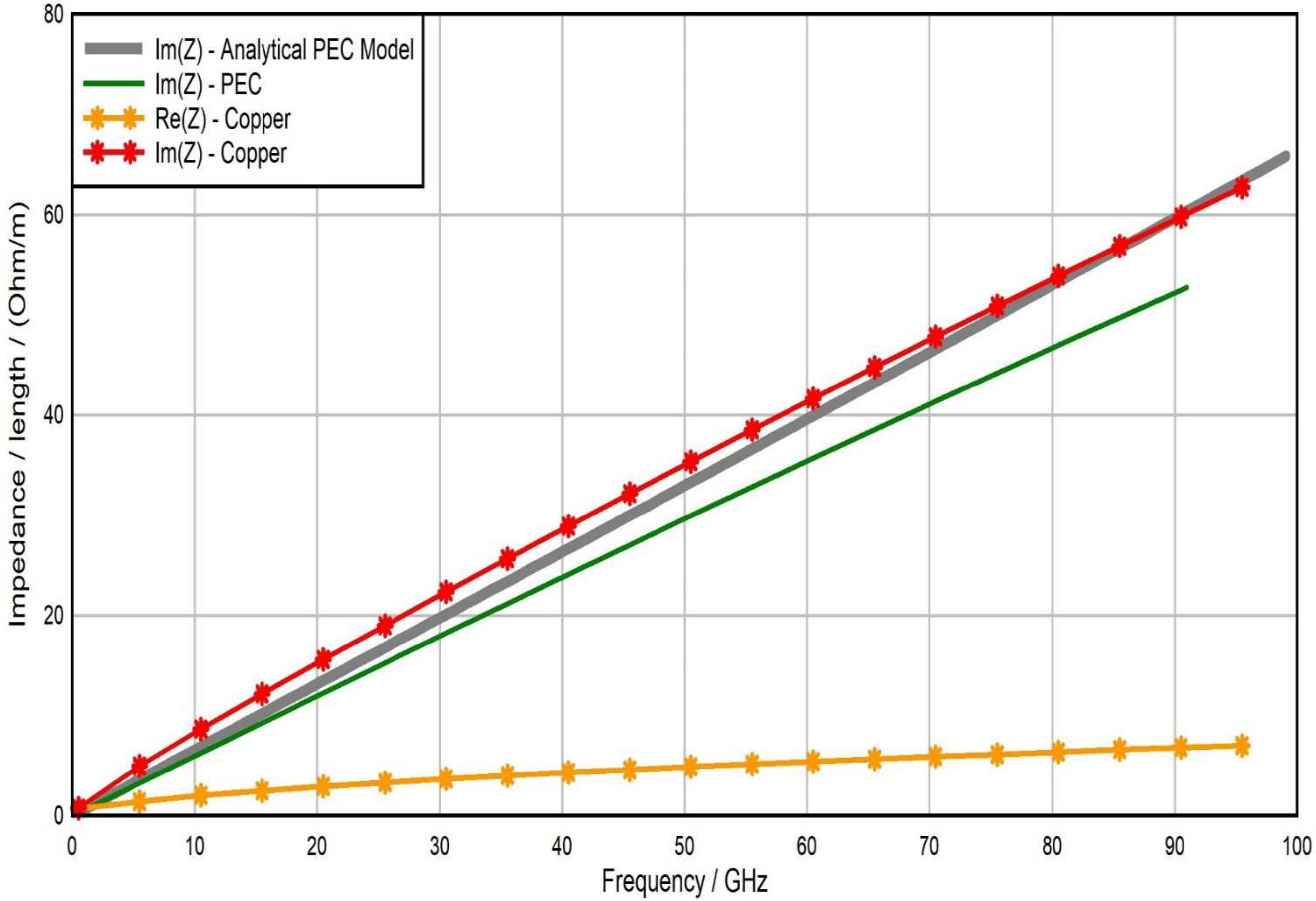


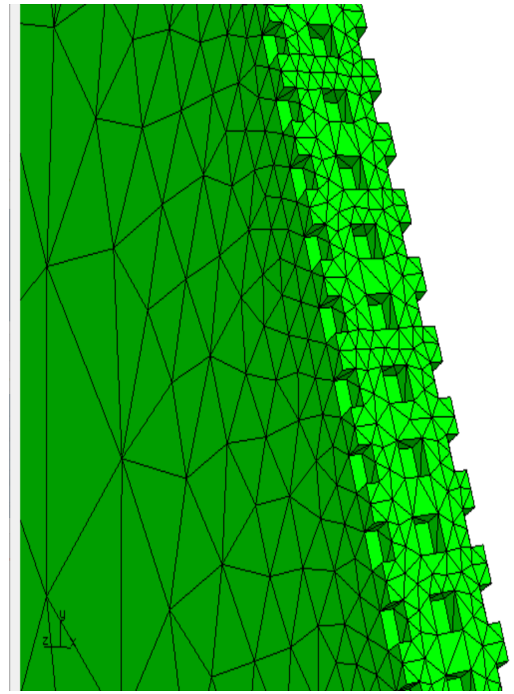


Cutoff frequency of highest waveguide mode ~500GHz

Impedance of rough resistive pipe

- Beam impedance: PEC vs. SIBC





Analytical PEC-model

$$Z_s = \frac{jg}{g+w} Z_0 \tan(k_o h)$$

$$Z_b = \frac{Z_s}{2\pi R} \frac{1}{1 + \frac{j\omega R}{2c} \frac{Z_s}{Z_0}}$$

Summary/Conclusion

estimation for Petra 4 based on NTW model

- short range wake resistive wall part + inductive part
- no synchronous mode in frequency range of interest
- no contribution from higher propagating modes

general approach

- regimes: shallow regime (XFEL) ~ roughness amplitude^2
  steep regime (Petra 4) ~ amplitude
- PEC: theoretical approaches rectangular surface profiles (1d) and shallow regime (2d)
- superposition of roughness and resistivity only for week roughness
- rectangular roughness underestimates effects in steep regime
- several numerical approaches:
  - “measurement” methods can resolve skin effect
  - coupled scattering parameters of thin slices + SIBC
    - (full geometry, verification of simpler models)